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Surface electromagnetic modes in layered conductors in a magnetic field

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A transfer-matrix approach is developed for studies of the collective electromagnetic modes in a semi-infinite layered conductor subjected to a quantizing external magnetic field perpendicular to the layers. The dispersion relations for the surface and bulk modes are derived. It is shown that the surface mode has a gap in the long-wavelength limit and exists only if the absolute value of the in-plane wave vector q exceeds the threshold value $\mathbf{q}^* = -1/(a \ln |\Delta|)$. Depending on the sign of the parameter $\Delta = (\varepsilon - \varepsilon_0)/(\varepsilon_0 + \varepsilon)$, the frequency of the surface mode $\omega_s(q, \Delta)$ goes either above (for $\Delta > 0$) or below (for $\Delta < 0$) the bulk-mode frequency $\omega(q, k) = \omega(q, k + 2\pi/a)$ for any value of k. At nonzero magnetic field H the bulk mode has a singular point $q_0(H)$ at which the bulk band twists in such a way that its top and bottom bounds swap. Small variations of q near this point change dramatically the shape of the dispersion function $\omega(q, k)$ in the variable k. The surface mode has no dispersion across the layers, since its amplitude decays exponentially into the bulk of the sample. Both bulk and surface modes have in the region $qa >> 1$ a similar asymptotic behavior $\omega \propto q^{1/2}$, but $\omega_s(q, \Delta)$ lies above or below $\omega(q, k)$, respectively, for $\Delta > 0$ and $\Delta < 0$ (a is the interlayer separation; ε_0 and ε stand for the dielectric constants of the media outside the sample and between the layers; q and k are the components of the wave vector in the plane and perpendicular to the layers, respectively).

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1. Introduction

The discovery of the quantum Hall effect in 1980 [1] has triggered intensive studies of a two-dimensional electron gas (2*D*EG) in an external quantizing magnetic field. These studies have since been extended to different types of artificially fabricated semiconducting and metallic superlattices (SL), organic conductors, and high-T_c layered superconductors. Numerous studies, in particular, have been devoted to the problem of collective plasma and electromagnetic waves in 2*D*EG and layered conductors as well as in SL in a high magnetic field. Generally, a three different physical cases should be distinguished in this problem: the case of classical SL, the case of quantum SL, and the case of layered conductors. In the first case constituent slabs of the SL are assumed to be so thick that one can neglect the electron energy quantization. The electromagnetic wave propagation in such SL is determined completely by Maxwell's equations and the appropriate boundary conditions. Quantum SL have small separation between conducting layers, and the electron dispersion across the layers in this case is due to the tunneling between neighboring layers. By layered conductors we shall understand a stack of 2*D* conducting planes separated by dielectric layers which prevent electrons from hopping between the neighboring planes. Layered conductors are realized in nature in the form of layered crystals such as dichalcogenides of transient metals, organic superconductors, and high-T_c cuprates. The high anisotropy of Tl- and Bi-based high-T_c cuprates [2], organic salts of $(TMTSF)_2X$ [3], and ET families [4] makes them, like dichalcogenides of transition metals [5], good layered conductors in the sense formulated above. It is evident, that layered conductors can also be fabricated artificially in the form of highly anisotropic SL. All these materials are well described by the model of conducting planes embedded in a dielectric matrix. This model has proved to be useful in studies of different types of plasma [6–10] and electromagnetic [11–20] waves in layered conductors, superconductors and superlattices. A quasi two-dimensional nature of the conductivity in layered conductors brings some specific features into calculations of the collective electromagnetic modes in them, especially in the presence of external magnetic field. Some new types of collective electromagnetic excitations have been predicted theoretically in a purely 2*D*EG in high magnetic fields under the conditions of the quantum and conventional Hall effects. Among them are surface polaritons [21,22], magnetoplasma oscillations [23], and quantum waves [24,25]. The variety of waves became richer in layered conductors. It is known that a quantizing magnetic field applied perpendicular to layers makes possible the propagation of the helicons across the layers in both the conventional [11–14] and quantum [14,26–28] Hall-effect regimes.

Real layered crystals and superlattices contain different types of defects within the layers as well as imperfections in their stacking which may give rise to new collective electromagnetic modes such as, for example, the magnetoimpurity waves [13] or various local modes [9,10,16]. The infinite crystal is yet another idealization of the theoretical treatment of the problem, since any sample in experiments has a surface which is known to be a «structural defect» that generates surface modes decreasing into the bulk of the sample. Surface plasma modes have been studied extensively in the model of a semi-infinite layered electron gas [7,8]. Surface electromagnetic waves have also been described in layered superconductors [15].

The purpose of this paper is to study the surface electromagnetic waves in layered conductors in a perpendicular quantizing magnetic field. The basic equations describing the electric field components on the layers, $E_\alpha(z_n) \equiv E_\alpha(n)$, were derived in our previous publication [14] and can be written as follows (see Appendix for details):

$$E_x(n) = \frac{4\pi i\omega}{c^2} \sum_{n'} G^x_{q_\omega}(n, n')[\sigma_{xx} E_x(n') + \sigma_{xy} E_y(n')],$$

$$E_y(n) = -\frac{4\pi i q_\omega^2}{\omega} \times \qquad (1)$$

$$\times \sum_{n'} G^y_{q_\omega}(n, n')[\sigma_{yy} E_y(n') + \sigma_{yx} E_x(n')]\varepsilon^{-1}(n)$$

(z_n is a discrete coordinate of a conducting plane along the z axis).

The Green's functions $G^\alpha_{q_\omega}(n, n') \equiv G^\alpha_{q_\omega}(z_n, z'_n)$ in Eq. (1) satisfy the following equations:

$$\left(\frac{\partial^2}{\partial z^2} - q_\omega^2(z)\right) G^x_{q_\omega}(z, z') = \delta(z - z') , \qquad (2)$$

$$\left(\frac{\partial^2}{\partial z^2} + U(\mathbf{q}, \omega, z)\frac{\partial}{\partial z} - q_\omega^2(z)\right) G^y_{q_\omega}(z, z') = \delta(z - z') , \qquad (3)$$

where

$$U(\mathbf{q},\ \omega,\ z) = \left(\frac{q}{q_\omega(z)}\right)^2 \varepsilon^{-1}(z)\, \frac{\partial\varepsilon(z)}{\partial z} . \qquad (4)$$

