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Triplet superconducting proximity effect in nonhomogeneous magnetic materials

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We show that quantum spin fluctuations in inhomogeneous conducting ferromagnets drastically affect the Andreev reflection of electrons and holes at a ferromagnet–superconductor interface. As a result, a strong long-range proximity effect appears, associated with electron–hole spin triplet correlations and persisting on a length scale typical for nonmagnetic materials but anomalously large for ferromagnets. For applications, an important consequence is that this long-range proximity effect permits to create superconducting quantum intetference devices with the Josephson magnetic junction of an anomalous large length.

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1. Introduction

In recent years much attention has been paid to normal conductor–superconductor (N/S) structures (for a review, see, e.g., Ref. [1]). Transport of electric charge in such systems is much affected by the existence of an energy gap in the spectrum of elementary excitations in the superconductor. As a result of the existence of the gap, electronic elementary excitations which freely propagate in the nonsuperconducting material cannot penetrate into the superconductor to a sufficient distance if their energy ε (measured from the Fermi level ε_F) is less than the superconductor energy gap Δ. A correlated transferring of two electrons accompanied by their pairing inside the superconductor is the only mechanism that provides a direct transmission of the charge into the superconducting condensate that is the ground state of the superconductor. The abovementioned two-electron transfer may be considered in terms of the conventional scattering scheme as a process of an electron–hole transformation of excitations inside the normal conductor that takes place at the boundary with the superconductor. This scattering (which is known as Andreev reflection) couples the incident electron and the reflected hole in such a way that their spins are oriented in opposite directions and their energies ($\pm\varepsilon$) are symmetrically positioned with respect to the Fermi energy («Andreev's hybrid»). Such a two-electron correlation which arises at the boundary with the superconductor persists inside the normal conductor to a distance $\mathcal{L}_\varepsilon$ from the superconductor, $\mathcal{L}_\varepsilon = \min(\hbar/\Delta p, \sqrt{\hbar D/\Delta p v_F})$, where $\Delta p = \varepsilon/v_F$, v_F is the Fermi velocity, and D is the diffusion coefficient. The destruction of the phase coherence arises due to the difference between the momenta of the electron (ε/v_F) and hole ($-\varepsilon/v_F$) components in the Andreev hybrid. The other peculiar feature of the Andreev hybrid, which is that the electron and hole spins have opposite directions, makes for sensitivity of the Andreev correlation to a magnetic field H controlling the spin splitting $\Delta E = \mu_B H$ (μ_B is the Bohr magneton) of the electron and hole energies in the hybrid.

It becomes necessary to take this splitting into account for the case of a ferromagnet–superconductor structure, where the interaction of the quasiparticle spin with the magnetization is of an exchange character and hence can be extremely large. Conservation of the electron–hole symmetry (that is, the symmetric positioning of their energies with respect to the Fermi level) causes an additional difference

$\Delta p = I_0/v_F$ (I_0 is the exchange energy of the ferromagnet) in the momenta of the electron and hole components of the hybrid that drastically decreases the penetration length $\mathcal{L}_\varepsilon$ by orders of magnitude (in this case the penetration length is $L_{I_0} = \sqrt{\hbar D/I_0}$ [2]). Such a shortening of the proximity effect has been actually observed in magnetic materials [3–7]. On the other hand, measurements carried out in recent works [8–11] demonstrate a long-range proximity effect in magnetic materials that is in an obvious contradiction with the general considerations discussed above. It has been pointed out [9] that spin triplet fluctuations in the electron–hole correlations caused by the spin–orbit interaction and electron–impurity scattering [12] cannot (by two orders of magnitude) explain the large effect observed in [8–11].

The main message of this paper is that in magnetically inhomogeneous materials (such as multidomain ferromagnets (F), inhomogeneous «cryptoferromagnetic» states imposed by the superconductor [13], F/S interfaces inducing electronic spin–flip processes [14]), strong quantum fluctuations of the electron and hole spins make the proximity effect less sensitive to the spin selection rule that applies to Andreev reflections. As a result, a strong long-range, spin-triplet proximity effect in F/S structures persists on a length scale typical for *nonmagnetic* materials*. We estimate the conductance of such an F/S structure to be of the same order of magnitude as the conductance measured in experiments [8–11]. Additional experiments with intentionally introduced magnetic inhomogeneities are needed to check the predicted effect quantitatively.

