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On massive photons inside a superconductor as follows from London and Ginzburg–Landau theory

R. de Bruyn Ouboter

Kamerlingh Onnes Laboratory, Leiden Institute of Physics, Leiden University
P.O. 9506, 2300 RA Leiden, The Netherlands
E-mail: r.de.bruijn.ouboter@umail.leidenuniv.nl

A.N. Omelyanchouk

B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine
47 Nauky Ave., Kharkiv 61103, Ukraine

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A phenomenological derivation in the frame of London`s and Ginzburg–Landau theories is given that photons behave inside a superconductor as if they have mass.

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The research presented in this paper is triggered by the reading the timelines in Wilczek's recent book [1] on "a beautiful question — finding nature's deep design" on p. 335: "1963 Philip Anderson [2] suggests the importance for particle physics of work on equations for massive photons that arose in work by the brothers Fritz and Heinz London [3] in 1935 and Lev Landau and Vitaly Ginzburg [4] in 1950".

A phenomenological derivation is given that photons behave inside a superconductor as if they have mass by comparison of the original first equation of the London`s and the equations for the electromagnetic field with the time-dependent relativistic Schrödinger equation. The photons move through a medium, the Ginzburg–Landau free energy density, inside the superconductor in which they acquire mass. The Compton wave length of the massive photons is equal to 2π times the London penetration depth and the mass of the photon is equal to $m = \hbar/c\lambda_L$.

The essential feature of superconductivity according F. London [3] is a condensation of a macroscopic number particles (bound electron pairs, with mass $2m_e$ and charge $2e$, first described by Cooper [5]) in the same single quasiparticle quantum state and obtained a fundamental relation for the generalized dynamical momentum $\mathbf{p}_s$ of the superconducting pairs,

$$\mathbf{p}_s = (2m_e)\mathbf{v}_s + (2e)\mathbf{A} = \hbar\nabla\phi, \tag{1}$$

in which $\mathbf{v}_s$ is the superfluid velocity, $\mathbf{A}$ is the vector potential, and ϕ is the phase of the macroscopic wave function. Cooper pairs behave like bosons. The superfluid current density $\mathbf{I}_s = \frac{n_s}{2}(2e)\mathbf{v}_s$ in which n_s is the superfluid density. Taking the curl of Eq. (1) the well known first relation of the London's from 1935 is obtained for in simply connected isolated superconductor:

$$\mathbf{A} = -\frac{2m_e}{2e}\mathbf{v} \tag{2}$$

or

$$\mu_0\mathbf{I}_s = -\frac{1}{\lambda_L^2}\mathbf{A} \tag{3}$$

in which λ_L is the London penetration depth

$$\lambda_L^2 = \frac{m_e}{\mu_0 n_s e^2}, \tag{4}$$

or

$$\mu_0\lambda_L^2 = \Lambda = \frac{m_e}{n_s e^2}. \tag{5}$$

Equation (3) is valid as long as $|\mathbf{p}_s|\zeta_{GL} = \hbar|\nabla\phi|\zeta_{GL} \ll \hbar$ in which ξ_{GL} is the Ginzburg–Landau (GL) coherence length. When the wave length $1/|\nabla\phi|$ is smaller or compatible to the coherence length the superconductivity disappears. According to Ginzburg–Landau [4] is their macroscopic theory reliable based on Eq. (3).

We restrict ourselves mainly to the case $T = 0$, and neglect normal currents. We like to remark that at $T = 0$ the London penetration depth

$$\lambda_L^2(0) = \frac{m_e}{\mu_0 n_e e^2} = \left(\frac{c}{\omega_p}\right)^2$$

in which the plasma frequency ω_p is defined by $\omega_p^2 \equiv n_e e^2/m_e\varepsilon_0$ (at $T = 0$ n_s goes to the electron density n_e).