Here $\varepsilon(z)$ is the dielectric constant of the matter between the layers, $\sigma_{\alpha\beta} \equiv \sigma_{\alpha\beta}(\mathbf{q},\ \omega,\ H)$ is the two-dimensional high-frequency conductivity tensor in an external magnetic field H; $\mathbf{q}$ stands for the wave vector, and $q_\omega(z)$ is defined by the equation

$$q_\omega^2(z) = q^2 - \frac{\omega^2}{c^2}\,\varepsilon(z) . \qquad (5)$$

2. The model and the basic equations

Consider a regular semi-infinite layered crystal in which conducting planes occupy positions at a discrete periodic set of points $z_n = na$ ($n = 0, 1, 2...$) along the z axis of the halfspace $z > 0$. We assume that the dielectric constants are different in the halfspaces: ε_0 at $z < 0$ and ε between the layers. The function $\varepsilon(z)$ can be written analytically with the help of the Heaviside step function

$$\varepsilon(z) = \varepsilon\theta(z) + \varepsilon_0\theta(-z) . \qquad (6)$$

It follows then from Eq. (5) that quantity $q_\omega^2(z)$ takes two different values in half spaces:

$$q_\omega^2(z) = \begin{cases} q_\omega^2 , & z > 0 , \\ \kappa_\omega^2 , & z < 0 , \end{cases} \qquad (7)$$

where $q_\omega^2 = q^2 - \varepsilon\omega^2/c^2$ and $\kappa_\omega^2 = q^2 - \varepsilon_0\omega^2/c^2$.

The Green's function $G^x_{q_\omega}(z, z')$ can be found with the help of the known general expression

$$G^x_{q_\omega}(z, z') = \frac{1}{W(\chi,\ \varphi)} \times$$

$$\times \{\theta(z - z')\, \chi(z)\, \varphi(z') + \theta(z' - z)\, \chi(z')\, \varphi(z)\} , \qquad (8)$$

where $\chi(z)$ and $\varphi(z)$ are two independent solutions of the differential operator in the left-hand side of Eq. (2), and $W(\chi,\ \varphi) = \varphi\chi' - \chi\varphi'$ is the Wronskian determinant.

Choosing $\chi(z) = \exp(-q_\omega z)$, $\varphi(z) = \cosh(q_\omega z) + (\kappa_\omega / q_\omega)\sinh(q_\omega z)$ for $z > 0$, we have

$$G^x_{q_\omega}(z, z') = -\frac{1}{2q_\omega}\left(e^{-q_\omega|z-z'|} + \delta_\omega\, e^{-q_\omega|z+z'|}\right), \quad z, z' > 0 , \tag{9}$$

where

$$\delta_\omega = (q_\omega - \kappa_\omega)/(q_\omega + \kappa_\omega) . \tag{10}$$

The Green's function $G^y_{q_\omega}(z, z') \equiv \hat{G}(z, z')$ in our model satisfies the following equation:

$$\left(\frac{\partial^2}{\partial z^2} - q^2_\omega(z)\right)\hat{G}(z,z') + \Delta_\omega\, \delta(z)\, \frac{\partial}{\partial z}\, \hat{G}(z,z') = \delta(z - z'). \tag{11}$$

The quantity Δ_ω is defined by the relation

$$\Delta_\omega = 2\left[\frac{q}{\overline{q}_\omega}\right]^2 \frac{\varepsilon - \varepsilon_0}{\varepsilon_0 + \varepsilon} , \tag{12}$$

where the following notations are adopted: $\overline{q}_\omega = q^2 - (\omega^2/c^2)\overline{\varepsilon}$, and $\overline{\varepsilon} = \frac{1}{2}(\varepsilon_0 + \varepsilon)$. The solution of Eq. (11) is trivially expressed in terms of the Green's function $G(z, z')$ that satisfies the very same equation but with $\Delta_\omega = 0$:

$$\hat{G}(z, z') = G(z, z') - \frac{\Delta_\omega}{1 + \Delta_\omega G'(0, 0)}\, G(z, 0)G'(0, z'), \tag{13}$$

where we have used the notation $G'(0, z') = \lim_{x\to 0} \partial G(x, z')/\partial x$.

Taking into account that $G(z, z') \equiv G^x_{q_\omega}(z, z')$ for $z, z' > 0$, we obtain from Eqs. (9) and (13) an exact formula for the Green's function $G^y_{q_\omega}(z, z')$ in the positive half space:

$$G^y_{q_\omega}(z, z') = -\frac{1}{2q_\omega}\left(e^{-q_\omega|z-z'|} + \hat{\Delta}_\omega\, e^{-q_\omega|z+z'|}\right), \quad z, z' > 0 . \tag{14}$$

We have introduced the notation

$$\hat{\Delta}_\omega = \delta_\omega + \frac{\Delta_\omega(1 - \delta^2_\omega)}{2 + \delta_\omega \Delta_\omega} . \tag{15}$$

Substituting Eqs. (9) and (14) into Eq. (1), we have

$$E_\alpha(n) = \sum^{\infty}_{\beta, n'=0} \hat{\sigma}_{\alpha\beta}(e^{-q_\omega a|n-n'|} + \hat{\Delta}^\alpha_\omega\, e^{-q_\omega a|n+n'|})\, E_\beta(n') , \tag{16}$$

where

$$\hat{\sigma}_{\alpha\beta} = -\frac{2\pi i\omega}{q_\omega c^2}\, \sigma_{\alpha\beta}(\mathbf{q}, \omega, H)V_{\alpha\beta} , \tag{17}$$

and $V_{\alpha\beta}$ is a matrix with the components $V_{11} = V_{12} = 1$, $V_{21} = V_{22} = -c^2q^2_\omega/\omega^2\varepsilon$. The quantity $\hat{\Delta}^\alpha_\omega$ takes two values: $\hat{\Delta}^x_\omega = \delta_\omega$ and $\hat{\Delta}^y_\omega = \hat{\Delta}_\omega$.

3. The transfer-matrix approach

To solve Eqs. (16) it is convenient to introduce new quantities

$$A_\alpha(n) = \sum_\beta \hat{\sigma}_{\alpha\beta}\left(\sum_{n' \le n} e^{-q_\omega a(n-n')}E_\beta(n') + \hat{\Delta}^\alpha_\omega \sum_{n'=0} e^{-q_\omega a(n+n')}E_\beta(n')\right) \tag{18}$$

and

$$B_\alpha(n) = \sum_\beta \hat{\sigma}_{\alpha\beta}\, (\sum_{n'>n} e^{-q_\omega a(n-n')}E_\beta(n')) . \tag{19}$$

The sum of $A_\alpha(n)$ and $B_\alpha(n)$ is exactly the electric field at the nth layer:

$$E_\alpha(n) = A_\alpha(n) + B_\alpha(n) . \tag{20}$$

Using Eqs. (18)-(20), one can easily obtain the recurrence relations

$$A_\alpha(n + 1) = e^{-q_\omega a}A_\alpha(n) + \sum_\beta \hat{\sigma}_{\alpha\beta}\, [A_\beta(n + 1) + B_\beta(n + 1)] , \tag{21}$$

$$B_\alpha(n + 1) = e^{q_\omega a}B_\alpha(n) - \sum_\beta \hat{\sigma}_{\alpha\beta}\, [A_\beta(n + 1) + B_\beta(n + 1)] . \tag{22}$$

