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1D Anderson model revisited: Band center anomaly for correlated disorder

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We study the band-center anomaly in the one-dimensional Anderson model with the disorder characterized by short-range positive correlations. Using the Hamiltonian map approach, we obtain analytical expressions for the localization length and the invariant measure of the phase variable. The analytical expressions are complemented by numerical data.

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72.15.Rn Localization effects (Anderson or weak localization);
05.40.–a Fluctuation phenomena, random processes, noise, and Brownian motion;
42.25.Dd Wave propagation in random media.

Keywords: Anderson localization, band-center anomaly.

1. Introduction

More than half a century has passed since the celebrated Anderson model (A-model) was introduced to prove the absence of diffusion of electrons in infinite lattices with disordered potentials [1]. To date, this model has been extensively studied due to its various non-trivial physical properties. First, it was proved that all electronic eigenstates in the 1D (one-dimensional) model are exponentially localized in infinite samples [2]. The characteristic length scale of such a spatial localization is called the localization length L_{loc}. Second, it was shown [3] that the knowledge of this length allows one to predict all transport properties for finite samples of size N, for any value of the key parameter which is the ratio L_{loc}/N (see, also, Ref. 4 and references therein). This result is known as the *single-parameter scaling* (SPS) and for a long time it was believed to be true for any eigenstate energy E within the allowed band $|E| < 2$, with the exception of energies close to the band edges, $E = \pm 2$ (for the normalized energy).

The first analytical expression for L_{loc} as a function of the energy E was given by Thouless [5] under the conditions of weak and uncorrelated disorder. However, shortly afterwards it was numerically found [6] that the Thouless expression for the discrete A-model is not correct for energies close to the band center, $E \simeq 0$. Although the discrepancy was quite small, a theoretical explanation was not found until the analytical studies published in [7,8]. It was established that the standard perturbation theory fails at the band center because it is based on the assumption of a non-degenerate spectrum of the Perron–Frobenius operator. An approximate expression for the localization length was then derived in Ref. 7 with the use of the degenerate perturbation theory. Later, with the use of a different approach, the band center anomaly was analytically resolved in Ref. 8.

It was understood that the mechanism of the band center anomaly can be ascribed to the resonance that emerges for $E = 0$ in the absence of disorder. The physical origin of this effect is that the shift μ of the phase of the wave function turns out to be $\pi/2$ for every lattice step. Therefore, even for weak disorder, the phases remain strongly correlated. Later on, the anomalous behavior of the localization length in a neighborhood of the band center was studied in [9]. Finally, a complete solution for the localization length in a whole vicinity of the band center was obtained in Ref. 10. This solution was derived with the Hamiltonian map approach, which is based on the mathematical correspondence between the 1D Anderson model and a classical parametric oscillator [11,12]. Further theoretical contributions are given in Ref. 8, were it was shown that similar resonances occur for $E = \pm 1$, as well as for all other resonant values of the energy, $E = 2\cos(\pi r)$, with r a rational number. However, only the resonances for $r = 1/2; 1; 2$ affect directly the localization length. As for other anomalies, one can expect that they influence the higher-order terms in the expansion of the localization length with respect to disorder. Expressions for the localization length at the band edges ($r = 1, 2$) can be found in Refs. 9 (see also [4]).

A second phase of research on the band-center anomaly began when the anomalies in the 1D Anderson model were related to the question of the validity of the single parameter scaling. The SPS theory, originally proposed in Ref. 13, provides a fundamental theoretical tool for the understanding of Anderson localization. For this reason an intense (and still ongoing) debate was sparked when it was shown that the SPS theory fails close to the band edges [14] and to the band center [15]. Since then, the nature of the anomalies in the 1D Anderson model has been analyzed with an eye to the foundations of the SPS theory [16].

More recently, the analysis of the band center anomaly has been extended to the case of the 1D Anderson model with *correlated* disorder [17,18]. In particular, in Ref. 18 it was shown how correlations of the disorder can either enhance or suppress the anomaly at the band center.