Combining the first London equation (3) with the Maxwell equation

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{I}_s,$$

the London's obtain [3]

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = \frac{\mathbf{A}}{\lambda_L^2}, \tag{6}$$

$$\nabla^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} = \frac{\mathbf{B}}{\lambda_L^2}, \tag{7}$$

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \frac{\mathbf{E}}{\lambda_L^2}. \tag{8}$$

In a static situation Eq. (7) leads to $\nabla^2\mathbf{B} = \mathbf{B}/\lambda_L^2$ which explains the Meissner effect.

The differential Eqs. (6)–(8) contain, respectively, only **A, B** and **E** and its spatial and time differentials of second order separately and the constants λ_L^2 and c^2.

However, the brilliant observation of Anderson [2] (1963) was that the Eqs. (6), (7) and (8) of the work of the London`s is also applicable to *the photon field inside the superconductor* with *massive* photons presented by the terms $\mathbf{A}/\lambda_L^2$, $\mathbf{B}/\lambda_L^2$ and $\mathbf{E}/\lambda_L^2$.

In this phenomenological description is for comparison written down the relativistic Schrödinger wave equation [6] for a free particle which shows the same structure and describes Bose particles, hence also photons with mass m and wave function ψ:

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{m^2c^2}{\hbar^2}\psi \equiv \frac{\psi}{\lambda_c^2} \tag{9}$$

in which $\lambda_c \equiv \hbar/mc$ is equals to the Compton wave length h/mc divided by 2π for a photon with mass m.

The Eqs. (6)–(9) are relativistic equations and have the same form, are mathematically identical and describe exactly the same phenomenon [1]. The squared lengths on the right hand sides of the Eqs. (6)–(8) λ_L^2, and in Eq. (9), $\lambda_c^2 \equiv \hbar^2/m^2c^2$ should be equal to each other

$$\lambda_L^2 = \lambda_c^2 = \hbar^2/m^2c^2,$$

or

$$\frac{mc^2}{\hbar} = \frac{c}{\lambda_L}, \tag{10}$$

or

$$m = \hbar/c\lambda_L. \tag{11}$$

Mass (11) is the mass of the photon inside the superconductor. This implies when penetrating the superconductor from outside into the bulk, superconductivity and photon mass arises in the same way. At $T = 0$, n_s goes to the electron density n_e so that the plasma frequency is equals to

$$\omega_p = \frac{c}{\lambda_L(0)} = \frac{mc^2}{\hbar}. \tag{12}$$

If we use $\lambda_L(0) = 500$ Å $= 5\cdot10^{-8}$ m we find

$$m = \hbar/c\lambda_L = \frac{10^{-34}}{3\cdot10^8\cdot5\cdot10^{-8}} = 7\cdot10^{-36}\ \text{kg}.$$

For comparison $m_e = 9{,}1\cdot10^{-31}$ kg and we find for $\omega_p = c/\lambda_L(0) = 6\cdot10^{15}$ Hz. For comparison the gap frequency $\omega_{\text{gap}} \approx 2\Delta(0)/\hbar \approx 10^{11}$–$10^{12}$ Hz.

The very small photon mass m implies a very large zero-point motion inside the superconductor.

These macroscopic phenomenological considerations are very academic since free space between the atoms in the superconductor is very limited for investigation. A photon in empty spaces moves at the speed of light, $v = c$, and has two transverse field components (**E** and **B**) perpendicular to each other and perpendicular to the direction of wave propagation. From the Eqs. (7) and (8) follows that a photon in motion inside a superconductor acquires also a third degree of freedom forward and back in the direction of motion (left and right, up and down and forward and back oscillations) leading to a particle with mass.

