GRAIN IN A PLASMA IN THE PRESENCE OF EXTERNAL ELECTRIC FIELD: KINETIC CALCULATION OF EFFECTIVE POTENTIAL AND IONIC DRAG FORCE

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Kinetic calculations of the effective grain potential are presented for the case of weakly-ionized plasma in the external electric field. The drag force associated with the ionic drift in the external field is found. It is shown that the absorption of electrons and ions by the grain can cause the change of the direction of the drag force.

1. Introduction

Theoretical description of various phenomena observed experimentally in dusty plasma (dusty structure formation, excitation of dust-acoustic waves, existence of spatial domains free of dust particles (voids), etc.) requires the knowledge of the explicit form of effective grain potentials. The numerical solution of the appropriate boundary-value problem, as well as numerical simulations of such a potential, does not give us necessary analytical expressions. Thus, one has to use approximate relations obtained within the framework of various models. Obviously, the more consistent description of plasma processes is used, the more fine details concerning the effective grain potentials are known. That is why it is very important to have the kinetic theory of grain screening in a plasma. The additional essential requirement is that the theory should take the electron and ion absorption by grains into account.

The kinetic theory of the effective grain potential with regard for the grain charging by a plasma current was proposed for the first time in Refs. [1–3]. With this purpose, the point sinks were introduced into the kinetic equations. Later on, the point sink model was substantiated on the basis of a consistent microscopic theory of dusty plasma [4, 5].

The proposed kinetic description turns out to be also efficient for theoretical studies of the effective potentials of moving grains [6–9]. The presence of the external force fields can be easily taken into account as well. This gives the possibility to generalize the kinetic description of the grain screening in a weakly ionized plasma exposed to an electric field, which was done in Ref. [10, 11] disregarding the influence of charging currents, to the case of absorbing grains. It is possible to expect that the absorption of plasma particles by grains can influence the asymptotic behavior of the potential (as it is observed in the case of immovable grains in a plasma without external electric field [1–3]) and lead to the qualitative changes of the polarization forces (as in the case of a grain moving in the Maxwellian plasma [6–9]). Obviously, the description of these effects within the kinetic model will give more details about the grain effective potential than that obtained in the fluid approximation [12]. In turn, this will provide more consistent calculations of the ionic drag force which are needed, in particular, for studies of the grain dynamics in the sheath region.

The purpose of the present paper is to give a kinetic description of the effective grain potential in a weakly ionized plasma under the presence of an external electric field taking the electron and ion absorption by grains into account.

The paper is organized in the following order. The basic set of equations is formulated in Section 2. Plasma dynamics is described by the Bhatnagar–Gross–Krook (BGK) kinetic equation [13], in which the point sinks of plasma particles are introduced. Such sinks naturally appear in the kinetic equations in the course of their derivation on the basis of the microscopic treatment [4, 5]. The formal solution of the problem is given in Section 3. In Section 4, we present specific calculations related to a weakly ionized plasma in an external electric field. The qualitative influence of the external electric field, as well as some analytical relations describing such an influence, are presented in Sec. 4.

2. Basic Set of Equations

The consistent derivation of the kinetic equations for plasma particles in the presence of grains gives the following result [4, 5]:

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{e_\alpha}{m_\alpha}\mathbf{E}(\mathbf{r},t)\frac{\partial}{\partial \mathbf{v}}+\frac{1}{m_\alpha}\mathbf{F}_\alpha^{\mathrm{ext}}(\mathbf{r},\mathbf{v},t)\frac{\partial}{\partial \mathbf{v}}\right\}\times$$

$$\times f_\alpha(\mathbf{r},\mathbf{v},t)=-f_\alpha(\mathbf{r},\mathbf{v},t)\int d\mathbf{v}'\int dq'\sigma_\alpha(q',\mathbf{v}-\mathbf{v}')\times$$

$$\times|\mathbf{v}-\mathbf{v}'|f_g(\mathbf{r},\mathbf{v}',q',t)+I_\alpha, \qquad (1)$$

where $f_\alpha(\mathbf{r},\mathbf{v},t)$ and $f_g(\mathbf{r},\mathbf{v},q,t)$ are the one-particle distribution functions normalized by volume for plasma particles and grains, respectively, $\alpha=e,i$ (electron and ion), I_α is the collision term describing the elastic scattering of electrons and ions by a grain and neutrals (if present), $\mathbf{F}_\alpha^{\mathrm{ext}}(\mathbf{r},\mathbf{v},t)$ is the external force field, $\sigma_\alpha(q,v)$ is the charging cross-section which is given for the collisionless plasma by