In this paper we apply the general results of Ref. 18 to the study of an interesting case which was numerically investigated in Ref. 19. The authors considered the 1D Anderson model with a specific kind of short-range correlated disorder; their numerical analysis revealed that the localization length at the band center could behave in unexpected ways for increasing values of the correlation length of the disorder. In particular, for weak disorder it was found that the localization length increases with the third power of the correlation length. Although the authors of Ref. 19 could not compute the localization length for very large values of the correlation length in the weak disorder regime, they did it for a relatively strong disorder and found that increasing the correlation length leads to more and more extended states at the band center.

Our analysis makes possible to understand the numerical results reported in Ref. 19. In fact, we were able to derive an analytic expression for the localization length at the band center in the case of weak disorder. The obtained expression reproduces the observed dependence of the localization length on the correlation length. In particular our results show that, as the correlation length is increased, the eigenstates do tend to be more delocalized at the band center.

We also found that longer correlation lengths tend to suppress the band center anomaly. Note that, in the context of the 1D Anderson model with correlated disorder, any anomaly must be defined as the discrepancy between the effective localization length and the value predicted by the IK-formula originally derived in Ref. 20 (see also [4] and references therein). Incidentally, this suppression of the band-center anomaly was the reason that stimulated some of us to conduct the research work published in [18].

The paper is organized as follows. In Sec. 2 we define the model under study. In Sec. 3 we summarize the main theoretical results that describe the band center anomaly when disorder is spatially correlated. In Sec. 4 these general results are applied to the model analyzed in Ref. 19. We draw our conclusions in Sec. 5.

2. Definition of the model

The tight-binding Anderson model has the form of the discrete stationary Schrödinger equation,

$$\vartheta\psi_{n+1} + \vartheta\psi_{n-1} + \varepsilon_n\psi_n = E\psi_n. \qquad (1)$$

Here ε_n are random site energies and ϑ is the parameter standing for the coupling between nearest sites. In this equation the eigenstates ψ_n and their energies E are fully determined by the properties of disorder ε_n; for this reason in what follows we put $\vartheta = 1$.

In order to define the site energies ε_n, we first generate a sequence $\{\eta_n\}$ of identically distributed independent random variables with the distribution,

$$p(\eta_n) = \begin{cases} 1 & \text{if} \;\; \eta_n \in [-1/2, 1/2] \\ 0 & \text{if} \;\; \eta_n \notin [-1/2, 1/2] \end{cases}.$$

The colored noise $\{\beta_n\}$, introduced in Ref. 19, is then obtained by filtering the white noise $\{\eta_n\}$ with an exponential weight function,

$$\beta_n = \sum_{m=-\infty}^{\infty} \eta_n \mathrm{e}^{-|n-m|/l_\mathrm{c}}. \qquad (2)$$

Finally, the site energies $\{\varepsilon_n\}$ are specified by the rescaling of the $\{\beta_n\}$ variables,

$$\varepsilon_n = \sigma \frac{\beta_n}{\sqrt{\langle \beta_n^2 \rangle}}. \qquad (3)$$

Here and in the rest of the paper we use the symbol $\langle\cdots\rangle$ to denote the average over disorder realizations. The parameter σ in Eq. (3) defines the intensity of the disorder and in this paper we consider the case of weak disorder,

$$\sigma^2 << 1. \qquad (4)$$

It is easy to see that, because $\langle \eta_n \rangle = 0$, one also has

$$\langle \varepsilon_n \rangle = 0.$$

Taking into account that the $\{\eta_n\}$ variables are independent, one can derive the binary correlator of the site energies from Eqs. (2) and (3). One obtains that

$$\langle \varepsilon_n \varepsilon_{n+l} \rangle = \sigma^2 \left[1 + |l| \tanh(k_\mathrm{c})\right] \mathrm{e}^{-k_\mathrm{c}|l|}$$

with

$$k_\mathrm{c} = \frac{1}{l_\mathrm{c}}.$$

The corresponding normalized correlator is

$$\mathcal{K}(l) = \frac{\langle \varepsilon_n \varepsilon_{n+l} \rangle}{\langle \varepsilon_n^2 \rangle} = \left[1 + |l| \tanh(k_c)\right] e^{-k_c|l|}. \qquad (5)$$

