DISPERSION RELATIONS FOR WAVES IN PLASMA AND BOGOLYUBOV IDEAS IN MANY-BODY THEORY

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On the basis of Bogolyubov reduced description method and quasirelativistic quantum electrodynamics, the kinetics of an electromagnetic field in an equilibrium plasma has been constructed. The calculation is carried out in the Hamilton gauge up to the second order of a generalized perturbation theory in interaction. Following Bogolyubov in his theory of superfluidity, the leading contribution to the Hamilton operator of the field is chosen with an additional term depending on the interaction. This allows us to discuss the kinetics of the field in the terms of photons in the plasma and plasmons. On the basis of the obtained material equation supplementing the Maxwell equations, plane electromagnetic waves have been considered. For the case of the Maxwell plasma, the obtained spectra and the attenuation coefficients give results which coincide with those in the standard theory. However, the developed approach allows one to avoid some difficulties of that theory. The method of construction of an effective Hamilton operator of the electromagnetic field in the plasma is proposed. On this basis, we have performed the renormalization of quasiparticle spectra which coincide finally with the spectra of waves in the system.

1. Introduction

The modern quasirelativistic theory of electromagnetic (EM) processes in a plasma medium is usually based on the introduction of the effective direct Coulomb interaction between charged particles (see, for example, [1]). This can be achieved in the Coulomb gauge, in which the scalar potential φ is equal to the Coulomb one φ_c, and the vector potential A_n is a transversal field $\mathrm{div}A = 0$. In fact, the Coulomb interaction is introduced instead of the longitudinal part of the vector potential A_n. This approach leads to some difficulties in the consideration of longitudinal freedom degrees of the system. They can be avoided in the Hamilton gauge, in which the scalar potential $\varphi = 0$ and the electromagnetic field is described by a vector potential with transversal and longitudinal parts. In this gauge, charged particles interact with one another only through the EM field. Then the transversal part of the vector potential describes EM waves, and the longitudinal part describes plasma oscillations. In quantum theory, this leads to photons in the medium and plasmons.

In this paper, we use the Hamilton gauge and build the kinetics of an EM field in the equilibrium plasma medium (bath) which describes the field by the average values of electric field and vector potential. In a certain sense in such an approach, we move in the reverse direction as compared with paper [2] (see also [3]). In that work, a system of particles which interact by Coulomb forces was investigated. Instead of the long-distance parts of the Coulomb interaction, they introduced an additional EM field which is described by a longitudinal vector potential and corresponds to plasma oscillations in the system.

Our investigation is based on the Bogolyubov reduced description method of nonequilibrium states [4] (see the review in [1]) and quasirelativistic quantum electrodynamics.

2. Bogolyubov's Reduced Description Method

An arbitrary state of the system is described by the statistical operator (SO) $\rho(t)$ that satisfies the Liouville equation

$$\dot{\rho}(t) = -\frac{i}{\hbar}[\hat{H}, \rho(t)] \equiv \mathbf{L}\rho(t); \qquad \hat{H} = \hat{H}_0 + \hat{H}_{\mathrm{int}} \tag{1}$$

($\hat{H}$ is the Hamilton operator, $\mathbf{L}$ is the Liouville operator; and $\hat{H}_0$ is the main contribution to $\hat{H}$). According to Bogolyubov [4], *in the presence of a few characteristic times in the system, its evolution passes through the corresponding stages.* At each stage, it is possible to describe the system by a relatively small set of reduced-description parameters $\eta_a(t)$ (RDPs) which are average values calculated with the SO $\rho(t)$

$$\eta_a(t) = \mathrm{Sp}\rho(t)\hat{\eta}_a. \tag{2}$$

We assume that the considered reduced description takes place at $t \geq 0$. According to *the Bogolyubov functional hypothesis*, the solution of Eq. (1) has the structure

$$\rho(t) = \rho(\eta(t)), \tag{3}$$

where the SO $\rho(\eta)$ does not depend on the initial value of the SO $\rho(t)$. The RDPs $\eta_a(t)$ satisfy the equation

$$\dot{\eta}_a(t) = L_a(\eta(t)), \qquad L_a(\eta) \equiv -\mathrm{Sp}\rho(\eta)\mathbf{L}\hat{\eta}_a. \tag{4}$$

The SO $\rho(\eta)$ of the system is a solution of the equations

$$\sum_a \frac{\partial\rho(\eta)}{\partial\eta_a} L_a(\eta) = \mathbf{L}\rho(\eta), \qquad \mathrm{Sp}\rho(\eta)\hat{\eta}_a = \eta_a. \tag{5}$$

Here, the first equation is the Liouville one (1) at the stage of reduced description, and the second is the definition of RDPs.

According to Bogolyubov [4], Eqs. (5) have at least two solutions for $\rho(\eta)$. One has to add a *boundary condition written in the terms of the evolution of the system in the natural direction of time* to these equations. As a boundary condition, we chose the functional hypothesis in the zero approximation in interaction written for an arbitrary initial state ρ_0 of the system. We take into account that, for an arbitrary initial state ρ_0, the system has a statistical operator of the form $\rho(\eta(t))$ only at long times $t \gg \tau_0$ (τ_0 depends on the SO ρ_0). Therefore, in the zero approximation in interaction (i.e. for the evolution with the main contribution $\hat{H}_0$ to the Hamilton operator $\hat{H}$), we have

$$e^{t\mathbf{L}_0}\rho_0 \xrightarrow[t\gg\tau_0]{} e^{t\mathbf{L}_0}\rho^{(0)}(\eta^{(0)}(0)) = \rho^{(0)}(\eta^{(0)}(t)) \tag{6}$$