A schematic illustration of Andreev reflection in the presence of a magnetic inhomogeneity is presented in Fig. 1. For convenience we consider the inhomogeneous magnetization of the material to be confined to a layer of finite width, close to the superconductor–ferromagnet interface. Such a layer serves as a magnetic spin-splitter for the incident electron (see Fig. 1). The composite scattering produced by the F/S boundary and the spin-splitter can then be separated into three scattering events: an incident electron with spin up crosses the inhomogeneous magnetic layer at point A and splits up into a coherent mixture of spin up ((e, ↑), see Fig. 1) and spin down (e, ↓) electronic states. These are subject to Andreev reflection at the F/S interface and are transformed into spin down (h, ↓) and spin up (h, ↑) hole states (see the dashed lines in Fig. 1). These two states encounter the magnetic

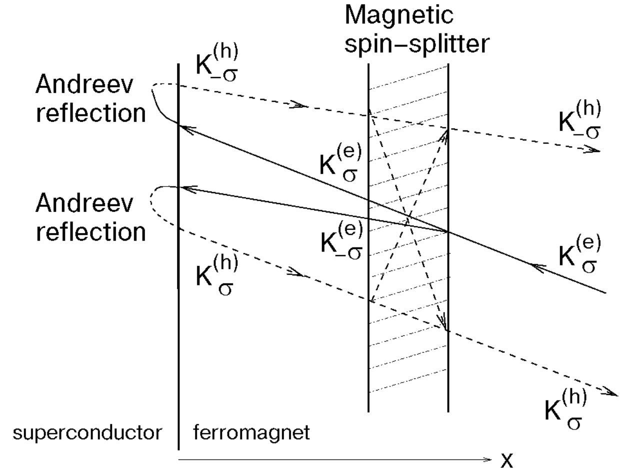

Fig. 1. Sketch of the composite scattering of an electron impinging on the interface between a magnetically inhomogeneous ferromagnet and a superconductor: Andreev reflection at the F/S interface is accompanied by spin-splitting scattering in the region of inhomogeneity (for clarity here assumed to be confined to a finite-width layer). As a result, the reflected hole is in a mixed state with both spin up and spin down components.

scatterer again at points B_1 and B_2, respectively, and experience further «spin-splitting». The final result of the composite scattering process is that the incoming electron is reflected in two hole-channels, one with spin up (h, ↑) and the other with spin down (h, ↓). One of the reflected hole channels has the same spin orientation as the incident electron.

Taking an alternative point of view one may consider the time reversed process when a Cooper pair propagates from the superconductor to the ferromagnet. Being in the singlet state at the moment of injection, the pair is then scattered into the triplet configuration by an inhomogeneously oriented magnetization in the ferromagnet. This singlet–triplet scattering is effective if the length scale of the inhomogeneity is of the order of L_{I_0}, the separation between the two Cooper pair electrons in the ferromagnet.

The above result of the two-channel magneto-Andreev scattering implies that the electron–hole correlation has a contribution that is unaffected by the magnetic exchange energy, which leads to a long-range «spin-triplet» proximity effect.

2. Formulation of the problem

We consider the conductance of a ferromagnet–superconductor structure schematically shown in Fig. 2 for a special case when magnetic spin scatter-

* A short formulation of this prediction was published as a Letter in [15]. Analogous prediction based on a somewhat different approach was simultaneously made in [16].