The exponential factor in the binary correlator (5) shows that the parameter l_c, introduced in Eq. (2), is essentially the correlation length for the site energies. The prefactor preceding the exponential term in Eq. (5), on the other hand, shows that the random site energies (3), strictly speaking, are not exponentially correlated. This would be the case only if the binary correlator (5) were of the form,

$$\mathcal{K}_e(l) = e^{-k_c|l|}. \qquad (6)$$

This may seem a pedantic remark; however, as we shall see, the linear prefactor in the binary correlator (5) plays an important role in accelerating the suppression of the band center anomaly for increasing values of l_c. It should be noted that in Ref. 19 the authors did not specify the analytic form of the binary correlator (5), relying instead on numerical evidence to show that it exhibits a roughly exponential decay for $|l| >> l_c$ (see Ref. 19).

3. The Hamiltonian map approach

The Hamiltonian map approach, introduced in [11,12], represents a useful tool to analyze the electronic states of the Anderson model. The method relies on the correspondence between the Anderson model (1) and a classical stochastic oscillator,

$$H = \frac{p^2}{2} + \frac{1}{2}\mu^2 x^2 \left[1 + \xi(t)\right]. \qquad (7)$$

In Eq. (7) the symbol $\xi(t)$ represents a succession of delta kicks of random strengths

$$\xi(t) = \sum_{n=-\infty}^{\infty} \xi_n \delta(t - n).$$

After integrating the dynamical equations of the parametric oscillator (7) over the period $T = 1$ between two kicks, one obtains the Hamiltonian map

$$\begin{aligned} x_{n+1} &= x_n \left[\cos\mu - \mu\xi_n \sin\mu\right] + p_n \frac{1}{\mu}\sin\mu, \\ p_{n+1} &= -\mu x_n \left[\sin\mu + \mu\xi_n \cos\mu\right] + p_n \cos\mu. \end{aligned} \qquad (8)$$

The correspondence between the models (1) and (7) emerges clearly if one eliminates the momenta from the map (8). In this way one obtains the equation

$$x_{n+1} + x_{n-1} + \mu\xi_n \sin\mu x_n = 2\cos\mu x_n. \qquad (9)$$

Equations (1) and (9) have the same structure; their comparison reveals that the parameters of the two models must obey the identities

$$E = 2\cos\mu \quad \text{and} \quad \varepsilon_n = \mu\xi_n \sin\mu.$$

The dynamics of the oscillator (7) is best studied with the use of action-angle variables (J_n, θ_n), which are related to the Cartesian coordinates by the relations

$$x_n = \sqrt{\frac{2J_n}{\mu}} \sin\theta_n,$$

$$p_n = \sqrt{2\mu J_n} \cos\theta_n.$$

In terms of these new variables the map (8) becomes

$$\begin{aligned} \theta_{n+1} &= \theta_n + \mu + \mu\xi_n \sin^2\theta_n + (\mu\xi_n)^2 \sin^3\theta_n \cos\theta_n + \\ &+ o\left(\sigma^2\right) \pmod{2\pi}, \end{aligned} \qquad (10)$$

$$J_{n+1} = J_n \left(1 - 2\mu\xi_n \sin\theta_n \cos\theta_n + (\mu\xi_n)^2 \sin^2\theta_n\right).$$

Note that the map (10) for the angular variable is an approximation, valid in the weak-disorder case (4). The Landau symbol in Eq. (10) represents neglected terms of order higher than the second in the perturbative parameter σ. For the sake of simplicity, in the rest of this paper we omit the symbol $o(\sigma^2)$; all identities must be interpreted as correct within the limits of the second-order approximation in the disorder strength.