$$\sigma_\alpha(q,v)=\pi a^2\left(1-\frac{2e_\alpha q}{m_\alpha v^2 a}\right)\theta\left(v^2-\frac{2e_\alpha q}{m_\alpha a}\right),$$

q is the grain charge which is an additional variable and, in the general case, can depend on time. Now let us apply Eq. (1) to the calculations of the effective grain potentials. In the case of a single immovable grain, $f_g(\mathbf{r},\mathbf{v}',q',t)=\delta(\mathbf{r})\delta(\mathbf{v}')\delta(q'-q)$, Eq. (1) reduces to

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{e_\alpha}{m_\alpha}\mathbf{E}(\mathbf{r},t)\frac{\partial}{\partial \mathbf{v}}+\frac{1}{m_\alpha}\mathbf{F}_\alpha^{\mathrm{ext}}(X,t)\frac{\partial}{\partial \mathbf{v}}\right\}\times$$

$$\times f_\alpha(X,t)=I_\alpha-v\sigma_\alpha(q,v)f_\alpha(X,t)\delta(\mathbf{r}), \qquad (2)$$

where X denotes $(\mathbf{r},\mathbf{v})$.

For the sake of simplicity, we do not use, in what follows, the collision term calculated in terms of correlation functions of microscopic fluctuations. Instead, we use a simple version of the model collision integral (simple Bhatnagar–Gross—Krook model) proposed in Ref. [13], namely

$$I_\alpha=-\nu_\alpha\left(f_\alpha(\mathbf{r},\mathbf{v},t)-\Phi_\alpha(\mathbf{v})\int d\mathbf{v}'f_\alpha(\mathbf{r},\mathbf{v}',t)\right). \qquad (3)$$

Here, ν_α is the effective collision frequency, $\Phi_\alpha(\mathbf{v})$ is the distribution function generated in the course of plasma particle collisions.

In view of the fact that the plasma particle absorption considerably suppresses the influence of a nonlinearity [14, 15], we can suggest that the sinks cause a small perturbation of the effective electric field and thus $f_\alpha(X,t)=f_{0\alpha}(\mathbf{v})+\delta f_\alpha(X,t)$. The linearized version of Eq. (2) reads

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_\alpha}\mathbf{F}_\alpha^{\mathrm{ext}}(X,t)\frac{\partial}{\partial \mathbf{v}}\right\}\delta f_\alpha(X,t)-$$

$$-\frac{e_\alpha}{m_\alpha}\nabla\Phi(\mathbf{r},t)\frac{\partial f_{0\alpha}(\mathbf{v})}{\partial \mathbf{v}}=-S_\alpha^{(0)}(\mathbf{v},t)\delta(\mathbf{r})-$$

$$-\nu_\alpha\left\{\delta f_\alpha(X,t)-\Phi_\alpha(\mathbf{v})\int d\mathbf{v}\delta f_\alpha(X,t)\right\}, \qquad (4)$$

where

$$S_\alpha^{(0)}(\mathbf{v},t)=v\sigma_\alpha(q(t),v)f_{0\alpha}(\mathbf{v})$$

is the intensity of the plasma particle sink, and $f_{0\alpha}(\mathbf{v})$ is the unperturbed distribution function. The potential $\Phi(\mathbf{r},t)$ is governed in this case by the Poisson equation

$$\Delta\Phi(\mathbf{r},t)=-4\pi q(t)\delta(\mathbf{r})\ -4\pi\sum_\alpha e_\alpha n_{0\alpha}\int d\mathbf{v}\delta f_\alpha(X,t). \qquad (5)$$

Here, $n_{0\alpha}$ is the unperturbed plasma particles density.

3. Effective Potential of Charged Grains (General Relations)

The solution of Eq. (4) is given by

$$\delta f_\alpha(X,t)=\frac{e_\alpha}{m_\alpha}\int\limits_{-\infty}^{t}dt'\int dX'\,W_\alpha(X,X';t-t')\times$$

$$\times\frac{\partial\Phi(\mathbf{r}',t')}{\partial\mathbf{r}'}\ \frac{\partial f_{0\alpha}(\mathbf{v}')}{\partial\mathbf{v}'}-$$

$$-\int\limits_{-\infty}^{t}dt'\int dX'\,W_\alpha(X,X';t-t')S_\alpha^{(0)}(\mathbf{v}',t')\delta(\mathbf{r}'), \qquad (6)$$