These equations may be recast in the matrix form:

$$\begin{pmatrix} A_\alpha(n + 1) \\ B_\alpha(n + 1) \end{pmatrix} = \sum_\beta \hat{T}_{\alpha\beta} \begin{pmatrix} A_\beta(n) \\ B_\beta(n) \end{pmatrix}, \tag{23}$$

where the transfer matrix $\hat{T}_{\alpha\beta}$ has been introduced by the definition

$$\hat{T}_{\alpha\beta} = \begin{pmatrix} (\delta_{\alpha\beta} + \hat{\sigma}_{\alpha\beta})\, e^{-q_\omega a} & \hat{\sigma}_{\alpha\beta}\, e^{q_\omega a} \\ -\hat{\sigma}_{\alpha\beta}\, e^{-q_\omega a} & (\delta_{\alpha\beta} - \hat{\sigma}_{\alpha\beta})\, e^{q_\omega a} \end{pmatrix} . \quad (24)$$

The transfer matrix satisfies the relation

$$\det \hat{T}_{\alpha\beta} = \hat{T}^{11}_{\alpha\beta}\hat{T}^{22}_{\alpha\beta} - \hat{T}^{12}_{\alpha\beta}\hat{T}^{21}_{\alpha\beta} = \delta_{\alpha\beta} . \quad (25)$$

As compared to the case of a one-component plasma oscillations in layered structures,which were discussed in papers [8,9] in terms of the transfer matrix of dimension 2×2, the matrix $\hat{T}_{\alpha\beta}$ given by Eq. (24) has a higher dimensionality (4×4) because of the two-component nature of the electromagnetic waves in the system under study.

Putting $n = 0$ in Eqs. (18) and (19), we arrive at the surface condition

$$A_\alpha(0) = \hat{\Delta}^\alpha_\omega B_\alpha(0) + \sum_\beta \hat{\sigma}_{\alpha\beta}(1 + \hat{\Delta}^\alpha_\omega)[A_\beta(0) + B_\beta(0)] . \quad (26)$$

Before turning to the surface-mode calculations it is instructive to address first the simpler case of an infinite layered conductor. In this case one can find the solution of the matrix equation (23) in the form

$$A_\alpha(n) = C_\alpha\, e^{ikan} , \quad B_\alpha(n) = D_\alpha\, e^{ikan} . \quad (27)$$

After substitution of these relations into Eq. (23), we have

$$\mathrm{Det}\, (\delta_{\alpha\beta}\, \hat{I} - \hat{T}_{\alpha\beta}\, e^{ika}) = 0 . \quad (28)$$

The symbol Det here stands for the determinant of the (4×4) matrix, while $\hat{I}$ is the (2×2) unit matrix.

Taking into account the condition given by Eq. (25), one can rewrite Eq. (28) in the form

$$\det\, (\delta_{\alpha\beta} \cos ka - \frac{1}{2}\, \mathrm{Tr}\, \hat{T}_{\alpha\beta}) = 0 \quad (29)$$

which, after the substitution of the transfer-matrix components, yields the dispersion relation

$$\det\, [\delta_{\alpha\beta} + \hat{\sigma}_{\alpha\beta}\, S(q, k, \omega)] = 0 , \quad (30)$$

where the structural form factor is given by

$$S(q, k, \omega) = \frac{\sinh\, (q_\omega a)}{\cosh\, (q_\omega a) - \cos\, (ka)} . \quad (31)$$

Different types of electromagnetic waves in infinite layered conductors have been studied on the basis of Eq. (30) under the the conditions of the conventional and quantum Hall effects, in particular, the magnetoimpurity waves [13] and the helicons and helicons–plasmons [14]. The surface breaks the translational invariance of Eq. (16) due to the term containing $\hat{\Delta}^\alpha_\omega$. Because of that, the surface mode has no dispersion across the layers, and its field components damp into the bulk of the layered conductor. We assume this damping to be exponential with a decrement γ and will find it below,

$$E_\beta(n + 1) = e^{-\gamma a} E_\beta(n) = \ldots = e^{-\gamma a n} E_\beta(0) . \quad (32)$$

This equation means that

$$A_\alpha(n) = A_\alpha(0)\, e^{-\gamma a n} , \quad B_\alpha(n) = B_\alpha(0)\, e^{-\gamma a n} . \quad (33)$$

The above relations have the very same exponential form as those in Eq. (27), so that we can find the dispersion relation for the surface mode immediately from Eq. (30) by the substitution $k \to i\gamma$. This yields

$$\det\, (\hat{S}^{-1}\delta_{\alpha\beta} - \hat{\sigma}_{\alpha\beta}) = 0 , \quad (34)$$

where the form factor $\hat{S}(q, \gamma, \omega) = S(q, i\gamma, \omega)$ is given by

$$\hat{S}(q, \gamma, \omega) = \frac{\sinh\, (q_\omega a)}{\cosh\, (q_\omega a) - \cosh\, (\gamma a)} . \quad (35)$$

To obtain an equation for the function $\gamma = \gamma(q_\omega , \omega)$ we proceed as follows. First, writing the condition $E_\alpha(n + 1) = e^{-\gamma a} E_\alpha(n)$ with the help of the transfer matrix and then putting $n = 0$, we arrive at the equation

$$\sum_\beta [(T^{11}_{\alpha\beta} + T^{21}_{\alpha\beta})A_\beta(0) + (T^{22}_{\alpha\beta} + T^{12}_{\alpha\beta})B_\beta(0)] =$$

$$= (A_\alpha(0) + B_\alpha(0))\, e^{-\gamma a} . \quad (36)$$

New using then Eq. (24) for the transfer-matrix components, we obtain from Eq. (36) a relation for the ratio A_α / B_α at the surface:

$$\frac{A_\alpha(0)}{B_\alpha(0)} = \Gamma(q, \omega, \gamma) = \frac{e^{q_\omega a} - e^{-\gamma a}}{e^{-\gamma a} - e^{-q_\omega a}} . \quad (37)$$

Combining this equation with the surface condition given by Eq. (26), we arrive at a pair of linear equations for the quantities $B_x(0)$ and $B_y(0)$, which have a nonzero solution if

$$\det\, [P_\alpha\, \delta_{\alpha\beta} - \hat{\sigma}_{\alpha\beta}] = 0 , \quad (38)$$

where

$$P_\alpha(q,\omega,\gamma) = \frac{1}{1+\hat{\Delta}^\alpha_\omega} - \frac{1}{1+\Gamma} - \hat{\sigma}_{\alpha\alpha} . \quad (39)$$