The inverse localization length (or Lyapunov exponent) $\lambda = 1/L_{\text{loc}}$ is

$$\lambda = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \ln\left|\frac{\psi_n}{\psi_{n-1}}\right|. \qquad (11)$$

For weak disorder, and away from the band edges, the Lyapunov exponent (11) can be written in terms of the action-angle variables as

$$\begin{aligned} \lambda = &\frac{\mu^2}{8}\langle \xi_n^2 \rangle \left[1 - 2\langle \cos(2\theta_n)\rangle + \langle \cos(4\theta_n)\rangle\right] - \\ &-\frac{\mu}{2}\langle \xi_n \sin(2\theta_n)\rangle. \end{aligned} \qquad (12)$$

The noise-angle correlator $\langle \xi_n \sin(2\theta_n)\rangle$ in Eq. (12) can be evaluated following the method introduced in Ref. 20. We first define the family of noise-angle correlators

$$q_l = \langle \xi_n e^{i2\theta_{n-l}} \rangle. \qquad (13)$$

After dropping the second-order terms in the θ-map (10), one can write

$$\theta_{n+1} = \theta_n + \mu + \mu\xi_n \sin^2\theta_n. \qquad (14)$$

With the use of the map (14) one can show that the correlators (13) obey the recursive relation

$$q_{l-1} = e^{i2\mu}\left[q_l + i2\mu\langle \xi_n^2 \rangle \langle e^{i2\theta_n} \sin^2\theta_n \rangle \mathcal{K}(l)\right].$$

After multiplying both sides of this equation by $e^{i2\mu(l-1)}$ and summing over the index l, one obtains

$$q_0 = i2\mu\langle \xi_n^2 \rangle \langle e^{i2\theta_n} \sin^2\theta_n \rangle \sum_{l=1}^{\infty} \mathcal{K}(l) e^{i2\mu l}.$$

The imaginary part of this identity gives

$$\langle\xi_n \sin(2\theta_n)\rangle = -\frac{\mu}{2}\langle\xi_n^2\rangle \times$$
$$\times\left\{\left[1-2\langle\cos(2\theta_n)\rangle+\langle\cos(4\theta_n)\rangle\right]\sum_{l=1}^{\infty}\mathcal{K}(l)\cos(2\mu l)+\right. \quad (15)$$
$$\left.+\left[2\langle\sin(2\theta_n)\rangle-\langle\sin(4\theta_n)\rangle\right]\sum_{l=1}^{\infty}\mathcal{K}(l)\sin(2\mu l)\right\}.$$

If the noise-angle correlator (15) is inserted in Eq. (12), the expression for the Lyapunov exponents becomes

$$\lambda = \frac{\sigma^2}{8\sin^2\mu}\left\{\left[1-2\langle\cos(2\theta_n)\rangle+\langle\cos(4\theta_n)\rangle\right]K(\mu)+\right.$$
$$\left.+\left[2\langle\sin(2\theta_n)\rangle-\langle\sin(4\theta_n)\rangle\right]Y(\mu)\right\}. \quad (16)$$

In Eq. (16) the term,

$$K(\mu) = 1+2\sum_{l=1}^{\infty}\mathcal{K}(l)\cos(2\mu l) \quad (17)$$

is the power spectrum of the disorder while

$$Y(\mu) = 2\sum_{l=1}^{\infty}\mathcal{K}(l)\sin(2\mu l) \quad (18)$$

is the sine transform of the binary correlator (5).