where $W_\alpha(X,X';t-t')$ satisfies the equation

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_\alpha}\mathbf{F}_\alpha^{\rm ext}(X,t)\frac{\partial}{\partial \mathbf{v}}\right\}W_\alpha(X,X';\tau)=$$

$$=-\nu_\alpha\left\{W_\alpha(X,X';\tau)-\Phi_\alpha(\mathbf{v})\int d\mathbf{v}W_\alpha(X,X';\tau)\right\} \quad (7)$$

with the initial condition

$$W_\alpha(X,X';0)=\delta(X-X'). \quad (8)$$

As is seen from Eqs. (7) and (8), the quantity $W_\alpha(X,X';\tau)$ is the phase density of probability of the particle transition from the phase point X' to the point X during the time period $\tau=t-t'$ in the system with no self-consistent particle interaction through the electric field.

Substituting Eq. (6) into Poisson equation (5) and performing the Fourier transformation yield

$$\Phi_{\mathbf{k}\omega}=\frac{4\pi q_\omega}{k^2\varepsilon(\mathbf{k},\omega)}-\frac{4\pi}{k^2\varepsilon(\mathbf{k},\omega)}\times$$

$$\times\sum_\alpha e_\alpha n_{0\alpha}\int d\mathbf{v}\int d\mathbf{v}'W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')S_{\alpha\omega}^{(0)}(\mathbf{v}'), \quad (9)$$

where $\varepsilon(\mathbf{k},\omega)$ is the dielectric response function

$$\varepsilon(\mathbf{k},\omega)=1-i\sum_\alpha\frac{4\pi e_\alpha^2 n_{0\alpha}}{k^2 m_\alpha}\times$$

$$\times\int d\mathbf{v}\int d\mathbf{v}'W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')\mathbf{k}\frac{\partial f_{0\alpha}(\mathbf{v}')}{\partial \mathbf{v}'}, \quad (10)$$

$$W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')=\int d\mathbf{R}e^{-i\mathbf{k}\mathbf{R}}\int_0^\infty d\tau e^{i\omega\tau}W_\alpha(X,X',\tau),$$

$$\mathbf{R}=\mathbf{r}-\mathbf{r}'.$$

In the stationary case where $q(t)=q$, Eq. (9) reduces to

$$\Phi_{\mathbf{k}}=\frac{4\pi q}{k^2\varepsilon(\mathbf{k},0)}-\frac{4\pi}{k^2\varepsilon(\mathbf{k},0)}\sum_\alpha e_\alpha n_{0\alpha}\times$$

$$\times\int d\mathbf{v}\int d\mathbf{v}'W_{\alpha\mathbf{k}}(\mathbf{v},\mathbf{v}')S_\alpha^{(0)}(\mathbf{v}'). \quad (11)$$

Here,

$$W_{\sigma\mathbf{k}}(\mathbf{v},\mathbf{v}')=W_{\sigma\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')\big|_{\omega=0},$$

$$\varepsilon(\mathbf{k},0)=1+\frac{k_D^2}{k^2},$$

$$k_D^2=\sum_\alpha k_\alpha^2,\qquad k_\alpha^2=\frac{4\pi e_\alpha^2 n_{0\alpha}}{T_\alpha}. \quad (12)$$

Thus, we have obtained the general relations describing the effective macroparticle potentials with regard to the electron and ion absorption by grains and the collisions of plasma particles with neutral gas molecules. These relations make it possible to recover all known analytical results for the effective grain potential. For example, in the case of an isotropic plasma with no external field, we have

$$W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')=\frac{i\delta(\mathbf{v}-\mathbf{v}')}{\omega-\mathbf{k}\mathbf{v}+i\nu_\alpha}-$$

$$-\frac{\nu_\alpha\Phi_\alpha(\mathbf{v})}{(\omega-\mathbf{k}\mathbf{v}+i\nu_\alpha)(\omega-\mathbf{k}\mathbf{v}'+i\nu_\alpha)}\times$$

$$\times\left[1-i\nu_\alpha\int d\mathbf{v}\frac{\Phi_\alpha(\mathbf{v})}{\omega-\mathbf{k}\mathbf{v}+i\nu_\alpha}\right]^{-1}, \quad (13)$$

which yields the stationary grain potential given by

$$\Phi(\mathbf{r})=\frac{qe^{-k_D r}}{r}+i\sum_\alpha 4\pi e_\alpha n_{0\alpha}\times$$

$$\times\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{e^{i\mathbf{k}\mathbf{r}}}{k^2+k_D^2}\frac{\int d\mathbf{v}\frac{v\sigma_\alpha(q,v)f_{0\alpha}(v)}{\mathbf{k}\mathbf{v}-i\nu_\alpha}}{1+i\nu_\alpha\int d\mathbf{v}\frac{\Phi_\alpha(\mathbf{v})}{\mathbf{k}\mathbf{v}-i\nu_\alpha}}. \quad (14)$$