Equations (34) and (38) form a closed system of equations for the surface mode. This system can, however, be recast into a simpler pair of equations. Indeed, comparing Eqs. (38) and (34), we see that $P_\alpha = \hat{S}^{-1}$. This condition gives an equation for $\gamma = \gamma(q_\omega, \omega)$:

$$(1+\hat{\Delta}^\alpha_\omega)\, e^{\gamma a} = \hat{\Delta}^\alpha_\omega\, e^{q_\omega a} + e^{-q_\omega a} . \quad (40)$$

Using this equation, we can exclude γ from the form factor $\hat{S}[q, \gamma = \gamma(q_\omega, \omega), \omega] \equiv \overline{S}(q,\omega)$ in Eq. (35), which yields the dispersion relation for the surface mode $\omega_s = \omega_s(q)$

$$\det\,(\delta_{\alpha\beta} - \hat{\sigma}_{\alpha\beta}(q,\omega)\,\overline{S}(q,\omega)) = 0 , \quad (41)$$

where

$$\overline{S}(q,\omega) = \left(\frac{1+\hat{\Delta}^\alpha_\omega}{2\hat{\Delta}^\alpha_\omega}\right)\frac{\hat{\Delta}^\alpha_\omega\, e^{q_\omega a} + e^{-q_\omega a}}{\sinh(q_\omega a)} . \quad (42)$$

The amplitudes of this surface mode decrease exponentially into the bulk of the layered conductor $E_\alpha(n) = e^{-\gamma a n} E_\alpha(0)$ with a decrement $\gamma = \gamma(q, \omega_s(q))$ given by

$$\gamma^\alpha(q) = \frac{1}{a}\ln\left(\frac{\hat{\Delta}^\alpha_\omega\, e^{q_\omega a} + e^{-q_\omega a}}{1+\hat{\Delta}^\alpha_\omega}\right) , \quad (43)$$

where $\omega = \omega_s(q)$.

Being a collective excitation of the finite layered conductor, the surface mode also decreases exponentially into the left half space $z < 0$ with a decrement $\kappa^2_\omega > 0$. This means that the condition $q^2 - (\omega^2/c^2)\varepsilon_0 > 0$ should hold, as well as the inequality $q^2 - (\omega^2/c^2)\varepsilon > 0$, which has been tacitly assumed in the course of all the above discussion. Therefore, these two constraints together with Eqs. (41)–(43) comprise a complete set of equations describing the surface electromagnetic mode in a layered conductor in an external magnetic field within the our approach. It is worthy of note that these dispersion relations are still rather general, since the 2D conductivity tensor that appears in them is as yet an arbitrary quantity. In the next section we will consider a Drude-like model for the conductivity of the 2DEG, leaving more complex models of the conductivity for further studies.

4. The surface mode

For further calculations a specific form for the in-plane conductivity tensor is required. Here we consider the simplest case of a two-dimensional electron gas in a perpendicular magnetic field. The conductivity tensor in this case has been calculated elsewhere (see [29] for the review) and has the following components:

$$\sigma_{xx} = \sigma_{yy} = \sigma_0\chi(1+\chi^2)^{-1} ,$$
$$\sigma_{xy} = -\sigma_{yx} = -\sigma_0 + \chi\sigma_{xx} , \quad (44)$$

where

$$\sigma_0 = \frac{Ne^2}{m\Omega} , \quad \chi = \frac{\nu - i\omega}{\Omega} , \quad (45)$$

$\Omega = eH/mc$ stands for the cyclotron frequency; $\nu = \tau^{-1}$ is the Landau level broadening due to the finite lifetime τ; and N is the two-dimensional electron density. Substituting the conductivity tensor of Eqs. (44) and (45) into the dispersion relations (41) and (34), we arrive at explicit equations for the dispersion relations of the surface, $\omega_s(q)$, and the bulk, $\omega(q)$, modes, which are nonetheless still intractable analytically without further approximations. The problem of the bulk electromagnetic modes within the approach taken here has been discussed in detail in Ref. 14 both numerically and analytically. In particular, the analytic solution was found for the dispersion relation of the bulk helicon-plasmon mode in the case $qa >> \sqrt{\varepsilon}\omega_*(\omega/\omega_p)$. The dimensionless quantity $\omega_* = \omega_p\, a/c$ is extremely small over a wide range of values of the constituent parameters typical for semiconducting superlattices, organic conductors, intercalated dichalcogenides of transition metals, and high-T_c superconductors. For example, for $a \simeq 10^{-7}$–10^{-5} cm and $\omega_p \simeq 10^{13}$ s^{-1}, ω_* is of the order of 10^{-4}–10^{-2} (ω_p is the plasma frequency of the 2D conducting layer determined by $\omega_p^2 = 4\pi Ne^2/ma$ and c is the speed of light). In this approximation $q_\omega a \approx \kappa_\omega a \approx qa$, so that, according to Eq. (10), $\delta_\omega \approx 0$, and Eqs. (12) and (15) yield $\hat{\Delta}^x_\omega = \delta_\omega \approx 0$, $\hat{\Delta}^y_\omega = \Delta_\omega/2 \simeq \Delta$, where

$$\Delta = \frac{\varepsilon - \varepsilon_0}{\varepsilon + \varepsilon_0} . \quad (46)$$

Under these conditions both form factors given by Eqs. (35) and (42) (for the bulk and surface mode, respectively) become frequency independent, and the inequalities $q^2 - (\omega^2/c^2)\varepsilon_0 > 0$ and $q^2 - (\omega^2/c^2)\varepsilon > 0$ hold automatically. Now setting the Landau level broadening $\nu = 0$, we find (see Ref. 14 for more details)

$$\omega_{(s)}^2 \approx 2qa \left[\frac{\Omega^2}{2qa + R_{(s)}\omega_*^2} + \frac{\omega_p^2}{4} \frac{R_{(s)}}{\varepsilon} \right], \quad (47)$$

where the factor R takes two different forms for the bulk and surface modes:

$$R = \frac{\sinh (qa)}{\cosh (qd) - \cos (ka)} \quad (48)$$

in case of a bulk mode, and

$$R_s = \left(\frac{1 + \Delta}{2\Delta} \right) \frac{\Delta e^{qa} + e^{-qa}}{\sinh (qa)} \quad (49)$$

in case of a surface mode. Note that the factor R in the formula for the bulk mode depends on the two projections of the wave vector, i.e., $R = R(q, k)$, where q is the in-plane wave vector, whereas k describes the dispersion of the bulk mode across the layers. The surface mode has no dispersion across the layers, and that is why $R_s = R_s(q, \Delta)$ depends only on q and the parameter Δ determined by Eq. (46), so that $\omega_s = \omega_s(q, \Delta)$). In case of the bulk mode, Eq. (47) describes a wave which is a combination of the helicon (first term) and plasmon (second term). The amplitude of the surface mode $\omega_s = \omega_s(q, \Delta)$ given by Eqs. (47) and (49) decreases into the bulk of a layered conductor according to the law

$$E_y(an) = E_y(0) \left(\frac{1 + \Delta}{\Delta e^{qa} + e^{-qa}} \right)^n . \quad (50)$$