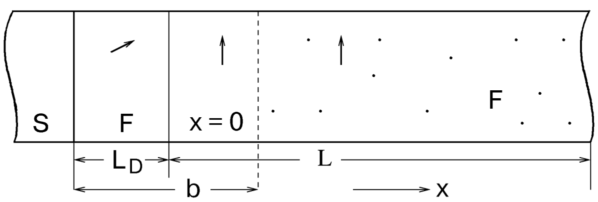

Fig. 2. Schematic view of an S/F structure with a magnetic domain wall at $x = 0$, a distance L_D from the S/F interface. Impurity scattering is assumed to occur out of the ballistic region (to the right of the vertical dashed line).

ing occurs within a distance, L_D , from the S/F interface that is shorter than the electronic mean free path l_0 (that is, $L_D << l_0$). This allows us to consider the Andreev reflection at the interface to occur with unit probability and to describe the magnetic spin scattering using the semiclassical Eilenberger equation [17], which can be readily solved in the ballistic transport regime. Proper boundary conditions for matching the solution of the Eilenberger equation to the appropriate solution in the diffusive part of the ferromagnet can also be formulated in this model and used to solve the Usadel equation [18], which is the appropriate equation in the diffusive transport regime. In this way, the excess conductance of the F/S boundary can be calculated.

Solving the model problem described, we find that a new type of superconducting ordering, corresponding to the triplet spin correlations

$$\mathcal{F}_{\sigma,\sigma}(\mathbf{r};\varepsilon) = \int \langle \psi_\sigma(\mathbf{r}, t)\psi_\sigma(\mathbf{r}, 0)\rangle \exp\left(\frac{i\varepsilon t}{\hbar}\right) dt \quad (1)$$

(here $\sigma = \uparrow, \downarrow$), is the source of the proximity effect at distances of order $\mathcal{L}_T >> L_{I_0}$, $\mathcal{L}_T \equiv \mathcal{L}_\varepsilon\big|_{\varepsilon=kT}$.

The Hamiltonian describing the system is written as follows.

$$\hat{H} = \int d\mathbf{r}\, \{\psi^+_\alpha(\mathbf{r})(\hat{\mathbf{p}}^2/2m - eV(\mathbf{r}))\, \psi_\alpha(\mathbf{r}) +$$

$$+ \Delta(\mathbf{r})\psi^+_\uparrow(\mathbf{r})\psi^+_\downarrow(\mathbf{r}) + \Delta^*(\mathbf{r})\psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r})\} + \mathbf{h}(\mathbf{r})\sigma_{\alpha\beta}\psi^+_\alpha(\mathbf{r})\psi_\beta(\mathbf{r}) \quad (2)$$

where $V(\mathbf{r})$ is the electrical potential; $\hat{\sigma}$ are Pauli matrices; $\alpha, \beta = (\uparrow, \downarrow)$ and summation with respect to double indices is assumed; $\mathbf{h}(\mathbf{r}) = I_0\, \mathbf{e}(\mathbf{r})$ is the inhomogeneous exchange energy in the ferromagnet ($\mathbf{e}(\mathbf{r})$ is a unit vector along the magnetization at point $\mathbf{r}$); the superconductor energy gap Δ and the ferromagnet exchange field $\mathbf{h}$ have nonzero values in complementary space regions: $\Delta \neq 0$, $\mathbf{h} = 0$ in the superconductor and $\mathbf{h} \neq 0$, $\Delta = 0$ in the ferromagnet.

We start with the 4×4 correlation functions

$$\hat{G}^<(1, 1') = i\langle \hat{\psi}^+(1)\hat{\psi}(1)\rangle \; ; \quad \hat{G}^>(1, 1') = - i\langle \hat{\psi}(1)\hat{\psi}^+(1)\rangle \quad (3)$$

where $\hat{\psi}^+(1) = (\psi^+_\uparrow(1), \psi_\downarrow(1), \psi^+_\downarrow(1), \psi_\uparrow(1))$ is the Nambu pseudo-spinor field (its variable is $1 \equiv \equiv (\mathbf{r}, t)$). Using the correlation functions Eq. (3) in the standard way (see, e.g., the review article [19]), one constructs the 8×8 Green's function in the 2×2 Keldysh and 4×4 Nambu spaces as follows:

$$\check{G} = \begin{pmatrix} \hat{G}^R & \hat{G}^K \\ \hat{0} & \hat{G}^A \end{pmatrix} \quad (4)$$

where $\check{G}^{R,A,K}$ are retarded (R), advanced (A), and Keldysh (K) 4×4 matrix Green's functions which include both the singlet $\hat{G}_{\sigma,-\sigma}$ and triplet $\hat{G}_{\sigma,\sigma}$ components of the normal as well as anomalous Green's functions:

$$\hat{G}^R = \Theta\,(t_1 - t_{1'})(\hat{G}^>(1,1') - \hat{G}^<(1, 1'))$$

$$\hat{G}^A = - \Theta\,(t_{1'} - t_1)(\hat{G}^>(1,1') - \hat{G}^<(1, 1')) \quad (5)$$

$$\hat{G}^K = \hat{G}^>(1,1') + \hat{G}^<(1, 1')\,.$$

Using Eqs. (2)–(5) one gets the Eilenberger equation for the matrix quasi-classical Green's function in the Wigner representation

$$\check{g}\,(\mathbf{n}, \mathbf{R}; \varepsilon) = (i/\pi)\check{\tau}_3 \int d\xi \check{G}\,(\mathbf{p}, \mathbf{R}; \varepsilon)\,, \quad (6)$$

where the 8×8 matrix $\check{\tau}_3$ is represented in a compact notation of the tensorial product of Pauli matrices as $\check{\tau}_3 = \hat{\sigma}_0 \otimes \hat{\sigma}_z \otimes \hat{\sigma}_0$; the integrand $\check{G}\,(\mathbf{p}, \mathbf{R}; \varepsilon)$ is Fourier-transformed Green's function Eq. (4) with respect to the coordinate and time differences; the space variable in the center-of-mass system is $\mathbf{R} = \mathbf{r} + \mathbf{r}'$; vector $\mathbf{n}$ is the unit vector along the momentum $\mathbf{p}$, and $\xi = p^2/2m - \varepsilon_F$. For reference we write out $\hat{g}^\alpha$ in full as follows:

$$\hat{g}^{\,\alpha} \equiv \begin{pmatrix} g^\alpha_{\uparrow\uparrow} & f^\alpha_{\uparrow\downarrow} & g^\alpha_{\uparrow\downarrow} & f^\alpha_{\uparrow\uparrow} \\ \bar{f}^\alpha_{\downarrow\uparrow} & \bar{g}^\alpha_{\downarrow\downarrow} & \bar{f}^\alpha_{\downarrow\downarrow} & \bar{g}^\alpha_{\downarrow\uparrow} \\ g^\alpha_{\downarrow\uparrow} & f^\alpha_{\downarrow\downarrow} & g^\alpha_{\downarrow\downarrow} & f^\alpha_{\uparrow\downarrow} \\ \bar{f}^\alpha_{\uparrow\uparrow} & \bar{g}^\alpha_{\uparrow\downarrow} & \bar{f}^\alpha_{\uparrow\downarrow} & \bar{g}^\alpha_{\uparrow\uparrow} \end{pmatrix} \quad (7)$$

where $\alpha = R, A, K$. The Eilenberger equation for $\check{g}(\mathbf{n}, \mathbf{R}; \varepsilon)$ reads

$$iv_F\mathbf{n}\frac{\partial}{\partial\mathbf{R}}\check{g} + [\varepsilon\check{\tau}_3 + \check{\Delta} + i\check{\sigma} - \check{h}, \check{g}] = 0 , \quad (8)$$

where $[\check{A}, \check{B}] = \check{A}\check{B} - \check{B}\check{A}$; the impurity induced self-energy $\check{\sigma}$ in the Born approximation is

$$\check{\sigma} = \frac{\pi}{2t_0}\int\frac{d\mathbf{n}}{4\pi}\check{g}(\mathbf{n}, \mathbf{R}, \varepsilon) , \quad (9)$$

t_0 is the impurity scattering time; the pairing potential $\check{\Delta}$ that determines the electron–hole correlations and the operator $\check{h} = \check{h}_\perp + \check{h}_z$ that describes the effect of the inhomogeneous magnetic moment on the spins of electrons and holes can be written as

$$\check{\Delta} = \hat{\sigma}_0 \otimes \hat{\sigma}_3 \otimes (\Delta^*\hat{\sigma}_- - \Delta\hat{\sigma}_+); \qquad \hat{\sigma}_\pm = \hat{\sigma}_1 \pm \hat{\sigma}_2 ;$$

$$\check{h}_\perp = \hat{\sigma}_0 \otimes (h_x\hat{\sigma}_1 \otimes \hat{\sigma}_0 + h_y\hat{\sigma}_2 \otimes \hat{\sigma}_0); \quad (10)$$

$$\check{h}_z = h_z\hat{\sigma}_0 \otimes \hat{\sigma}_3 \otimes \hat{\sigma}_3 .$$