A massless photon moves at the speed of light in vacuum and moves into a bulk superconductor through a medium, a field inside the superconductor (the Ginzburg–Landau free energy density) of which the symmetry is broken and the photon acquires mass. We start by investigating the penetration depth in the Ginzburg–Landau theory (1950) [4]. We write down a modern version of the second Ginzburg–Landau equation, an equation also present in the theory of F. and H. London (1935) [3]:

$$\mathbf{I}_s = \frac{i(2e)\hbar}{2(2m_e)}(\psi_s^*\nabla\psi_s - \psi_s\nabla\psi_s^*) - \frac{(2e)^2}{2m_e}|\psi_s|^2\mathbf{A} = \\ = (2e)|\psi_s|^2\frac{\hbar}{2m_e}\left[\hbar\nabla\phi - \frac{2e}{\hbar}\mathbf{A}\right] = (2e)|\psi_s|^2\mathbf{v}_s \tag{13}$$

from which follows Eq. (1). Taking the curl of Eqs. (1) and (2) is obtained. This Eq. (13) follows in form from both the relativistic [8] and the nonrelativistic [6,8] expression, in which $|\psi_s|^2 = n_s/2$ is the pair density in modern language. In the GL theory the difference in the free energy density $F\left[|\psi_s|^2, T\right]$ can be written as

$$\Delta F = F_s(|\psi_s|^2, T) - F_n(T) = \alpha(T)|\psi_s|^2 + \frac{1}{2}\beta|\psi_s|^4 + \ldots \tag{14}$$

in which β is positive at all temperatures and $\alpha(T) < 0$ for $T < T_c$. The equilibrium superconductive state is $\partial\Delta F/\partial|\psi_s|^2 = 0$, hence $\partial\Delta F/\partial|\psi_s|^2 = \alpha + \beta|\psi_s|^2 = 0$, or $|\psi_s|^2_{\text{equil}} = -\alpha/\beta = |\alpha|/\beta$ and $\Delta F_{\text{equil}} = -\frac{1}{2}|\alpha||\psi_s|^2_{\text{equil}}$. We find the GL penetration depth λ_{GL} by

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{I}_s = -\mu_0\frac{n_s}{2}2e\mathbf{v}_s = \\ = \mu_0\frac{|\alpha|}{\beta}\frac{(2e)^2}{2m_e}\mathbf{A} = \frac{\mathbf{A}}{\lambda_{GL}^2} \tag{15}$$

in which

$$\lambda_{GL}^2 = \frac{2m_e}{(2e)^2\mu_0|\psi_s|^2}.$$

If $|\psi_s|^2_{\text{equil}} = n_s/2$ this equation is identical with Eq. (4), the London penetration depth, and

$$\frac{m^2c^4}{\hbar^2} = \frac{c^2}{\lambda_L^2} = \frac{(2e)^2c^2}{2m_e}\mu_0|\psi_s|^2. \tag{16}$$

In the spirit of the "Higgs mechanism" and the principle of broken symmetry which starts when the temperature is lowered from $T > T_c$ to $T < T_c$ we plot the relevant Ginzburg–Landau free energy density F versus $|\psi_s|^2$, which is often called the "Higgs field", for both $T > T_c$ and $T < T_c$ when the symmetry of the "Higgs field" is broken (Fig. 1).

The lowest potential energy density for $T < T_c$ corresponds to a finite displacement of a non-zero value of $|\psi_s|^2$. There is a small bump in the bottom of the curve, the presence of this bump forces the symmetry to break as the "field" cools from $T > T_c$ to $T < T_c$ and a valley appears

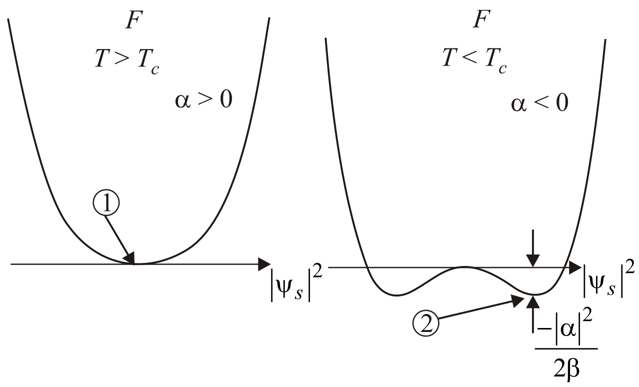

Fig. 1. The relevant GL free energy density F versus $|\psi_s|^2$ for $T > T_c$ and $T < T_c$.