We see from this equation that the field decays into the bulk of the sample in such a way that $E_y(an)$ becomes exponentially small for $qa >> 1$:

$$E_y(an) \approx E_y(0) \left(\frac{1 + \Delta}{\Delta} \right)^n e^{-qan} . \quad (51)$$

In this limit the factor R_s becomes a constant. $R_s \simeq 1 + \Delta$, and the dispersion relation of the surface wave becomes very simple:

$$\omega_s(q, \Delta) \simeq \left[\Omega^2 + \omega_p^2 \left(\frac{1 + \Delta}{2\varepsilon} \right) qa \right]^{1/2} . \quad (52)$$

Such a square-root dispersion relation is typical for films, as is clear, since the electromagnetic field of the surface wave is nonzero only at the interface layer in the limit $qa >> 1$. The dispersion of the surface mode $\omega_s(q, \Delta)$ for arbitrary qa is given by

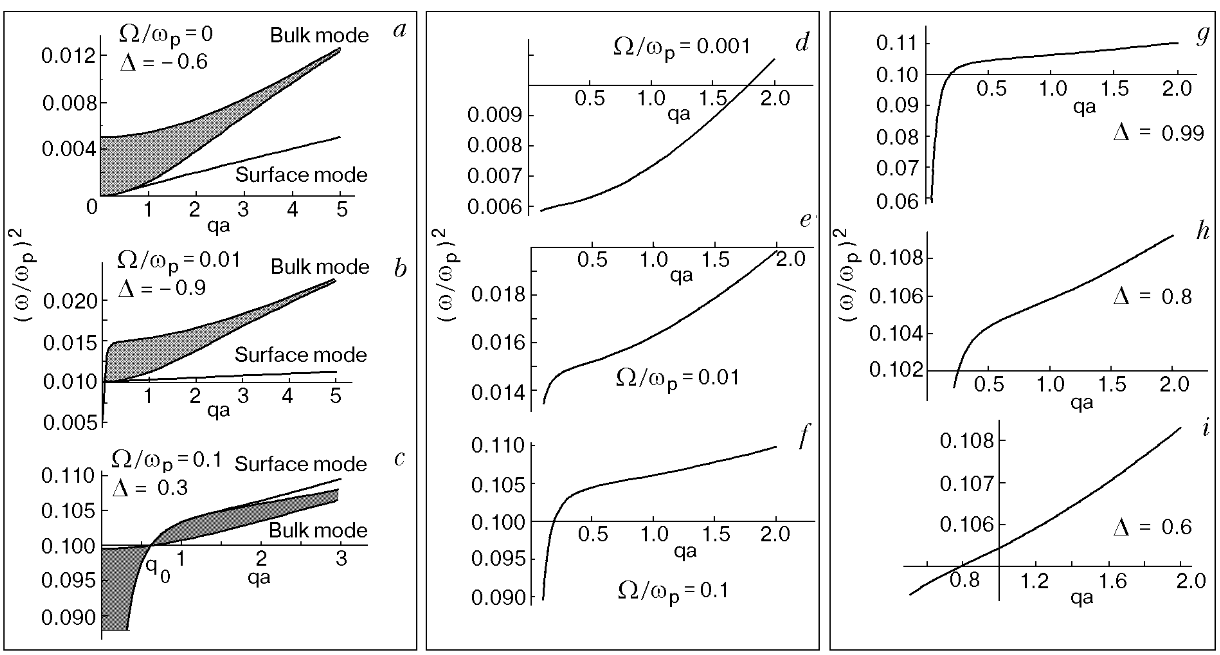

Fig. 1 The dispersion relation of the surface mode given by Eqs. (47), (49) and taken at $\omega_* = \omega_p a/c = 0.001$, $\sqrt{\varepsilon} = 10$ for different values of the parameters Δ and Ω/ω_p (*a–c*) (the darkened area denotes the bulk mode band determined by Eqs. (47) and (48), and q_0 marks the singular point of the bulk mode). The same at $\Delta = 0.99$ for three different values of the parameter Ω/ω_p (*d–f*) and at $\Omega/\omega_p = 0.1$ for three different values of the parameter Δ (*g–i*). ω_p is the plasma frequency; Ω stands for the cyclotron frequency; Δ is determined by Eq. (46).

Eqs. (47) and (49) and is shown in Fig. 1 (*a–i*) for different values of the parameters Δ and Ω. The gray area in Fig. 1 (*a–c*) marks the bulk wave band, which lies between its upper ($\omega_+(q) = \omega(q, ka = 0)$) and lower ($\omega_-(q) = \omega(q, ka = \pi)$) boundaries. The surface mode does exist only for $q > q^*$, where the threshold value q^* is given by the relation $q^*a = -\ln|\Delta|$. This relation follows immediately from Eq. (43) for $q_\omega \approx q$, which implies that the inequality $|\Delta e^{qa} + e^{-qa}| > 1 + \Delta$ should hold. When $\Delta > 0$ the surface mode goes above the bulk wave band, whereas for negative Δ the function $\omega_s(q, \Delta)$ continues below the bulk wave band.

Therefore, we see that two conditions are required for the surface mode propagation: (i) the dielectric constant outside the layered conductor, ε_0 , should differ from the corresponding quantity ε between the layers; (ii) the wave vector q should exceed the threshold value q^*. Figs. 1 (*d–f*) display the deformations of the surface wave dispersion with increasing external magnetic field. The dependence of $\omega_s(q, \Delta)$ on the parameter Δ is shown in Figs. 1 (*g–i*). As one can see in Figs. 1 (*a–c*), the width of the bulk mode band decreases with increasing qa, so that the upper, $\omega_+(q)$, and the lower, $\omega_-(q)$, bounds merge in the limit $qa \to \infty$. For finite but large $qa > 1$ the dispersion across the layers is negligible, since $R \approx 1$, and in this case $\omega(q, k)$ takes, according to Eqs. (47) and (48), the simple form

$$\omega(q, k) \simeq \left[\Omega^2 + \omega_p^2\left(\frac{1}{2\varepsilon}\right)qa\right]^{1/2}. \qquad (53)$$