(the Liouville operator $\mathbf{L}_0$ corresponds to the Hamiltonian $\hat{H}_0$; here, $\eta_a^{(0)}(t)$ depends on ρ_0). The SO $\rho^{(0)}(\eta)$ is the leading contribution to $\rho(\eta)$ and is a solution of Eqs. (5) in the zero approximation in interaction. The parameters $\eta_a^{(0)}(t)$ satisfy the equation

$$\dot{\eta}_a^{(0)}(t) = L_a^{(0)}(\eta^{(0)}(t)), \quad L_a^{(0)}(\eta) \equiv -\mathrm{Sp}\rho^{(0)}(\eta)\mathbf{L}_0\hat{\eta}_a, \tag{7}$$

which follows from (4). In our investigation, the operators $\hat{\eta}_a$ of the reduced-description parameters $\eta_a(t)$ satisfy the Peletminsky–Yatsenko condition [1]

$$\mathbf{L}_0\hat{\eta}_a = -i\sum_a c_{ab}\hat{\eta}_b \tag{8}$$

(c_{ab} are some coefficients). Relation (7) yields obviously

$$\eta_a^{(0)}(t) = \sum_b e_{ab}^{it\mathbf{c}}\,\eta_b^{(0)}(0). \tag{9}$$

It is possible to find the initial condition $\eta_a^{(0)}(0)$ from (6), by multiplying it by $\hat{\eta}_a$ and taking trace Sp of both sides of this formula

$$\eta_a^{(0)}(0) = \mathrm{Sp}\rho_0\hat{\eta}_a. \tag{10}$$

Then condition (6) can be written in the form

$$e^{t\mathbf{L}_0}\rho_0 \xrightarrow[t\gg\tau_0]{} e^{t\mathbf{L}_0}\rho^{(0)}(\mathrm{Sp}\rho_0\hat{\eta}) = \rho^{(0)}(e^{it\mathbf{c}}\mathrm{Sp}\rho_0\hat{\eta}). \tag{11}$$

The SO $\rho^{(0)}(\eta)$ satisfies the equations

$$\sum_{a,b} \frac{\partial\rho^{(0)}(\eta)}{\partial\eta_a} i\, c_{ab}\eta_b = \mathbf{L_0}\rho^{(0)}(\eta), \quad \mathrm{Sp}\rho^{(0)}(\eta)\hat{\eta}_a = \eta_a. \tag{12}$$

Condition (11) is the functional hypothesis taken in the zero approximation of perturbation theory for an arbitrary initial state ρ_0. Following [1], we call it the ergodic relation. The evolution in (11) with the Liouville operator $\mathbf{L_0}$ of free evolution cannot lead the system to an equilibrium; therefore, the SO $\rho^{(0)}$ is a quasiequilibrium one.

From relation (11), it is possible to get the necessary boundary condition for Eqs. (5), by replacing ρ_0 by $\rho(\eta)$ and η by $e^{-it\mathbf{c}}\eta$. We have

$$\lim_{\tau\to+\infty} e^{\tau\mathbf{L}_0}\rho\left(e^{-i\tau\mathbf{c}}\eta\right) = \rho^{(0)}(\eta). \tag{13}$$

Writing this relation in an integral form and taking the Liouville equation (5) into account, we get [1] the integral equation for $\rho(\eta)$:

$$\rho(\eta) = \rho^{(0)}(\eta) + \int_0^{+\infty} d\tau e^{\tau\mathbf{L}_0}\{\mathbf{L}_{\mathrm{int}}\rho(\eta) -$$

$$-\sum_a \frac{\partial\rho(\eta)}{\partial\eta_a}\tilde{L}_a(\eta)\}_{\eta\to e^{-i\tau\mathbf{c}}\eta}, \tag{14}$$

where the function $\tilde{L}_a(\eta)$ is defined by the formula

$$\tilde{L}_a(\eta) = -\mathrm{Sp}\rho(\eta)\mathbf{L}_{\mathrm{int}}\hat{\eta}_a \tag{15}$$

(the Liouville operator $\mathbf{L}_{\mathrm{int}}$ corresponds to $\hat{H}_{\mathrm{int}}$). This equation is solvable within perturbation theory in interaction (first, it was obtained by Peletminsky and Yatsenko (see [1])).

3. Kinetics of an EM Field in the Equilibrium Plasma

We neglect the influence of an EM field on the equilibrium plasma subsystem, by considering only low-energy processes. The Hamilton operator of the system is chosen here in the form

$$\hat{H} = \hat{H}_s + \hat{H}_b + \hat{H}_1 + \hat{H}_2, \tag{16}$$

where

$$\hat{H}_s = \frac{1}{8\pi}\int d^3x\{\hat{E}^2(x) + \hat{B}^2(x)\},$$

$$\hat{H}_1 = -\frac{1}{c}\int dx \hat{A}_n(x)\,\hat{j}_n(x),$$

$$\hat{H}_2 = \frac{1}{2c^2}\int dx \hat{A}^2(x)\,\hat{\chi}(x). \tag{17}$$

In the Hamilton gauge, the Hamilton operator of the bath $\hat{H}_b$ is the operator of kinetic energy of particles. Therefore, the average values of plasma operators can be calculated by the Wick–Bloch–de Dominicis theorem. The operators of the EM field in (17) are given by the formulas $\hat{B}_n = \mathrm{rot}_n\hat{A}$, $\hat{E}_n = -\frac{1}{c}\dot{\hat{A}}_n$, and the vector potential $\hat{A}_n(x)$ has longitudinal and transversal components. The interaction between plasma particles is realized only through the EM field. The operator $\hat{j}_n(x)$ is the operator of current, and the auxiliary operator $\hat{\chi}(x)$ is expressed through the plasma frequency operator

$$\hat{\chi}(x) = \sum_a \frac{e_a^2}{m_a}\hat{\rho}_a(x) = \hat{\Omega}^2(x)/4\pi \tag{18}$$

($\hat{\rho}_a(x)$ is the operator of mass density; m_a and e_a are, respectively, the mass and the charge of particles of the a-th component of the bath ($e_a = z_a e$, $e > 0$)).