It is important to stress that Eq. (16) contains averages which can be evaluated only if one knows the invariant distribution $\rho(\theta)$ for the angle variable. For most values of the energy in the Anderson model, the map (10) ensures that the angle variable has a uniform distribution

$$\rho(\theta) = \frac{1}{2\pi}.$$

Making use of this distribution in Eq. (16), one obtains the IK-formula originally derived in Ref. 20,

$$\lambda_{IK} = \frac{\sigma^2}{8\sin^2(\mu)}K(\mu). \quad (19)$$

Equation (19) gives the inverse localization length for the 1D Anderson model with weak correlated disorder. It represents a generalization of the Thouless formula, to which it reduces for uncorrelated disorder.

For the case of interest here, however, Eq. (19) cannot be applied, because the invariant measure is modulated, not uniform. This is due to the fact that $E \simeq 0$ corresponds to $\mu \simeq \pi/2$ and for this value of the μ parameter the angle map (10) has almost periodic orbits of period 4, which ultimately lead to the modulation of $\rho(\theta)$.

The invariant distribution $\rho(\theta)$ close to the band center can be obtained with the method introduced in [12] for the case of uncorrelated disorder and extended in [18] to the case of correlated disorder. In this approach one first considers the fourth iterate of the map (10) with $\mu \simeq \pi/2$; the continuum limit is then taken and the map is replaced with a stochastic differential equation for $\theta(t)$. The invariant measure for θ is eventually obtained by solving the stationary Fokker–Planck equation associated to the stochastic differential equation previously derived. The interested reader can find a detailed explanation of the derivation in Ref. 18.

After lengthy calculations, one obtains that the invariant measure at the exact band center is

$$\rho(\theta) = \frac{1}{2\mathbf{K}(\alpha)}\frac{1}{\sqrt{4-2\alpha^2\left[1-\cos(4\theta)\right]}} \quad (20)$$

with

$$\alpha = \sqrt{\frac{K(\pi/2)}{K(0)+K(\pi/2)}}. \quad (21)$$

The corresponding inverse localization length is

$$\lambda = \frac{\sigma^2}{4}\left\{\left[K(0)+K\left(\frac{\pi}{2}\right)\right]\frac{\mathbf{E}(\alpha)}{\mathbf{K}(\alpha)}-K(0)\right\}. \quad (22)$$

In Eqs. (20) and (22) the symbols $\mathbf{K}(k)$ and $\mathbf{E}(k)$ represent the complete elliptic integrals of the first and second kinds. Equations (20) and (22) are the main theoretical results that will be used below to analyze the Anderson model with the short-range correlated disorder introduced in Ref. 19.

4. The band center anomaly in the Sales-de Moura model

When the binary correlator of the site energies in the Anderson model (1) has the form (5), the power spectrum of the disorder takes the form

$$K(\mu) = \frac{\sinh^3(k_c)}{\cosh(k_c)\left[\cosh(k_c)-\cos(2\mu)\right]^2}. \quad (23)$$

From Eqs. (22) and (23), one easily obtains that the inverse localization length is equal to

$$\lambda = \frac{\sigma^2}{2\sinh(2k_c)}\left\{2\left[1+\cosh^2(k_c)\right]\frac{\mathbf{E}(\alpha_1)}{\mathbf{K}(\alpha_1)}-\left[1+\cosh(k_c)\right]^2\right\}, \quad (24)$$

where the parameter (21) assumes the specific value

$$\alpha_1 = \frac{\sinh^2(k_c)}{\left[1+\cosh(k_c)\right]\sqrt{2\left[1+\cosh^2(k_c)\right]}}. \quad (25)$$

Equation (24) gives the inverse localization length at the exact band center for the Anderson model (1) with the Sales-de Moura correlated disorder (3).