In the collisionless limit ($\nu_\alpha\to 0$), this relation is simplified to

$$\Phi(\mathbf{r})=\frac{qe^{-k_D r}}{r}+i\sum_\alpha 4\pi e_\alpha n_{0\alpha}\times$$

$$\times\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{e^{i\mathbf{k}\mathbf{r}}}{k^2+k_D^2}\int\frac{v\sigma_\alpha(q,v)f_{0\alpha}(v)}{\mathbf{k}\mathbf{v}-i0}d\mathbf{v}, \quad (15)$$

and thus we have

$$\Phi(\mathbf{r})=\frac{qe^{-k_D r}}{r}-\frac{\widetilde{Q}}{r}g(k_D r), \quad (16)$$

where

$$g(x)=e^{-x}\mathrm{Ei}(x)-e^{x}\mathrm{Ei}(-x),$$

$$\widetilde{Q}=\frac{2\pi}{k_D}\sum_\alpha e_\alpha n_{0\alpha}\int_0^\infty dv\,v^2\sigma_\alpha(q,v)f_{0\alpha}(v). \quad (17)$$

At $k_D r \gg 1$, Eq. (16) yields the well-known result

$$\Phi(r) \simeq -\frac{2\widetilde{Q}}{k_D r^2},$$

i.e. the effective potential is of the dipole type.

In the strongly collisional limit ($\nu_\alpha \gg k s_\alpha$, $s_\alpha = \sqrt{T_\alpha/m_\alpha}$), Eq. (14) reduces to a superposition of the screened and unscreened potentials

$$\Phi(\mathbf{r}) = (q+\widetilde{S})\frac{e^{-k_D r}}{r} - \frac{\widetilde{S}}{r}\,, \tag{18}$$

which is in agreement with the results obtained in terms of the drift-diffusion approximation [1, 16]. Here,

$$\widetilde{S} = \sum_\alpha \widetilde{S}_\alpha = \sum_\alpha \frac{e_\alpha n_{0\alpha}}{k_D^2 D_\alpha}\int d\mathbf{v}\, S_\alpha^{(0)}(\mathbf{v}),$$

$D_\alpha = T_\alpha/(m_\alpha \nu_\alpha)$ is the diffusion coefficient. When deriving (18), we have put $\Phi_\alpha(\mathbf{v}) = f_{0\alpha}(\mathbf{v})$. Thus, we see that, in the case of the dusty plasma which can be treated as an open system, the stationary screened potential considerably depends on the details of the plasma dynamics, in contrast to the case of a screened potential for the ordinary plasma.

4. Effective grain potential in the weakly ionized plasma exposed to an external electric field

If an external electric field $\mathbf{E}_0 = (0,0,E_0)$ is present, then Eq. (2) generates an equation for the unperturbed distribution function, i.e.

$$\frac{e_\alpha}{m_\alpha}\mathbf{E}_0\frac{\partial f_{0\alpha}(\mathbf{v})}{\partial \mathbf{v}} = -\nu_\alpha\left\{f_{0\alpha}(\mathbf{v}) - \Phi_\alpha(\mathbf{v})\int d\mathbf{v}' f_{0\alpha}(\mathbf{v}')\right\}, \tag{19}$$

and $W_\alpha(X, X';\tau)$ [see Eq. (7)] satisfies the equation

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha}\mathbf{E}_0\frac{\partial}{\partial \mathbf{v}}\right\} W_\alpha(X,X';\tau) =$$

$$= -\nu_\alpha\left\{W_\alpha(X,X';\tau) - \Phi_\alpha(\mathbf{v})\int d\mathbf{v}\, W_\alpha(X,X';\tau)\right\}. \tag{20}$$