We chose the main contribution $\hat{H}_0$ to the Hamilton operator of the system $\hat{H}$ in the form

$$\hat{H}_0 = \hat{H}_\mathrm{f} + \hat{H}_b,$$

$$\hat{H}_\mathrm{f} = \frac{1}{8\pi}\int d^3x\{\hat{E}^2(x) + \hat{B}^2(x) + \Omega^2\hat{A}^2(x)/c^2\}, \tag{19}$$

where $\hat{H}_\mathrm{f}$ can be considered as the Hamilton operator of a free EM field in the bath $\hat{H}_\mathrm{f}$ ($\Omega^2 \equiv \mathrm{Sp_b}w\hat{\Omega}^2$, i.e. Ω is the plasma frequency; w is the equilibrium SO of the bath, $\mathrm{Sp_b}$ is the trace over bath states). This understanding of the Hamiltonian $\hat{H}_\mathrm{f}$ follows from the procedure of canonical quantization. Canonical commutation relations have the form

$$[\hat{A}_n(x), \hat{A}_l(x')] = 0, \quad [\hat{A}_n(x), \hat{\pi}_l(x')] = i\hbar\delta_{nl}\delta(x - x'),$$

$$[\hat{\pi}_n(x), \hat{\pi}_l(x')] = 0, \tag{20}$$

where $\hat{\pi}_n(x) = -\hat{E}_n(x)/4\pi c$ are the generalized momenta corresponding to $\hat{A}_n(x)$ as generalized coordinates of the field. Usual steps lead to a representation of the vector potential operator through the creation and annihilation operators $c^+_{\alpha k}$, $c_{\alpha k}$,

$$\hat{A}_n(x) = c\sum_{\alpha k}\left(\frac{2\pi\hbar}{V\omega_{\alpha k}}\right)^{1/2} e_{\alpha kn}(c_{\alpha k} + c^+_{\alpha,-k})e^{ikx};$$

$$[c_{\alpha k}, c_{\alpha' k'}] = 0, \quad [c_{\alpha k}, c^+_{\alpha' k'}] = \delta_{\alpha,\alpha'}\delta_{kk'} \tag{21}$$

and transform the Hamilton operator $\hat{H}_\mathrm{f}$ in the following way:

$$\hat{H}_\mathrm{f} = \sum_{\alpha k}\hbar\omega_{\alpha k}\left(c^+_{\alpha k}c_{\alpha k} + \frac{1}{2}\right) \tag{22}$$

(V is the volume of the system, and $e_{\alpha kn}$ are vectors of polarization ($\alpha = 1, 2, 3$)). Formulas (21), (22) introduce transversal (with $\alpha = 1, 2$) and longitudinal (with $\alpha = 3$) excitations in the system with the dispersion laws

$$\omega_{\alpha k} = \omega_k \quad (\alpha = 1, 2), \quad \omega_{3k} = \Omega;$$

$$\omega_k \equiv (c^2k^2 + \Omega^2)^{1/2}. \tag{23}$$

These excitations are photons in the medium and plasmons, respectively (see another approach to their introduction in [5]). Therefore, using the operator (19) as the main contribution to the Hamilton operator of the system allows us to discuss all processes in it in the terms of photons in the medium and plasmons. Extracting the last term in the expression for $\hat{H}_\mathrm{f}$ from operator $\hat{H}_2$, we form a new interaction between the introduced excitations and charged particles:

$$\hat{H} = \hat{H}_0 + \hat{H}_\mathrm{int}, \quad \hat{H}_\mathrm{int} = \hat{H}_1 + \hat{H}'_2,$$

$$\hat{H}'_2 = \frac{1}{8\pi c^2}\int d^3x\hat{A}^2(x)\{\hat{\Omega}^2(x) - \Omega^2\};$$

$$\hat{H}_1 \sim \lambda, \quad \hat{H}_2' \sim \lambda^2 \tag{24}$$

(operator $\hat{H}_1$ is defined in (17), λ is a small parameter of the theory). Our introduction of the main Hamiltonian $\hat{H}_{\text{f}}$ corresponds to the role of processes with characteristic frequency Ω in plasma (see, for example, [6]). Perturbation theory with the Hamiltonian $\hat{H}_{\text{int}}$ as interaction gives results which can be obtained in usual perturbation theory with the summation of a class of its contribution. *The same idea was proposed by Bogolyubov in his theory of superfluid Bose gas* [7].