One can gain physical insight into the behavior of the inverse localization length (24) by considering the limit cases of a very short and very long correlation length l_c. In the limit $l_c \to 0$, i.e., for $k_c \to \infty$, the Lyapunov exponent (24) reduces as expected to the Derrida–Gardner form [8,12],

$$\lambda = \frac{\sigma^2}{4}\left[2\frac{\mathbf{E}(1/\sqrt{2})}{\mathbf{K}(1/\sqrt{2})} - 1\right]. \qquad (26)$$

In the opposite limit, $l_c \to \infty$ (which corresponds to $k_c \to 0$), the inverse localization length decays as

$$\lambda \simeq \frac{\sigma^2}{32 l_c^3}. \qquad (27)$$

Equation (27) shows that, for increasing values of the correlation length l_c, the localization length increases with the third power of the correlation length, as numerically observed in Ref. 19.

It is interesting to compare the inverse localization length (24) with the expression given by the Eq. (19). When one considers the Anderson model with site energies (3), for $E = 0$ the IK-formula gives

$$\lambda_{IK} = \frac{\sigma^2}{8}\frac{\sinh^3(k_c)}{\cosh(k_c)\left[1+\cosh(k_c)\right]^2}. \qquad (28)$$

For $l_c \to 0$, Eq. (28) reduces to

$$\lambda_{IK} = \frac{\sigma^2}{8},$$

which coincides with the prediction of the Thouless formula and differs from the Derrida–Gardner result (26). On the other hand, in the limit $l_c >> 1$ Eq. (19) reproduces the correct result (27). This implies that the band-center anomaly is gradually suppressed as the correlation length increases.

This effect is confirmed by the numerical data, as shown by Fig. 1. The data obtained for $l_c = 0$ (uncorrelated disorder) shows that, at the band center, the Lyapunov exponent exhibits a pronounced dip (the anomaly) which, however, is essentially suppressed already for $l_c = 1$.

To understand the physical reason behind the suppression of the band-center anomaly, it is useful to consider how the invariant measure $\rho(\theta)$ changes as l_c increases. In the present case, at the band center the invariant measure for the phase variable is given by Eq. (20) with the parameter α taking the value (25). For $l_c = 0$ the invariant distribution takes the limit form

$$\rho(\theta) = \frac{1}{2\mathbf{K}\left(\frac{1}{\sqrt{2}}\right)\sqrt{3+\cos(4\theta)}} \qquad (29)$$

which coincides with the result derived in [12] for the case of uncorrelated disorder. For $l_c >> 1$, on the other hand, one obtains

$$\rho(\theta) = \frac{1}{2\pi}\left[1 - \frac{1}{64 l_c^4}\cos(4\theta) + \ldots\right]. \qquad (30)$$

Equation (30) shows that the invariant distribution becomes quickly uniform for increasing values of the correlation length l_c. This explains why the anomaly is suppressed and the standard formula (19), derived under the assumption of a flat distribution for the angle variable, becomes valid.

The band-center anomaly is not suppressed only if the site energies have correlations of the form (5). As shown in [18], a similar effect occurs for disorder with exponentially decaying, *positive* correlations. One should also add that exponentially decaying correlations with *alternating* sign can have the opposite effect and *enhance* the band-center anomaly [18].

We conclude our analysis of the Anderson model (1)–(3) with a remark on the difference between a truly exponentially correlated disorder with the correlator of the form (6), and the Sales-de Moura disorder with the correlator (5). The power spectrum corresponding to the exponential correlator (6) is