Comparing this result with the Eq. (52), we arrive at the conclusion that in the region $qa >> 1$ the surface mode frequency exceeds the appropriate value of the bulk wave $\omega_s(q, \Delta) > \omega(q, k)$ for $\Delta > 0$ and goes below $\omega(q, k)$ for negative Δ. The dependence of $\omega(q, k)$ on k for different values of qa is shown in Figs. 2 (*a–f*). In the case of zero magnetic field $\Omega = 0$ the collective excitation of the system in question is a bulk plasmon whose upper, $\omega_+(q)$, and lower, $\omega_-(q)$, boundaries (given by Eq. (47) with $R \equiv R_+ = \coth(qa/2)$ and $R \equiv R_- = \tanh(qa/2)$, respectively) approach each other

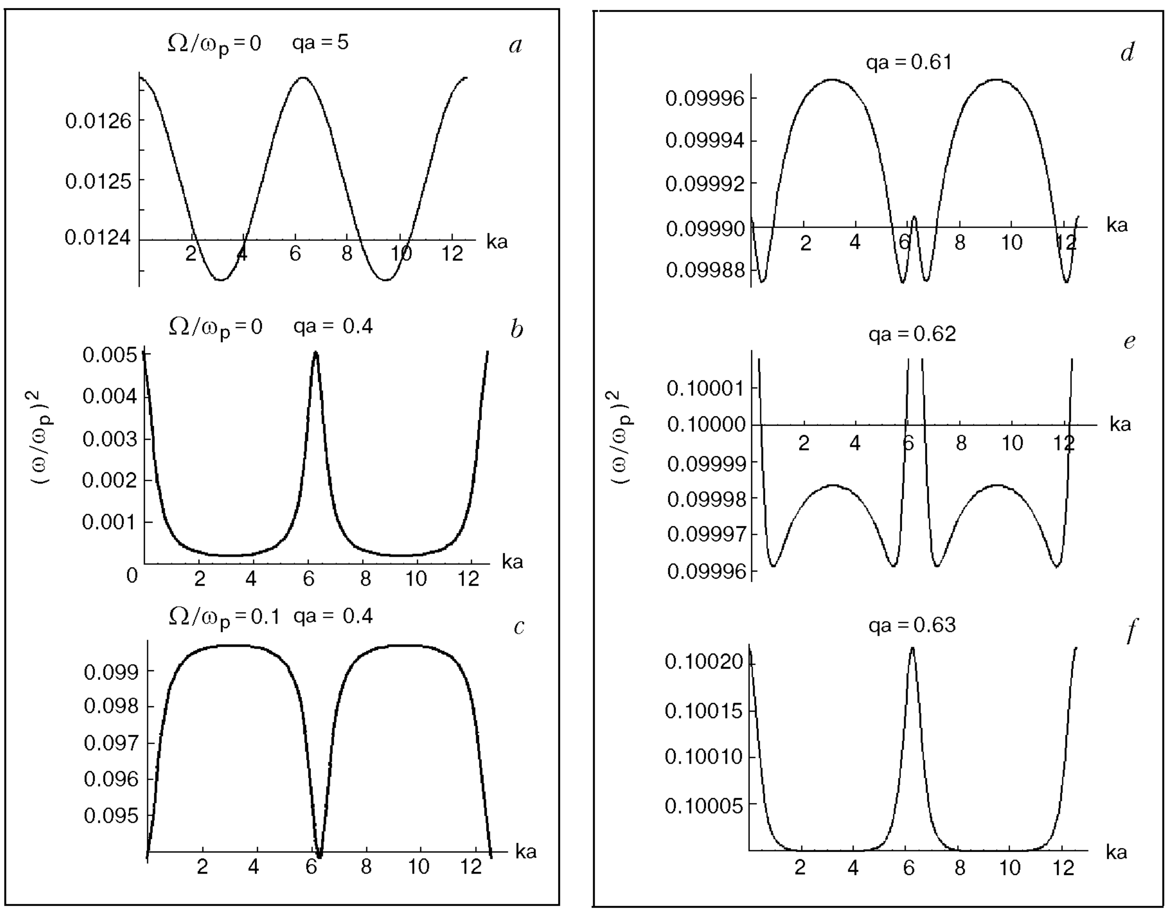

Fig. 2 The dispersion relation of the bulk mode given by Eqs. (47) and (48) and taken at $\omega_* = \omega_p a/c = 0.001$, $\sqrt{\varepsilon} = 10$ and $\Delta = 0.3$ in zero magnetic field for different values of the parameters qa and Ω/ω_p (*a–c*) and at $\Omega/\omega_p = 0.1$ for three different values of the parameter qa near the singular point q_0 of the bulk mode (*d–f*). Notation as in Fig. 1.

but never cross, as one can see in Figs. 1 (a–c). The evolution of the quantity $\omega(q, k)^2$ in this case is shown in Figs. 2 (a–c). In the case $\Omega = 0$, $qa = 5$ (see Fig. 2,a) the bulk mode is narrow, and $\omega(q, k)^2$ displays a sinelike behavior as a function of k. The band width becomes one order of magnitude wider at $qa = 0.4$ and the shape of the dispersion in Fig. 2,b becomes strongly non-sinusoidal. At nonzero magnetic field the function $\omega(q, k)^2$, shown in Fig. 2,c, differs in shape from that in Fig. 2,b taken at $\Omega = 0$. The physical reason for this difference is illustrated by Figs. 1,a and 1,c, from which we see that at $\Omega \neq 0$ the decrease in qa results in a change of the bulk transverse dispersion below some singular point, marked as q_0 in Fig. 1,c. At this point $\omega_+(q_0) = \omega_-(q_0)$, and below $q_0 = q_0(H)$ the upper and the lower boundaries swap: $\omega_+(q) < \omega_-(q)$. The equation for $q_0(H)$ in explicit form is

$$\omega_*^2\Omega^2 = \left(\frac{\omega_p}{2\sqrt{\varepsilon}}\right)^2 [(2q_0\, a)^2 + 4q_0\, a\omega_*^2 \coth(q_0\, a) + \omega_*^4]. \tag{54}$$

Analysis of this equation shows that it has a solution q_0 under the condition $\Omega > \omega_p/2\sqrt{\varepsilon}$. The function $\omega(q, k)^2$ experiences the most dramatic changes with respect to the variable k in the narrow vicinity of the singular point $q = q_0(H)$. These changes are illustrated by Figs. 2,d–f.