We construct a reduced description of the EM field in the equilibrium plasma by the average values of the electric field and the vector potential $\eta_a(t)$: $E_n(x,t), A_n(x,t)$. At the reduced description, the SO $\rho(\eta) \equiv \rho(E,A)$ satisfies relations of type (5):

$$\text{Sp}\rho(E,A)\hat{E}_n(x) = E_n(x),$$

$$\text{Sp}\rho(E,A)\hat{A}_n(x) = A_n(x). \tag{25}$$

According to (19) and (20), these RDPs satisfy the Peletminsky–Yatsenko condition (8)

$$\frac{i}{\hbar}[\hat{H}_0, \hat{A}_n(x)] = -c\,\hat{E}_n(x),$$

$$\frac{i}{\hbar}[\hat{H}_0, \hat{E}_n(x)] = -c\,\text{rot}_n\text{rot}\hat{A}(x) + \frac{\Omega^2}{c}\hat{A}_n(x), \tag{26}$$

and the kinetics of an EM field in the equilibrium plasma can be built on the basis of integral equation (14). The SO in the zero approximation, $\rho^{(0)}(\eta)$, can be written as

$$\rho^{(0)}(E,A) = w\rho_q(E,A), \quad \text{Sp}\rho_q(E,A)\hat{E}_n(x) = E_n(x),$$

$$\text{Sp}\rho_q(E,A)\hat{A}_n(x) = A_n(x), \tag{27}$$

where w is the equilibrium SO of the bath, and $\rho_q(E,A))$ is the quasiequilibrium SO of the EM field. This expression for $\rho^{(0)}(E,A)$ is in accordance with *the Bogolyubov principle of the spatial weakening of correlations.* In other terms, the ergodic relation (11) in the problem under study is the *Bogolyubov condition of the complete correlation weakening* [4]

$$e^{\tau\mathbf{L}_0}\rho_0 \xrightarrow[\tau\gg\tau_0]{} e^{\tau\mathbf{L}_0} w\rho_q(E,A). \tag{28}$$

This relation takes into account that the free evolution of the system brakes correlations between subsystems of the field and of the plasma (the arbitrary SO ρ_0 describes a nonequilibrium EM field in the equilibrium plasma). Using the operator of free evolution $e^{t\mathbf{L}_0}$ in the above-presented formula is only a tool to express this and has no influence on the domain of applicability of the developed theory. However, *according to Bogolyubov, the boundary condition for the Liouville equation must be written in the terms of the evolution in the natural direction of time.* The necessary boundary condition for the Liouville equation (5) follows from (28) by the substitution $\rho_0 \to \rho(E,A)$. Finally, we note that, for the calculation of the right-hand sides $L_a(\eta)$ of the equations for RDPs up to the second order of perturbation theory, we do not need a specific expression for the SO $\rho_q(E,A)$, and it is enough to use the last formulas from (27).

To derive the evolution equation for the RDP, it is convenient to use the Schrödinger equations of motion for the operators of the EM field [1]

$$\dot{\hat{E}}_n(x) = c\,\text{rot}_n\text{rot}\hat{A}(x) - 4\pi\hat{J}_n(x),$$

$$\dot{\hat{A}}_n(x) = -c\,\hat{E}_n(x), \tag{29}$$

where the EM current $\hat{J}_n(x)$ is defined as

$$\hat{J}_n(x) \equiv \hat{j}_n(x) - \frac{1}{c}\hat{A}_n(x)\hat{\chi}(x). \tag{30}$$

Averaging this relation with the SO $\rho(E,A)$ of the system gives the time equations for the RDPs

$$\dot{E}_n(x,t) = c\,\text{rot}_n\text{rot}A(x,t) - 4\pi J_n(x,E(t),A(t)),$$

$$\dot{A}_n(x,t) = -c\,E_n(x,t), \tag{31}$$

where the average current

$$J_n(x,E,A) = \text{Sp}\rho(E,A)\hat{J}_n(x) \tag{32}$$

is introduced. In fact, relations (31) are average Hamilton equations, because $A_n(x,t)$ are the average generalized coordinates, and $E_n(x,t)$ are proportional to the average generalized momentum of the field. To calculate the average current in (31), we need a solution of the integral equation (14). The simple consideration gives [8]

$$\rho(E,A) = w\rho_q(E,A) + \frac{1}{c\hbar}\int\limits_{-\infty}^{0} d\tau \int dx[\hat{A}_n(x,\tau)\times$$

$$\times\hat{j}_n(x,\tau), w\rho_q(E,A)] + O(\lambda^2), \tag{33}$$

where the operators

$$\hat{j}_n(x,\tau) = e^{-\tau \mathbf{L}_0}\hat{j}_n(x), \quad \hat{A}_n(x,\tau) = e^{-\tau \mathbf{L}_0}\hat{A}_n(x) \qquad (34)$$

are introduced into the interaction picture. The longitudinal and transversal parts

$$\hat{A}^l_{nk} \equiv \hat{A}_{mk}\tilde{k}_m\tilde{k}_n, \quad \hat{A}^t_{nk} \equiv \hat{A}_{mk}\delta^t_{mn};$$

$$\tilde{k}_n \equiv k_n/k, \quad \delta^t_{mn} \equiv \delta_{mn} - \tilde{k}_m\tilde{k}_n. \qquad (35)$$

of the Fourier components of the operator $\hat{A}_n(x,\tau)$ are given by the formulas

$$\hat{A}^l_{nk}(\tau) = \hat{A}^l_{nk}\cos(\Omega\tau) - \frac{c}{\Omega}\hat{E}^l_{nk}\sin(\Omega\tau),$$

$$\hat{A}^t_{nk}(\tau) = \hat{A}^t_{nk}\cos(\omega_k\tau) - \frac{c}{\omega_k}\hat{E}^t_{nk}\sin(\omega_k\tau). \qquad (36)$$

They follow from relations (26) which give

$$\ddot{\hat{A}}^l_k(\tau) + \Omega^2\hat{A}^l_k(\tau) = 0, \; \hat{A}^l_k(0) = \hat{A}^l_k, \; \dot{\hat{A}}^l_k(0) = -c\,\hat{E}^l_k;$$

$$\ddot{\hat{A}}^t_k(\tau) + \omega_k^2\hat{A}^t_k(\tau) = 0, \; \hat{A}^t_k(0) = \hat{A}^t_k, \; \dot{\hat{A}}^t_k(0) = -c\,\hat{E}^t_k. \qquad (37)$$