In the ferromagnet at distances much greater than the free path length l_0 (the dirty limit) the Eilenberger equation (8) reduces to the Usadel equation for the symmetric part of the Green's function $\check{G} \equiv \langle\check{g}\rangle$ ($\langle\ldots\rangle$ denotes an average over the directions of electron/hole momenta):

$$\check{G}^\alpha \equiv \begin{pmatrix} G^\alpha_{\uparrow\uparrow} & F^\alpha_{\uparrow\downarrow} & G^\alpha_{\uparrow\downarrow} & F^\alpha_{\uparrow\uparrow} \\ \overline{F}^\alpha_{\downarrow\uparrow} & \overline{G}^\alpha_{\downarrow\downarrow} & \overline{F}^\alpha_{\downarrow\downarrow} & \overline{G}^\alpha_{\downarrow\uparrow} \\ G^\alpha_{\downarrow\uparrow} & F^\alpha_{\downarrow\downarrow} & G^\alpha_{\downarrow\downarrow} & F^\alpha_{\uparrow\downarrow} \\ \overline{F}^\alpha_{\uparrow\uparrow} & \overline{G}^\alpha_{\uparrow\downarrow} & \overline{F}^\alpha_{\uparrow\downarrow} & \overline{G}^\alpha_{\uparrow\uparrow} \end{pmatrix} \quad (11)$$

This equation reads as follows:

$$D\frac{d}{dx}\left(\check{G}\frac{d\check{G}}{dx}\right) + [(\varepsilon\check{\tau}_3 - \check{I}_0), \check{G}] = 0 , \quad (12)$$

where $\check{I}_0 = I_0\hat{\sigma}_0 \otimes \hat{\sigma}_3 \otimes \hat{\sigma}_3$; when writing Eq. (12) we took into account the fact that at distances $x \gtrsim l_0$ from the boundary the ferromagnet is assumed to be homogeneous with the magnetic moment parallel to the z axis ($h_z = I_0$). Here and below we also assume all quantities in the structure to vary only along the x axis which is perpendicular to the F/S interface.

3. Boundary conditions for the Usadel equation

In order to find boundary conditions for the Usadel equation in the diffusive region we solve the Eilenberger equation (8) at distances $x \lesssim b << l_0$ (see Fig. 2), where the impurity scattering term ($\propto \check{\sigma}$) is negligible (we assume the superconductor to be pure as well; that is, $\xi_0 = \hbar v_F/|\Delta| << l_0$). In this case Eq. (8) reduces to the following linear equation:

$$iv_Fn_x\frac{d}{dx}\check{g} + [(\varepsilon\check{\tau}_3 + \check{\Delta}(x) - \check{h}(x)), \check{g}] = 0 . \quad (13)$$

Deep inside the superconductor ($x \to -\infty$) we use the conventional boundary conditions for the Eilenberger equation (8):

$$g_{\sigma,\sigma} = \frac{\varepsilon}{\sqrt{\varepsilon^2 - |\Delta|^2}} , \quad f_{\sigma,-\sigma} = \frac{\Delta}{\sqrt{\varepsilon^2 - |\Delta|^2}} ,$$

$$\overline{f}_{\sigma,-\sigma} = -\frac{\Delta^*}{\sqrt{\varepsilon^2 - |\Delta|^2}} , \quad \overline{g}_{-\sigma,-\sigma} = -\frac{\varepsilon}{\sqrt{\varepsilon^2 - |\Delta|^2}} , \quad (14)$$

$$g_{\sigma,-\sigma} = \overline{g}_{\sigma,-\sigma} = f_{\sigma,\sigma} = \overline{f}_{\sigma,\sigma} = 0 .$$

As we consider the case that the transparency of the superconductor–ferromagnet interface is equal to 1 (that is the Andreev reflection takes place in the absence of the normal reflection) the boundary condition at the F/S interface $x = -L_D$ (see Fig. 2) is the continuity of Green's functions, that is

$$\check{g}(-L_D - 0) = \check{g}(-L_D + 0) . \quad (15)$$

We solve the ballistic linear Eilenberger equation (13) in the superconductor, where $\mathbf{h} = 0$, $\Delta = \Delta_0$, and in the ferromagnet, where $\Delta = 0$ $\mathbf{h} = \mathbf{h}(x)$, matching the solutions at the F/S interface.