Integrating Eq. (19) results in

$$f_{0\alpha}(\mathbf{v}) = \int_{-\infty}^{v_z} dv_z' \beta_\alpha \Phi_\alpha(\mathbf{v}_\perp, v_z')\exp(-\beta_\alpha(v_z - v_z')),$$

$$\beta_\alpha > 0, \tag{21}$$

$$f_{0\alpha}(\mathbf{v}) = -\int_{v_z}^{\infty} dv_z' \beta_\alpha \Phi_\alpha(\mathbf{v}_\perp, v_z')\exp(-\beta_\alpha(v_z - v_z')),$$

$$\beta_\alpha < 0\,, \tag{22}$$

where

$$\beta_\alpha = \frac{1}{v_\alpha}, \quad v_\alpha = \frac{e_\alpha E_0}{m_\alpha \nu_\alpha}\,.$$

The solution of Eq. (20) for $v_\alpha > 0$ is given by

$$W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') = \frac{\beta_\alpha}{\nu_\alpha}\Bigg\{\delta(\mathbf{v}_\perp - \mathbf{v}_\perp')\theta(v_z - v_z')\times$$

$$\times\exp\left[-i\psi_\alpha(\mathbf{v}_\perp, v_z')\right] + \nu_\alpha W_{\alpha\mathbf{k}\omega}(\mathbf{v}')\times$$

$$\times\int_{-\infty}^{v_z} dv_z''\,\Phi_\alpha(\mathbf{v}_\perp, v_z'')\exp\left[-i\psi_\alpha(\mathbf{v}_\perp, v_z'')\right]\Bigg\}\exp[i\psi_\alpha(v_z)]\,, \tag{23}$$

where

$$\psi_\alpha(\mathbf{v}_\perp, v_z) = \frac{\beta_\alpha}{\nu_\alpha}\left\{(\omega - \mathbf{k}_\perp\mathbf{v}_\perp + i\nu_\alpha)v_z - \frac{k_z v_z^2}{2}\right\}, \tag{24}$$

$$W_{\alpha\mathbf{k}\omega}(\mathbf{v}') = \frac{\beta_\alpha}{\nu_\alpha}\int_{-\infty}^{\infty} dv_z \exp\left[i\psi_\alpha(\mathbf{v}_\perp', v_z) - i\psi_\alpha(\mathbf{v}_\perp', v_z')\right]\times$$

$$\times\theta(v_z - v_z')\Bigg\{1 - \beta_\alpha\int d\mathbf{v}\int_{-\infty}^{v_z} dv_z''\,\Phi_\alpha(\mathbf{v}_\perp, v_z'')\times$$

$$\times\exp\left[i\psi_\alpha(\mathbf{v}_\perp, v_z) - i\psi_\alpha(\mathbf{v}_\perp, v_z'')\right]\Bigg\}^{-1}. \tag{25}$$

For $v_\alpha < 0$, we have

$$W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') = -\frac{\beta_\alpha}{\nu_\alpha}\Bigg\{\delta(\mathbf{v}_\perp - \mathbf{v}_\perp')\theta(v_z' - v_z)\times$$

$$\times\exp\left[-i\psi_\alpha(\mathbf{v}_\perp, v_z')\right] + \nu_\alpha W_{\alpha\mathbf{k}\omega}(\mathbf{v}')\times$$

$$\times \int_{v_z}^{\infty} dv_z'' \, \Phi_\alpha(\mathbf{v}_\perp, v_z'') \exp\left[-i\psi_\alpha(\mathbf{v}_\perp, v_z'')\right] \Bigg\} \exp[i\psi_\alpha(v_z)], \tag{26}$$

where

$$W_{\alpha\mathbf{k}\omega}(\mathbf{v}') = -\frac{\beta_\alpha}{\nu_\alpha} \int_{-\infty}^{\infty} dv_z \exp\left[i\psi_\alpha(\mathbf{v}_\perp', v_z) - i\psi_\alpha(\mathbf{v}_\perp', v_z')\right] \times$$

$$\times \theta(v_z' - v_z) \Bigg\{ 1 + \beta_\alpha \int d\mathbf{v} \int_{v_z}^{\infty} dv_z'' \, \Phi_\alpha(\mathbf{v}_\perp, v_z'') \times$$

$$\times \exp\left[i\psi_\alpha(\mathbf{v}_\perp, v_z) - i\psi_\alpha(\mathbf{v}_\perp, v_z'')\right] \Bigg\}^{-1}. \tag{27}$$