The results of our calculation of the average EM current $J_n(x,E,A)$ can be expressed through the retarded Green function of currents $G_{nl}(x,t)$. It is defined by the formula

$$G_{nl}(x,t) = -\frac{i}{\hbar}\theta(t)\mathrm{Sp}_b w[\hat{j}_n(x,t),\hat{j}_l(0)] =$$

$$= \int \frac{d^3k\,d\omega}{(2\pi)^4}G_{nl}(k,\omega)e^{i(kx-\omega t)}. \qquad (38)$$

In the considered problem, the plasma is an isotropic medium, and the function $G_{nl}(k,\omega)$ has the structure

$$G_{n\,l}(k,\omega) = G^t(k,\omega)\delta^t_{nl} + G^l(k,\omega)\tilde{k}_n\tilde{k}_l, \qquad (39)$$

where the scalar functions $G^t(k,\omega)$, $G^l(k,\omega)$ are its transversal and longitudinal parts. In these terms, the calculation of the average current with the help of (32)-(36) gives

$$J_n(x,E,A) = \int dx'\sigma_{nl}(x-x')E_l(x')+$$

$$+\int dx'\lambda_{nl}(x-x')A_l(x') + O(\lambda^3), \qquad (40)$$

where the Fourier transformed functions $\sigma_{nl}(x)$ and $\lambda_{nl}(x)$ have the form

$$\sigma_{nl}(k) = -\frac{\mathrm{Im}G^t\,(k,\omega_k)}{\omega_k}\delta^t_{nl} - \frac{\mathrm{Im}G^l\,(k,\Omega)}{\Omega}\tilde{k}_n\tilde{k}_l,$$

$$\lambda_{nl}(k) = -\frac{1}{c}\{\chi\delta_{nl} + \mathrm{Re}G^t(k,\omega_k)\delta^t_{nl}+$$

$$+\mathrm{Re}G^l(k,\Omega)\tilde{k}_n\tilde{k}_l\} \qquad (41)$$

($\chi \equiv \mathrm{Sp}_b w\hat{\chi}(0)$; $\mathrm{Sp}_b w\hat{j}_n(0) = 0$). The functions $\sigma_{nl}(k)$ and $\lambda_{nl}(k)$ are the conductivity and the magnetic susceptibility of the equilibrium plasma and determine its EM properties, taking the spatial dispersion into account. Equations (31) together with the material equation (40) give a closed set of equations for an EM field in the plasma. However, they do not look like the usual Maxwell equations because of the presence of the longitudinal part of the vector potential. This is a consequence of the absence of a time dispersion in the material equation (40). One can see this, by applying the standard procedure (see, for example, [1]) based on the time Fourier transformation, which gives this material equation as the Ohm law

$$J_n(k,\omega) = \sigma_{nl}(k,\omega)E_l(k,\omega),$$

$$\sigma_{nl}(k,\omega) \equiv \sigma_{nl}(k) - i\frac{c}{\omega}\lambda_{nl}(k) \qquad (42)$$

(according to (31), $A_n(k,\omega) = -icE_n(k,\omega)/\omega$).

4. EM Waves in the Equilibrium Plasma

The obtained equations (31) together with the material equation (40) can be divided into equations for longitudinal and transversal fields. In the terms of the Fourier components, the equations for the longitudinal field are

$$\dot{E}^l_k = -4\pi\{\sigma^l(k)E^l_k + \lambda^l(k)A^l_k\},$$

$$\dot{A}^l_k = -cE^l_k \qquad (43)$$

and give the following time equation for E^l_k:

$$\ddot{E}^l_k + 4\pi\sigma^l(k)\dot{E}^l_k - 4\pi c\lambda^l(k)E^l_k = 0$$

(here, $\sigma^l(k)$ and $\lambda^l(k)$ are, respectively, the longitudinal parts of the functions $\sigma_{nl}(k)$ and $\lambda_{nl}(k)$ from (41)). We find a solution of this equation in the form $E^l_k \sim e^{-itz_l}$

that gives $z_l = \pm\,\omega_l(k) - i\gamma_l(k)$, where $\omega_l(k), \gamma_l(k)$ are the dispersion law and the damping rate for longitudinal waves

$$\omega_l(k) = 2\sqrt{-\pi^2\sigma^l(k)^2 - \pi c\lambda^l(k)},$$

$$\gamma_l(k) = 2\pi\sigma^l(k) \qquad (44)$$

for $-\pi\sigma_l(k)^2 - c\lambda_l(k) \geq 0$. In the opposite case $-\pi\sigma_l(k)^2 - c\lambda_l(k) < 0$, longitudinal waves do not exist. The dispersion law $\omega_l(k)$ corrects the frequency Ω of longitudinal oscillations described by the Hamilton operator $\hat{H}_{\rm f}$ (see (23)), which is considered here as the main contribution to the Hamilton operator of the system. In accordance with [2, 3], we deal here with plasmons.