5. Summary and conclusions

We have given a transfer-matrix theory for the collective electromagnetic modes of a semi-infinite layered conductor subjected to a quantizing external magnetic field. We started from Eqs. (1)–(3), describing the electromagnetic field in a stack of conducting layers embedded in a dielectric matrix within a model which ignores the interlayer electron hopping and assumes neither periodicity of the layer stacking nor uniformity of the dielectric constant across the layers. To apply these equations to the case of a uniform layered conductor placed in the halfspace $Z > 0$ we first calculated Green's functions in this halfspace which, in a model where the dielectric constant $\varepsilon(z) = \varepsilon\theta(z) + \varepsilon_0\,\theta(-z)$, are given by Eqs. (9) and (14). Putting then these Green's functions into Eqs. (1), we reformulated the eigenvalue problem in the matrix form of Eq. (23) and introduced the transfer matrix by Eq. (24). This transfer matrix has a higher dimensionality (4×4) than the analogous transfer matrix (2×2) used before in Refs. 8,9 for studies of the plasma collective modes in layered electron gas. Within the transfer-matrix approach we then found dispersion relations for the bulk (Eq. (30)) and surface (Eqs. (34) and (35)) modes, valid for an arbitrary form of the $2D$ conductivity tensor of a layer placed in an external magnetic field. Since Eqs. (1) are written in terms of the field components at the layers it may create the wrong impression that our approach does not take into account the field dynamics between the conducting planes. To rule out this suspicion we gave an alternative derivation of the transfer matrix in the Appendix B which is based on Maxwell's equations between the layers and boundary conditions at the conducting planes.

The bulk modes have dispersion both within and across the layers and have been discussed earlier in Refs. 13,14. The surface mode exponentially damps into the bulk of the layered conductor and has no dispersion across the layers. Its dispersion relation along the layers is determined by two equations (41) and (42), while the damping decrement is given by Eq. (43). Generally, these equations are rather complicated to be solved analytically, but for a Drude-like conductivity tensor of the form given by Eqs. (44) and (45) for $\nu = 0$ and under the condition $qa \gg \sqrt{\varepsilon}\omega_*(\omega/\omega_p)$ the surface mode frequency $\omega_s = \omega_s(q, \Delta)$ is given analytically by Eqs. (47) and (49). The quantity ω_* is extremely small for real layered conductors (of the order of 10^{-4}–10^{-2}), so that the above inequality does not place severe restrictions on the magnitude of the wave vector qa. The corresponding calculations for the bulk, $\omega(q, k)$, and surface, $\omega_s(q, \Delta)$, modes are plotted in Figs. 1, 2 for different values of the parameter Δ (see Eq. (46)) and cyclotron frequency Ω. At zero magnetic field the bulk mode $\omega(q, k)$ given by Eqs. (47) and (48) becomes a well-known plasmon of a layered conductor, the bandwidth of which in respect to k grows narrower with increasing qa, as Fig. 1,a illustrates. The surface plasmon mode shown in Figs. 1,a–i lies below or above the bulk plasmon band, depending on the sign of the Δ, and starts at the threshold value wave vector $q^* = -(1/a)\ln|\Delta|$, as was first found in Ref. 7. In case of nonzero magnetic field a bulk collective mode in a layered conductor became a mixture of the helicon and plasmon, with a dispersion relation given by Eqs. (47) and (48). The corresponding surface mode $\omega_s(q, \Delta)$ is determined by Eqs. (47) and (49). It has the very same threshold q^* in q and continues below the bulk mode band for $\Delta < 0$ and above it for $\Delta > 0$ (see Figs. 1,a–c). The dependence of the shape of the surface mode dispersion

$\omega_s(q, \Delta)$ on the magnetic field Ω and parameter Δ is shown in Figs. 1(d-i). It is seen in these figures, as well as in Figs. 1,a–c, that $\omega(q, k)^2$ becomes a linear function of q at large values of the quantity qa. The appropriate asymptotic expressions for the surface and bulk waves in the limit $qa >> 1$ are given by Eqs. (52) and (53). From these equations it is clear seen that $\omega(q, k) > \omega_s(q, \Delta)$ for $\Delta < 0$ and $\omega(q, k) < \omega_s(q, \Delta)$ for $\Delta > 0$. According to Eq. (46), $q^* \to 0$ if $\varepsilon \to \varepsilon_0$, i.e., in the case when the optical densities of the left and right halfspaces are close in magnitude. For example, $q^*a \approx 0.10005$ for $\Delta = 0.99$, and $q^*a \approx 0.1053$ for $\Delta = 0.9$. In the limit $\omega_* << qa << 1$ (which holds if Δ close to unity) we have from Eqs. (47) and (49) the simple formula

$$\omega_s^2(q, \Delta) \approx \Omega^2 + \frac{\omega_p^2}{4\varepsilon}\left(\frac{1+\Delta}{\Delta}\right)[(1+\Delta) + qa(\Delta - 1)] \ . \tag{55}$$

Thus the surface mode has a gap at $qa << 1$ even if the cyclotron frequency (the external magnetic field) goes to zero. This is also seen in Fig. 1,d where the ratio Ω/ω_p is taken as small as 0.001. The numerical analysis shows a negligible deformation of the curve in Fig. 1,d for smaller values of the parameter Ω/ω_p , down to zero.

The bulk mode $\omega(q, k)$ with respect to the variable k is a periodic function with period $2\pi/a$ which has a different shape depending on the value of qa as shown in Figs. 2,a–f. The width of the bulk mode grows wider with decreasing qa. In an external magnetic field under the condition $\Omega > \omega_p /2\sqrt{\varepsilon}$ the bulk mode twists at some wave vector $q_0 = q_0(H)$, so that its upper bound $\omega_+(q) = \omega(q, ka = 0)$ becomes greater than the lower bound $\omega_-(q) = \omega(q, ka = \pi)$ for $q < q_0(H)$. This transmutation of the bulk mode band in an external magnetic field is seen especially clearly in Fig. 1,c. The shape of the bulk dispersion across the layers $\omega(q, k)$ experiences dramatic changes in the vicinity of the point $q = q_0(H)$, as is displayed in Figs. 2,d–f. The dependence of the bulk and surface modes frequencies on the distance between the layers a is given in fact (for fixed values of q and k) by Figs. 1 and 2, since these plots show the dependences of the above modes on qa and ka. The surface mode frequency in the limit $a \to \infty$ is given by Eq. (52), where one should take into account the plasma frequency dependence on a: $\omega_p^2 = 4\pi Ne^2/ma$ (N is the electron density per unit area of a 2D conducting sheet and m stands for the effective mass of the electron). The decrease of the plasma frequency in this limit also favors the appearance of the twisting point $q_0(H)$, since the inequality $\Omega > \omega_p /2\sqrt{\varepsilon}$ is satisfied at lower H. In the opposite limit $a \to 0$, the surface mode disappears because its wave vector threshold value $q^* \propto 1/a \to \infty$.