Substituting Eqs. (21)–(27) in Eq. (10) for the dielectric response function $\varepsilon(\mathbf{k}, \omega)$ yields

$$\varepsilon(\mathbf{k}, \omega) = 1 + \sum_\alpha \frac{k_\alpha^2}{\varkappa_\alpha^2(\mathbf{k}) G_\alpha(\mathbf{k}, \omega)} \times$$

$$\times \int_0^\infty dy \exp(-y) W\left(\frac{\omega + i\nu_\alpha - k_z v_\alpha y}{\varkappa_\alpha(\mathbf{k}) s_\alpha}\right), \tag{28}$$

where

$$G_\alpha(\mathbf{k}, \omega) = \frac{1}{\omega + i\nu_\alpha} \left[\omega + i\nu_\alpha W\left(\frac{\omega + i\nu_\alpha}{\varkappa_\alpha(\mathbf{k}) s_\alpha}\right)\right],$$

$$\varkappa_\alpha^2(\mathbf{k}) \equiv \varkappa_\alpha^2 = k^2 + \frac{i k_z v_\alpha \nu_\alpha}{s_\alpha^2}, \tag{29}$$

$$W(z) = 1 - z \exp\left(-\frac{z^2}{2}\right) \int_0^z dy \, \exp\left(\frac{y^2}{2}\right) +$$

$$+ i \left(\frac{\pi}{2}\right)^{1/2} z \exp\left(-\frac{z^2}{2}\right). \tag{30}$$

With the accuracy up to the notation, Eqs. (28) and (29) recover the result obtained in Ref. [10].

For $|k_z v_\alpha| \ll \nu_\alpha$, Eq. (28) is simplified to

$$\varepsilon(\mathbf{k}, \omega) = 1 + \sum_\alpha \frac{k_\alpha^2}{\varkappa_\alpha^2 G_\alpha(\mathbf{k}, \omega)} W\left(\frac{\omega + i\nu_\alpha}{\varkappa_\alpha(\mathbf{k}) s_\alpha}\right) =$$

$$= 1 + \sum_\alpha \frac{k_\alpha^2}{\varkappa_\alpha^2} \frac{(\omega + i\nu_\alpha) W\left(\frac{\omega + i\nu_\alpha}{\varkappa_\alpha s_\alpha}\right)}{\omega + i\nu_\alpha W\left(\frac{\omega + i\nu_\alpha}{\varkappa_\alpha s_\alpha}\right)}. \tag{31}$$

In the drift-diffusion approximation ($\omega \ll \nu_\alpha$), we have

$$\varepsilon(\mathbf{k}, \omega) \simeq 1 + i \sum_\alpha \frac{\omega_{p\alpha}^2}{\nu_\alpha(\omega - k_z v_\alpha + i k^2 D_\alpha)}, \tag{32}$$

where $\omega_{p\alpha}^2 = 4\pi e_\alpha^2 n_\alpha / m_\alpha$.

In the stationary case ($\omega = 0$),

$$\varepsilon(\mathbf{k}, 0) = 1 - i \sum_\alpha \frac{\omega_{p\alpha}^2}{\nu_\alpha(k_z v_\alpha - i k^2 D_\alpha)}. \tag{33}$$

For the further calculations, we need to know the explicit forms of the quantities describing the contribution of the absorption processes to the effective potential, namely, to the second terms in Eqs. (9) and (11). We denote these by $\Phi_{\mathbf{k}\omega}^{(s)}$ and $\Phi_{\mathbf{k}}^{(s)}$, i.e.,

$$\Phi_{\mathbf{k}\omega}^{(s)} = \frac{4\pi Q_\omega^{(s)}}{k^2 \varepsilon(\mathbf{k}, \omega)}, \tag{34}$$

where

$$Q_\omega^{(s)} = \sum_\alpha e_\alpha n_{0\alpha} \int d\mathbf{v} \int d\mathbf{v}' \, W_{\alpha\mathbf{k}\omega}(\mathbf{v}, \mathbf{v}') \, S_{\alpha\omega}^{(0)}(\mathbf{v}'). \tag{35}$$

With regard for the explicit form of $W_{\alpha\mathbf{k}\omega}(\mathbf{v}, \mathbf{v}')$, one obtains

$$Q_\omega^{(s)} = -i \sum_\alpha \frac{e_\alpha n_{0\alpha}}{G_\alpha(\mathbf{k}, \omega)} \int d\mathbf{v} \frac{S_{\alpha\omega}^{(0)}(\mathbf{v})}{\omega - \mathbf{k}\mathbf{v} + i\nu_\alpha} \times$$

$$\times \left[1 - W\left(\frac{\omega - \mathbf{k}\mathbf{v} + i\nu_\alpha}{\sqrt{i k_z v_\alpha \nu_\alpha}}\right)\right],$$