Equations for the transversal field have a structure similar to that of (43),

$$\dot{E}^t_k = k^2cA^t_k - 4\pi\{\sigma^t(k)E^t_k + \lambda^t(k)A^t_k\},$$

$$\dot{A}^t_k = -cE^t_k \qquad (45)$$

and give the following time equation for E^t_k:

$$\ddot{E}^t_k + 4\pi\sigma^t(k)\dot{E}^t_k + \{(kc)^2 - 4\pi c\lambda^t(k)\}E^t_k = 0$$

($\sigma^t(k)$ and $\lambda^t(k)$ are, respectively, the transversal parts of the functions $\sigma_{nl}(k)$ and $\lambda_{nl}(k)$ from (41)). We find a solution of this equation in the form $E^t_k \sim e^{-itz_t}$ that gives $z_t = \pm\,\omega_t(k) - i\gamma_t(k)$, where $\omega_t(k), \gamma_t(k)$ are the dispersion law and the damping rate for transversal waves

$$\omega_t(k) = \sqrt{(kc)^2 - 4\pi^2\sigma^t(k)^2 - 4\pi c\lambda^t(k)}$$

$$\gamma_t(k) = 2\pi\sigma^t(k) \qquad (46)$$

for $(kc)^2 - 4\pi^2\sigma_t(k)^2 - 4\pi c\lambda_t(k) \geq 0$. In the opposite case $(kc)^2 - 4\pi^2\sigma_t(k)^2 - 4\pi c\lambda_t(k) < 0$, transversal waves do not exist. The dispersion law $\omega_t(k)$ corrects the frequency of transversal oscillations ω_k described by the Hamilton operator $\hat{H}_{\rm f}$ (see (23)). In accordance with [2, 9], we deal here with photons in the plasma. This result coincides with our previous one obtained in the Coulomb gauge [8].

5. Electromagnetic Waves in the Maxwell Plasma

The developed theory allows one to avoid some difficulties inherent in the standard theory at the calculation of dispersion laws and damping rates of EM waves in the equilibrium plasma (see the standard approach, e.g., in [10]). Here, an analysis of the obtained results is given for the case of the Maxwell plasma, i.e. for the classical ideal gas of charged particles. The consideration is based on a spectral representation of the Green function (38)

$$G_{nl}(k,\omega) = \frac{1}{2\pi\hbar}\int\limits_{-\infty}^{\infty} d\omega' \frac{I_{nl}(k,\omega')}{\omega - \omega' + i0}(e^{\hbar\,\omega'/T} - 1), \qquad (47)$$

where the correlation function of the currents is defined by the formula

$$I_{nl}(x,t) = {\rm Sp}_b w\hat{j}_l(0)\hat{j}_n(x,t) \qquad (48)$$

(see, for example, [1]). According to the definition of Maxwell plasma, its particles are completely described by the Maxwell distribution. The correlation function of this system $I_{nl}(k,t)$ can be written in the form

$$I_{nl}(k,t) = \sum_a e_a^2 \int d^3v v_n v_l f_a(v) e^{i\mathbf{kv}t},$$

$$f_a(v) \equiv n_a\left(\frac{m_a}{2\pi T}\right)^{3/2} e^{-\frac{m_a v^2}{2T}}. \qquad (49)$$

Taking this expression and formula (47) into account, we obtain the transversal and longitudinal parts of the Green function

$$G^t(k,\omega) = \sum_a \frac{e_a^2}{2k^2T}\int d^3v \frac{\{k^2v^2 - (\mathbf{kv})^2\}(\mathbf{kv})f_a(v)}{\omega - (\mathbf{kv}) + i0},$$

$$G^l(k,\omega) = \sum_a \frac{e_a^2}{k^2T}\int d^3v \frac{(\mathbf{kv})^3 f_a(v)}{\omega - (\mathbf{kv}) + i0}. \qquad (50)$$

After the standard calculation [10], we obtain

$$G^t(k,\omega) = -\sum_a \chi_a F(\frac{\omega}{kv_a\sqrt{2}}) - \chi,$$

$$G^l(k,\omega) = -\sum_a \chi_a(\frac{\omega}{kv_a})^2\{1 + F(\frac{\omega}{kv_a\sqrt{2}})\} - \chi, \qquad (51)$$

where the function

$$F(x) = \frac{x}{\sqrt{\pi}}\int\limits_{-\infty}^{+\infty} dz \frac{e^{-z^2}}{z - x - i0} \qquad (52)$$

was introduced ($v_a^2 \equiv T/m_a$, $\chi_a \equiv e_a^2 n_a/m_a$; some authors [6, 9] prefer to use the function $J_+(x) = -F(x/\sqrt{2})$. Function (52) has the following properties:

$$\mathrm{Re}F(x) = -2x^2 + \frac{4}{3}x^4 + O(x^6) \quad (x \ll 1),$$

$$\mathrm{Re}F(x) = -1 - \frac{1}{2x^2} - \frac{3}{4x^4} + O(x^{-6}) \quad (x \gg 1),$$

$$\mathrm{Im}F(x) = \pi^{1/2} x e^{-x^2} \tag{53}$$

(see, for example, [10]).

According to (51) and (53), the attenuation constants of EM waves in the Maxwell plasma are given by the exact expressions

$$\gamma_l(k) = \left[\frac{\pi}{8}\right]^{1/2} \sum_a \frac{\Omega_a^2}{\Omega} \left[\frac{\Omega}{kv_a}\right]^3 e^{-\frac{1}{2}\left[\frac{\Omega}{kv_a}\right]^2},$$

$$\gamma_t(k) = \left[\frac{\pi}{8}\right]^{1/2} \sum_a \frac{\Omega_a^2}{\omega_k} \left[\frac{\omega_k}{kv_a}\right]^3 e^{-\frac{1}{2}\left[\frac{\omega_k}{kv_a}\right]^2} \tag{54}$$

(here, $\Omega_a^2 \equiv 4\pi\chi_a$). In the developed nonrelativistic theory, $v_a \ll c$, and the inequality $kv_a \ll \omega_k$ is true (see (23)). Therefore, $\gamma_t(k) \approx 0$ in the considered theory. However, in a consequent relativistic theory, the damping of transversal EM waves in a collisionless plasma is absent [10].