in the curve. The lowest point in the curve corresponds to a non-zero value of the scalar "field". The photon in the superconductor does interact with the Ginzburg–Landau or "Higgs field", it interacts with this field, gains energy, slows down, the "field" dragged on the photon and the interaction with the particle photon and the field is manifested as a resistance of the photon particle acceleration. When the photon particle moves at constant velocity it is not affected by the "field" and $\partial\Delta F/\partial|\psi_s|^2 = 0$. The Ginzburg–Landau "field" is a scalar field with no directions. During the cooling of the superconductor from $T > T_c$ to $T < T_c$ each photon inside the superconductor acquires an energy mc^2.

We now consider the solution of the Eqs. (6)–(9) inside the bulk superconductor

$$\omega = \frac{E_{\mathbf{k}}}{\hbar} = \sqrt{\frac{m^2c^4}{\hbar^2} + \mathbf{k}^2c^2} = \frac{mc^2}{\hbar}\sqrt{1 + \lambda_L^2\mathbf{k}^2} \tag{17}$$

in which $E_{\mathbf{k}} = \hbar\omega = \hbar 2\pi\nu$ is the relativistic energy ($E_p^2 = m^2c^4 + p^2c^2$) and $p = \hbar k = \hbar 2\pi/\lambda$ is the relativistic momentum. We have plotted ω versus k for the photon mass m inside the bulk superconductor and for a photon in empty space $c = \omega/k$ (Fig. 2) for two cases: a type I and a type II superconductors. Inside the bulk superconductor the massive photon particle has a group velocity $v_g = \partial\omega/\partial k$ of the associated wave.

We remarked already that if $|\nabla\phi| = 2\pi/\lambda < 1/\xi_{GL}$ superconductivity exists [10], and if $|\nabla\phi| = 2\pi/\lambda > 1/\xi_{GL}$ superconductivity ceases to exist [10]. For k values smaller than $1/\xi_{GL}$ the wave-packet of the photon behaves massive, contrary to the opposite case for k values larger than $1/\xi_{GL}$. It should be possible in principle to observe this transition from the superconductive state $|\nabla\phi| < 1/\xi_{GL}$ to the state of anomalous conductivity $|\nabla\phi| > 1/\xi_{GL}$ with a quantum foam-like structure (hence from the m to the $\omega/k = c$ state of photons) in a superconducting layer in an external radiation field $|\nabla\phi| = 2\pi/\lambda$ of which the frequency increases.

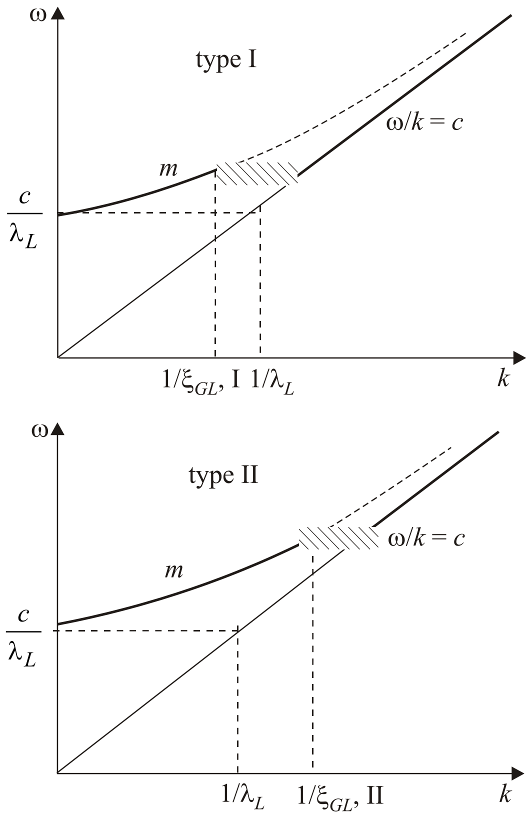

Fig. 2. ω versus k for the photon mass m inside the bulk superconductor and for a photon in empty space $c = \omega/k$ for two cases: a type I and a type II superconductors.

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