$$\mathrm{Re}\left(\sqrt{i k_z v_\alpha \nu_\alpha}\right) > 0. \tag{36}$$

Notice that, in the case of collisional plasma, the problem of the appropriate approximation for the charging cross-section $\sigma_\alpha(q, v)$ is not yet solved (rigorously speaking, by introducing any cross-section, we assume that the incoming particle flux in course of its movement from the infinity to the scattering center is disturbed by this center only). Therefore, the description of the grain charging is usually done in terms of charging currents [17, 18] or in self-consistent kinetic simulations [19, 20]. With the dependence of charging currents on the collision frequency and other plasma parameters being known, it is possible to propose reasonable approximations for the integral terms which include the charging cross-sections. In particular, such an idea was used in Ref. [21] in order to calculate the effective grain potential in the case of a weakly collisional regime of the grain charging. Namely, it was assumed in [21] that

$$\int d\mathbf{v} v\sigma_i(q,v)F(\mathbf{v}) = \sigma_{0i}\int d\mathbf{v} vF(\mathbf{v}), \tag{37}$$

where $F(\mathbf{v})$ is an arbitrary (but regular) function, and σ_{0i} is the effective cross-section taken from the relation for the charging current

$$J_i = n_i\int d\mathbf{v} v\sigma_i(v)f_{0i}(v) = n_i\sigma_{0i}\int d\mathbf{v} vf_{0i}(v). \tag{38}$$

A similar approximation can be used in the case under consideration. Moreover, as follows from the comparison of the consistent calculations of the effective grain potential in the case of the low-collisional regime with the appropriate estimates on the basis of the relations for charging currents, the approximation of the type

$$\int d\mathbf{v} v\sigma_\alpha(q,v)F(\mathbf{v}) = \frac{J_\alpha}{n_\alpha}\int d\mathbf{v} F(\mathbf{v}) \tag{39}$$

is even more efficient. Here, J_α is the charging current generated by particles of the α species.

So doing, we are led to the following relations for the effective charge associated with the grain charging:

$$Q^{(s)}_\omega = i\sum_\alpha e_\alpha J_{\alpha\omega}\int_0^\infty dy\,\frac{1}{G_\alpha(\mathbf{k},\omega)}\frac{\exp(-y)}{\omega - k_z v_\alpha y + i\nu_\alpha}\times$$

$$\times\left[1 - W\left(\frac{\omega - k_z v_\alpha y + i\nu_\alpha}{\varkappa_\alpha(k)s_\alpha}\right)\right]. \tag{40}$$

As follows from Eq. (36) and (40) for $\nu_\alpha \gg ks_\alpha, k_z v_\alpha$, we have

$$Q^{(s)}_\omega = -i\sum_\alpha \frac{e_\alpha}{G_\alpha(\mathbf{k},\omega)}\,\frac{1}{(\omega + i\nu_\alpha)}\times$$

$$\times\left[1 - W\left(\frac{\omega + i\nu_\alpha}{\sqrt{ik_z v_\alpha\nu_\alpha}}\right)\right] J_{\alpha\omega},\ J_{\alpha\omega} = n_\alpha\int d\mathbf{v} S^{(0)}_{\alpha\omega}(\mathbf{v}). \tag{41}$$

In the stationary case ($\omega = 0$), Eq. (41) reduces to

$$Q^{(s)} = -\sum_\alpha \frac{e_\alpha}{\nu_\alpha G_\alpha(\mathbf{k},\omega)}\left[1 - W\left(\sqrt{\frac{i\nu_\alpha}{k_z v_\alpha}}\right)\right] J_\alpha,$$

$$G_\alpha(\mathbf{k},\omega) \simeq W\left(\frac{i\nu_\alpha}{\varkappa_\alpha(\mathbf{k})s_\alpha}\right). \tag{42}$$

If the condition $|\varkappa_\alpha(\mathbf{k})s_\alpha| \ll \nu_\alpha$ is satisfied, then

$$G_\alpha(\mathbf{k},0) \simeq \frac{k^2 s_\alpha^2 + ik_z v_\alpha\nu_\alpha}{\nu_\alpha^2}. \tag{43}$$

Obviously, for $k_z v_\alpha \ll \nu_\alpha$, we have

$$Q^{(s)} \simeq -\sum_\alpha \frac{e_\alpha}{\nu_\alpha G_\alpha(\mathbf{k})}\, J_\alpha. \tag{44}$$