$$K_e(\mu) = \frac{\sinh(k_c)}{\cosh(k_c) - \cos(2\mu)}, \qquad (31)$$

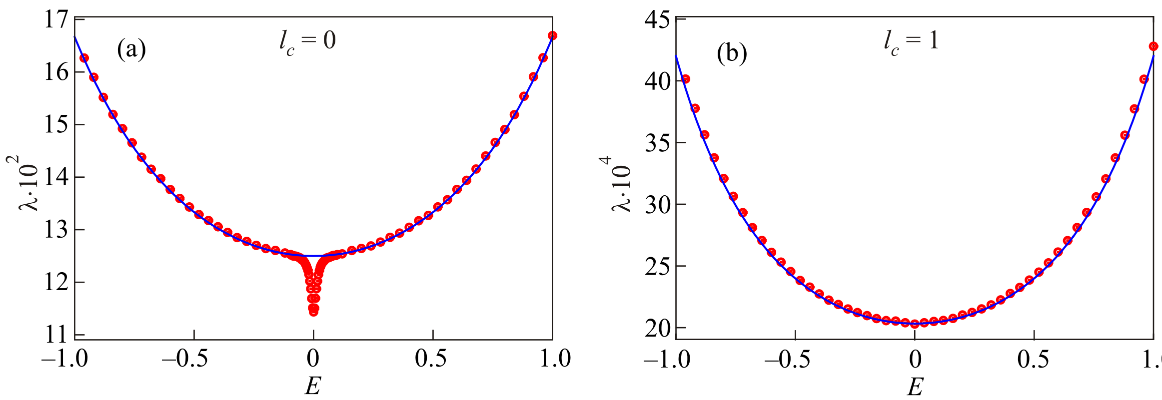

Fig. 3. (Color online) The inverse localization length $\lambda = L_{\text{loc}}^{-1}$ versus the energy E for two different values of correlation length: (a) $l_c = 0$ and (b) $l_c = 1$. Red circles represent numerical data, while blue continuous curves correspond to Eq. (19). The intensity of the disorder in numerical simulations was set to $\sigma^2 = 0.01$.

that leads to the inverse localization length,

$$\lambda = \frac{\sigma^2}{4}\left[2\coth(k_c)\frac{\mathbf{E}(\alpha_2)}{\mathbf{K}(\alpha_2)} - \frac{\sinh(k_c)}{\cosh(k_c)-1}\right] \tag{32}$$

with

$$\alpha_2 = \sqrt{\frac{1}{2}\left[1 - \frac{1}{\cosh(k_c)}\right]}. \tag{33}$$

In the limit $l_c \to 0$, Eq. (32) tends to the anomalous form (26). For $l_c >> 1$, however, Eq. (32) becomes

$$\lambda \simeq \frac{\sigma^2}{16 l_c}. \tag{34}$$

Comparing the limit forms (27) and (34), it is easy to see that the localization length increases *linearly* with the correlation length l_c when the disorder is exponentially correlated in the strict sense, whereas it increases with the l_c^3 when the disorder has correlations of the form (5).

This behavior of the localization length is matched by the corresponding behavior of the invariant measure in the limit of large correlation length. When the power spectrum of the disorder has the form (31), the invariant distribution is given by Eq. (20) with the parameter α assuming the value (33). For $l_c = 0$, one recovers the distribution (29). When $l_c >> 1$, however, the invariant distribution assumes the form

$$\rho(\theta) = \frac{1}{2\pi}\left[1 - \frac{1}{16 l_c^2}\cos(4\theta) + \ldots\right]. \tag{35}$$

Comparing the asymptotic forms (30) and (35), one can see that when the correlator has the form (5) the invariant distribution becomes uniform much faster for increasing l_c than in the case for disorder with the exponential correlations (6).

5. Summary

In this paper we demostrated how the approach, developed in [18] for the band-center in the Anderson model, can be applied to the quite specific case of correlated disorder studied in Ref. 19. Our analytical results, obtained for a weak disorder, allow one to explain the anomalous relation $L_{\rm loc} \propto l_c^3$ between the localization length and the correlation length which was numerically observed in [19]. Our analysis shows that this dependence is not completely due to the exponential decay of the correlations, as was claimed, but has to be partly ascribed to the polynomial prefactor in the binary correlator (5).

We showed that the band-center anomaly is suppressed for increasing values of the correlation length l_c when the binary correlator has either the form (5) or (6). The localization length at the band center increases with l_c in both cases; however, the polynomial prefactor in the binary correlator (5) results in a quantitative difference between the two types of disorder. Specifically, it gives rise to a faster suppression of the anomaly for increasing l_c and generates an unusual dependence $L_{\rm loc} \propto l_c^3$.

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