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Appendix A

In this Section we derive the wave equations (1) within the framework of a model of conducting planes embedded in a dielectric background. To this end we direct the z axis perpendicular to the layers and assume that a constant external magnetic field H is also directed along this axis. We suppose that the permeability of the substance between the layers is equal to unity, $\mu = 1$, and assume its dielectric constant, $\varepsilon = \varepsilon(z)$, to be a function of z.

Under these assumptions, Maxwell's equations, written in terms of the electric field $\mathbf{E}$,

$$\nabla\,(\mathrm{div}\,\mathbf{E}) - \Delta\mathbf{E} = -\frac{\varepsilon}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \frac{4\pi}{c^2}\frac{\partial\mathbf{J}}{\partial t} \ , \tag{A1}$$

after the substitution of the wave in the form

$$E_l = E_l(\mathbf{q}, z, \omega)\exp\,[i(\mathbf{q}\boldsymbol{\rho} - \omega t)] \ , \quad l = x, y, z \tag{A2}$$

take the form

$$-\mathbf{q}(\mathbf{q}\mathbf{E}_\perp) + i\mathbf{q}\left(\frac{\partial}{\partial z}E_z\right) + \left(q_\omega^2 - \frac{\partial^2}{\partial z^2}\right)\mathbf{E}_\perp = -\frac{4\pi i\omega}{c^2}\mathbf{J}_\perp \ , \tag{A3}$$

$$E_z = -\frac{1}{q_\omega^2}\frac{\partial}{\partial z}(i\mathbf{q}\mathbf{E}_\perp) \ , \tag{A4}$$

$$q_\omega^2(z) = q^2 - \frac{\omega^2}{c^2}\varepsilon(z) \ . \tag{A5}$$

Here $\boldsymbol{\rho}$, $\mathbf{q}$ and ω are the in-plane coordinate, the wave vector, and the frequency of the collective mode; $\mathbf{E}_\perp$ and $\mathbf{J}_\perp$ are the in-plane field and current, respectively.

Choosing $\mathbf{q}$ to be parallel to the y axis, we arrive at the following set of equations:

$$\left(\frac{\partial^2}{\partial z^2} - q_\omega^2\right)E_x = \frac{4\pi i\omega}{c^2}J_x \ , \tag{A6}$$

$$\left(\frac{\partial^2}{\partial z^2} - q_\omega^2\right) E_y + U(q,\omega,z)\frac{\partial}{\partial z} E_y = -\frac{4\pi i q_\omega^2}{\omega\varepsilon(z)} J_y , \tag{A7}$$

$$E_z = -\frac{iq}{q_\omega^2}\frac{\partial E_y}{\partial z} , \tag{A8}$$

$$U(q,\omega,z) = \left(\frac{q}{q_\omega(z)}\right)^2 \varepsilon^{-1}(z)\frac{\partial\varepsilon(z)}{\partial z} . \tag{A9}$$

Thus we see that all three components of the electric field are determined by the two equations (A7) and (A6), which can be rewritten in the form of Eqs. (1) with the help of the constitutive equation relating the in-plane current with the field components:

$$J_\alpha = \sum_{\beta,n} \sigma_{\alpha\beta}(\mathbf{q},\omega,H)\,\delta(z-z_n)\,E_\beta(\mathbf{q},\omega,z) . \tag{A10}$$

The δ functions in Eq. (A10) take into account that currents flow only within the conducting planes $z = z_n$, and $\sigma_{\alpha\beta}(\mathbf{q},\omega,H)$ stands for the conductivity tensor of a $2D$ layer in a perpendicular magnetic field. In this connection, note that only derivatives of the background dielectric constant enter Eq. (A9).

Appendix B

In this Appendix an alternative derivation for the transfer matrix and the dispersion relation (30) for the bulk mode is given. The method is based directly on the calculation of the electromagnetic field between the conducting layers and matching them with the appropriate boundary conditions at the layers. Equations (A6)–(A9) in the bulk of the layered conductor may be rewritten in the form

$$\left(\frac{\partial^2}{\partial z^2} - q_\omega^2\right) E_\alpha = \sum_{\beta,n} \delta(z-z_n)\tilde{\sigma}_{\alpha\beta} E_\beta , \tag{B1}$$

where

$$\tilde{\sigma}_{\alpha\beta} = -\frac{4\pi i\omega}{c^2}\sigma_{\alpha\beta}(\mathbf{q},\omega,H)V_{\alpha\beta} , \tag{B2}$$

$V_{\alpha\beta}$ is a matrix with the components $V_{11} = V_{12} = 1$, $V_{21} = V_{22} = -c^2q_\omega^2/\omega^2\varepsilon$. Writing the solution of Eq.(B1) between the nth and the neighboring layer in the form

$$E_\alpha(n) = C_\alpha(n)\,e^{-q_\omega(z-z_n)} + D_\alpha(n)\,e^{q_\omega(z-z_n)} \tag{B3}$$

and using the boundary conditions at the layer

$$E_\alpha(z_n+0) = E_\alpha(z_n-0) \tag{B4}$$

and

$$\frac{\partial}{\partial z}E_\alpha(z_n+0) - \frac{\partial}{\partial z}E_\alpha(z_n-0) = \sum_\beta \tilde{\sigma}_{\alpha\beta} E_\beta(z_n) , \tag{B5}$$

we have

$$\begin{pmatrix} C_\alpha(n+1) \\ D_\alpha(n+1) \end{pmatrix} = \sum_\beta \tilde{T}_{\alpha\beta}\begin{pmatrix} C_\beta(n) \\ D_\beta(n) \end{pmatrix}, \tag{B6}$$

$$\tilde{T}_{\alpha\beta} = \begin{pmatrix} (\delta_{\alpha\beta} + \hat{\sigma}_{\alpha\beta})\,e^{-q_\omega a} & \hat{\sigma}_{\alpha\beta}\,e^{q_\omega a} \\ -\hat{\sigma}_{\alpha\beta}\,e^{-q_\omega a} & (\delta_{\alpha\beta} - \hat{\sigma}_{\alpha\beta})\,e^{q_\omega a} \end{pmatrix} . \tag{B7}$$

Note that the transfer matrix $\tilde{T}_{\alpha\beta}$ in Eq. (B7) differs from $\hat{T}_{\alpha\beta}$ of Eq. (24) (because of the difference in definition of the coefficients $A_\alpha(n)$, $B_\alpha(n)$ in Eqs. (18) and (19) from $C_\alpha(n)$ and $D_\alpha(n)$ in Eq. (B3)). Nonetheless, $\mathrm{Tr}\,\tilde{T}_{\alpha\beta} = \mathrm{Tr}\,\hat{T}_{\alpha\beta}$, and the dispersion relation (29) remains the same in both approaches.

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