Thus, in the drift-diffusion approximation ($\nu_\alpha \gg ks_\alpha, k_z v_\alpha$), we obtain

$$\Phi_\mathbf{k} = \frac{4\pi q}{k^2\varepsilon(\mathbf{k},0)} - \frac{4\pi}{k^2\varepsilon(\mathbf{k},0)}\sum_\alpha \frac{e_\alpha}{\nu_\alpha G_\alpha(k)} J_\alpha\,. \tag{45}$$

Neglecting the field influence on electrons ($\mathbf{v}_e = 0$), i.e. assuming that electrons can be described by the Boltzmann distribution, we can put

$$\varepsilon(\mathbf{k},0) = 1 + \frac{k_e^2}{k^2} + \frac{k_i^2}{k^2 + ik_z v_i\nu_i/s_i^2},\quad G_e(\mathbf{k}) = \frac{k^2 s_e^2}{\nu_e^2}\,. \tag{46}$$

With the potential $\Phi_\mathbf{k}$ being known, we can calculate the force acting on the grain due to the polarization of the medium

$$\mathbf{F} = -q\frac{\partial\Phi(\mathbf{r})}{\partial\mathbf{r}}\bigg|_{\mathbf{r}=0} = -iq\int\frac{d\mathbf{k}}{(2\pi)^3}\,\mathbf{k}\Phi_\mathbf{k}\,. \tag{47}$$

If the external field is absent, then $\Phi_\mathbf{k}$ depends on the squared $\mathbf{k}$, and the force acting on the particle is equal to zero. The situation crucially changes if particle fluxes occur in the plasma. Taking the explicit form of $\varepsilon(\mathbf{k},0)$ and $G_i(\mathbf{k})$ into account, we find

$$F_z = -\frac{q}{2\pi^2}\int d\mathbf{k}\frac{ik_z}{k^2\varepsilon(\mathbf{k},0)}\left[q - \sum_\alpha \frac{e_\alpha}{\nu_\alpha G_\alpha(\mathbf{k})} J_\alpha\right]. \tag{48}$$

For small velocities v_i, Eq. (48) reduces to

$$F_z = \frac{qv_i}{2\pi^2 D_i}\int d\mathbf{k}\frac{k_z^2}{k^2(k^2 + k_D^2)}\times$$

$$\times\left[\frac{k_D^2\tilde{S}_i}{k^2} + \frac{k_i^2}{k^2 + k_D^2}\left(q - \frac{k_D^2\tilde{S}}{k^2}\right)\right]. \tag{49}$$

In terms of J_α, $\tilde{S} = \sum\tilde{S}_\alpha = \sum e_\alpha J_\alpha/(k_D^2 D_\alpha)$.

After the integration, we have

$$F_z = \frac{qv_i}{6D_i k_D}\left\{k_i^2(q-\tilde{S}) + 2k_D^2\tilde{S}_i\right\} \ . \quad (50)$$

We see that if there is no absorption by the grain ($\tilde{S}_\alpha = 0$), then the drag force is positive, i.e. it acts in the direction of the ion flux velocity. However, if the condition

$$k_i^2\left(q-\tilde{S}\right) + 2k_D^2\tilde{S}_i > 0 \quad (51)$$

is satisfied and $q < 0$, then the force becomes negative, as it was found in kinetic simulations in Ref. [22]. Equations (50)–(51) reproduce the results obtained earlier in Refs. [7, 9].

5. Summary and Conclusions

The constituent kinetic theory of dusty plasma is used to calculate the effective grain potential for the case of a weakly ionized plasma in an external electric field. The grain charging by plasma currents is taken into account with regard for elastic collisions of plasma particles with neutrals within the framework of the BGK collision term.

The drag force associated with the ionic drift in the external field is found. It is shown that the absorption of electrons and ions by the grain can cause the change of the direction of the drag force.

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ПОРОШИНКА В ПЛАЗМІ
У ПРИСУТНОСТІ ЗОВНІШНЬОГО
ЕЛЕКТРИЧНОГО ПОЛЯ: КІНЕТИЧНИЙ РОЗРАХУНОК
ЕФЕКТИВНОГО ПОТЕНЦІАЛУ ТА ІОННОЇ СИЛИ ОПОРУ

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Р е з ю м е

Представлено кінетичні розрахунки ефективного потенціалу порошинки для випадку слабоіонізованої плазми у зовнішньому електричному полі. Знайдено силу опору, яка пов'язана з дрейфом іонів у зовнішньому полі. Показано, що поглинання електронів та іонів порошинкою може привести до зміни напрямку сили опору.