Note that, in the standard theory [1,10], the dispersion laws of EM waves in a plasma and their damping rates are calculated from the equations

$$\varepsilon_t(k, z_t(k))\, z_t(k)^2 = c^2k^2 \quad (z_t(k) \equiv \omega_t(k) - i\gamma_t(k)),$$

$$\varepsilon_l(k, z_l(k)) = 0 \quad (z_l(k) \equiv \omega_l(k) - i\gamma_l(k)). \tag{55}$$

These formulas contain the transversal and longitudinal permittivities $\varepsilon_t(k,\omega)$ and $\varepsilon_l(k,\omega)$ of the plasma. In the self-consistent field approximation based on the Vlasov equation for the Maxwell plasma, they have the form [10]

$$\varepsilon_t(k,\omega) = 1 + \sum_a \frac{\Omega_a^2}{\omega^2} F\Big(\frac{\omega}{kv_a\sqrt{2}}\Big),$$

$$\varepsilon_l(k,\omega) = 1 + \sum_a \frac{\Omega_a^2}{(kv_a)^2} \Big\{1 + F\Big(\frac{\omega}{kv_a\sqrt{2}}\Big)\Big\}. \tag{56}$$

Equation (55) can be solved only approximately with respect to $\gamma_l(k)$. The result obtained by this way [10] (the Landau damping rate $\gamma_l^L(k)$) differs from the above exact formula (54) by the multiplier $\gamma_l^L(k) = e^{-3/2}\gamma_l(k)$. However, the standard expression for $\gamma_l^L(k)$ is valid for $kv_a \ll \Omega$. In this situation, the damping rate $\gamma_l^L(k)$ is exponentially small, and this multiplier is not important. Note also that there is a problem with the usual solution of Eqs. (55) and (56) related to the substitution of complex values $z_t(k), z_i(k)$ with negative imaginary part in the function $F(x)$ from (52). The simple substitution brakes the rule of pole passing in the function $F(x)$. In our approach, this problem does not exist, because we substitute only real values in this function (see formulas (41), (44), (46), (51)). A different approach to the Landau damping theory is discussed in [11].

For small wave vectors, formulas (44), (46), and (53) give the dispersion laws of EM waves in the system

$$\omega_l(k)^2 = \Omega^2 + 3\sum_a \Omega_a^2 \frac{(kv_a)^2}{\Omega^2} + O(k^4) \quad (kv_a \ll \Omega),$$

$$\omega_t(k)^2 = \omega_k^2 + \sum_a \Omega_a^2 \frac{(kv_a)^2}{\omega_k^2} + O(k^4) \quad (kv_a \ll \omega_k) \tag{57}$$

which coincide with the known results [9], [10]. In the considered nonrelativistic case, $v_a \ll c$, and the inequality $kv_a \ll \omega_k$ is true always (see (23)).

In the case of large wave vectors, longitudinal waves do not exist because, according to (53), the expression under root in (44) becomes negative. For transversal waves, a simplification of the dispersion relation (46) on the basis of formulas (53) is impossible due to the inequality $\omega_k > kv_a$.

6. Effective Hamilton Operator of an EM Field in the Equilibrium Plasma

Here, we construct the effective Hamiltonian of an EM field in the equilibrium plasma $\hat{H}_{\mathrm{ef}}$ on the basis of the definition proposed in [12]. The SO of the field subsystem is given by the formula

$$\rho_{\mathrm{f}}(E, A) = \tilde{\mathrm{S}}\mathrm{p}_b \rho(E, A), \tag{58}$$

where $\tilde{\mathrm{S}}\mathrm{p}_b$ is the trace over states of the bath, which gives an operator in the space of EM field states (see details in [12]). Formula (33) for the SO of the system yields

$$\rho_{\mathrm{f}}(E, A) = \rho_q(E, A) + O(\lambda^2). \tag{59}$$

In accordance with (1), (19), and (24), the evolution equation for the SO $\rho_{\rm f}(E(t),A(t))$ can be obtained from the identity

$$\frac{\partial\rho_{\rm f}(E(t),A(t))}{\partial t}=-\frac{i}{\hbar}[\hat{H}_{\rm f},\rho_{\rm f}(E(t),A(t))]-$$

$$-\frac{i}{\hbar}\tilde{\rm S}{\rm p}_b[\hat{H}_{\rm int},\rho(E(t),A(t))]. \quad (60)$$

According to [12], the last term of this equation contains values of the type $\hat{A}\rho_{\rm f}+\rho_{\rm f}\hat{A}^+$ ($\hat{A}$ is some operator), which give contributions to the effective Hamilton operator $\hat{H}_{\rm ef}$ of the form $i\hbar(\hat{A}-\hat{A}^+)/2$. The above-obtained results lead to the second-order contributions to $\hat{H}_{\rm ef}$.

Note that the relations

$$\tilde{\rm S}{\rm p}_b[\hat{H}_2,\rho(E,A)]=O(\lambda^3),\quad \tilde{\rm S}{\rm p}_b[\hat{H}_1,\rho(E,A)]=$$

$$=-\frac{i}{\hbar}\int\limits_{-\infty}^{0}d\tau\tilde{\rm S}{\rm p}_b\left[\hat{H}_1,[\hat{H}_1(\tau),w\rho_q(E,A)]\right]+O(\lambda^3) \quad (61)$$

are true. Using the Jacobi identity, it is easy to see that the last term below gives the commutator of an operator with the SO of the field $\rho_{\rm f}(E,A)$:

$$\tilde{\rm S}{\rm p}_b\left[\hat{H}_1,[\hat{H}_1(\tau),w\rho_q]\right]=\frac{1}{2}\tilde{\rm S}{\rm p}_b\left[\hat{H}_1,[\hat{H}_1(\tau),w\rho_q]\right]+$$

$$+\frac{1}{2}\tilde{\rm S}{\rm p}_b\left[\hat{H}_1(\tau),[\hat{H}_1,w\rho_q]\right]+\frac{1}{2}\tilde{\rm S}{\rm p}_b\left[w\rho_q,[\hat{H}_1(\tau),\hat{H}_1]\right]. \quad (62)$$

This identity allows us to rewrite Eq. (60) in the form

$$\frac{\partial\rho_{\rm f}(E(t),A(t))}{\partial t}=-\frac{i}{\hbar}[\hat{H}_{\rm ef},\rho_{\rm f}(E(t),A(t))]+$$

$$+\mathbf{L}_{\rm dis}\rho_{\rm f}(E(t),A(t))+O(\lambda^3), \quad (63)$$

where the effective Hamilton operator of the EM field and the dissipative Liouville operator are introduced by formulas

$$\hat{H}_{\rm ef}=\hat{H}_{\rm f}-\frac{1}{2c}\int dx\{J_n^{(2)}(x,\hat{A},\hat{E})\hat{A}_n(x)+$$

$$+\hat{A}_n(x)J_n^{(2)}(x,\hat{A},\hat{E})\},$$

$$\mathbf{L}_{\rm dis}\rho_{\rm f}=-\frac{i}{2\hbar}\int\limits_0^\infty d\tau\left(\tilde{\rm S}{\rm p}_b\left[\hat{H}_1,[\hat{H}_1(\tau),\rho_{\rm f}w]\right]+\right.$$

$$\left.+\tilde{\rm S}{\rm p}_b\left[\hat{H}_1(\tau),[\hat{H}_1,\rho_{\rm f}w]\right]\right) \quad (64)$$

($J_n^{(2)}(x,E,A)$ is the second-order contribution to the function $J_n(x,E,A)$ given by expression (40)).

The effective Hamilton operator of an EM field in the medium $\hat{H}_{\rm ef}$ is a quadratic form in the operators of generalized coordinates $\hat{A}_n(x)$ and momenta $\hat{\pi}_n(x)$ of the field. By using *the Bogolyubov transformation* [7], it can be written in the form

$$\hat{H}_{\rm ef}=\sum_{\alpha,k}\hbar\omega_\alpha(k)\left(\tilde{c}^+_{\alpha k}\tilde{c}_{\alpha k}+\frac{1}{2}\right);$$

$$\omega_\alpha(k)=\omega_t(k)\quad(\alpha=1,2),\quad \omega_3(k)=\omega_l(k). \quad (65)$$

Here, the spectra $\omega_t(k)$, $\omega_l(k)$ coincide with the dispersion laws of transversal and longitudinal EM waves in the plasma given by the formulas (44), (46), $\tilde{c}^+_{\alpha k}$, $\tilde{c}_{\alpha k}$ are new Bose operators of creation and annihilation with the usual commutation relations (21). The usual expression for the operators of vector potential $\hat{A}_n(x)$ in terms of operators (21) is also valid. The expression for new Bose operators in terms of old ones (21) can be given too. So, in this section, we have performed a renormalization of quasiparticle spectra introduced by our choice of the leading contribution $\hat{H}_{\rm f}$ (19) to the Hamilton operator of the EM field. Moreover, in the third approximation of perturbation theory, the effective Hamiltonian will contain terms which describe the interaction between quasiparticles.

7. Conclusion

Bogolyubov's ideas in many-body theory allow one to build the kinetics of an electromagnetic field in an equilibrium plasma in the terms of photons in the medium and plasmons. The proposed approach allows one to avoid some difficulties related to the calculation of the dispersion laws and the damping rates of EM waves in the plasma. The effective Hamilton operator obtained in this paper for quasiparticles gives renormalized quasiparticle spectra which coincide with the spectra of EM waves. The consideration is based on the following Bogolyubov's ideas in many-body theory: the method of reduced description of nonequilibrium states (stages of evolution of a nonequilibrium system, the functional hypothesis, boundary condition to the Liouville equation), idea of the leading contribution to the Hamilton operator of a system, and the principle of spatial correlation weakening.

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ДИСПЕРСІЙНІ СПІВВІДНОШЕННЯ ДЛЯ ХВИЛЬ У ПЛАЗМІ ТА ІДЕЇ БОГОЛЮБОВА У ТЕОРІЇ БАГАТЬОХ ТІЛ

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Р е з ю м е

На основі методу скороченого опису Боголюбова і квазірелятивістської квантової електродинаміки побудовано кинетику електромагнітного поля в рівноважній плазмі. Обчислення проводиться в калібровці Гамільтона до другого порядку узагальненої теорії збурень за взаємодією. Наслідуючи Боголюбова в його теорії надплинності, основний внесок в оператор Гамільтона поля вибирається з додатком, який залежить від взаємодії. Це дозволяє обговорювати кінетику електромагнітного поля в термінах фотонів у плазмі та плазмонів. На основі отриманого матеріального рівняння додаткового до рівнянь Максвелла розглянуто плоскі електромагнітні хвилі. Для випадку максвеллівської плазми отримані закони дисперсії та декременти згасання хвиль дають результати, які збігаються зі стандартною теорією. Однак розвинутий підхід дозволяє позбутися деяких труднощів цієї теорії. Запропоновано метод побудови ефективного оператора Гамільтона електромагнітного поля у плазмі. На цій основі виконано перенормування спектрів квазічастинок, які у підсумку збігаються зі спектрами хвиль у системі.