In order to find the solution in the ferromagnet one needs to know the detailed character of the magnetic inhomogeneity. A quantitative theory can be formulated only in case the magnetic structure is known in the experiment of interest. In the absence of any precise information about the magnetic structure of the samples used in existing experiments, we turn to illustrative examples of magnetic disorder and restrict ourselves to making only qualitative comparisons with experiments. We will consider two such examples.

1. The spin-splitting magnetic scattering is due to a multidomain structure with the magnetizations in the magnetic domain near the F/S interface

(L_D region in Fig. 2) and the rest of the ferromagnet being collinear but of opposite directions; the width L_{DW} of the domain wall between them is small compared with the ballistic magnetic length $L_h = \hbar v_F / I_0$. In this case the spin-flip takes place inside the domain wall with a probability amplitude proportional to the parameter $\lambda_D = L_{DW}/L_h$.

2. The magnetizations in the magnetic domain near the F/S interface and the rest of the ferromagnet are noncollinear, with the domain wall between them of a negligible small width. In this case the spin-slip takes place due to Rabi oscillations in the L_D region, the probability amplitude of that being proportional to the noncollinearity of the magnetizations in the neighboring domains $\lambda_R = \sqrt{h_x^2 + h_y^2}/I_0$.

Inserting dimensionless variables into Eq. (13), one sees that solutions of this equation in the ferromagnet region are controlled by the parameter λ_D for case 1 and by λ_R for case 2; this allows us to develop a perturbation theory in $\lambda_D << 1$ and $\lambda_R << 1$. Therefore, while solving this equation inside the ferromagnet we assume the probability of the spin-flip scattering to be small.

For both cases, rather simple but cumbersome perturbation-theory calculations show that far inside the ferromagnet, where the magnetization is already homogeneous but one is still in the ballistic region (at the distance b in Fig. 2), the retarded and advanced parts of the Green's function $\hat{g}^{(\alpha)}$, $\alpha = (R, A)$ can be written as follows:

$$\hat{g}_a^{(\alpha)}(b) = \hat{\tau}_a \tilde{g}_s^{\alpha}(b) + \hat{\tau}_s \tilde{g}_a^{\alpha}(b) + (\hat{\tau}_s - \hat{1}) \operatorname{sign} p_x$$
$$\hat{g}_s^{(\alpha)}(b) = \hat{\tau}_s \tilde{g}_s^{\alpha}(b) + \hat{\tau}_a \tilde{g}_a^{\alpha}(b) + \hat{\tau}_a \operatorname{sign} p_x \tag{16}$$

where $\hat{g}_{a,s}^{(\alpha)} = \hat{g}^{(\alpha)}(p_x) \mp \hat{g}^{(\alpha)}(-p_x)$ are the antisymmetric and symmetric parts of the Green's functions;

$$\tilde{g}_{a,s}^{\alpha} = \begin{pmatrix} g_{\uparrow\uparrow}^{(a,s)} & 0 & g_{\uparrow\downarrow}^{(a,s)} & 0 \\ 0 & \bar{g}_{\downarrow\downarrow}^{(a,s)} & 0 & \bar{g}_{\uparrow\downarrow}^{(a,s)} \\ g_{\downarrow\uparrow}^{(a,s)} & 0 & g_{\downarrow\downarrow}^{(a,s)} & 0 \\ 0 & \bar{g}_{\uparrow\downarrow}^{(a,s)} & 0 & \bar{g}_{\uparrow\downarrow}^{(a,s)} \end{pmatrix}$$

where $g_{\sigma,\sigma'}^{(a,s)} = ½ (g_{\sigma\sigma'}(p_x) \pm g_{\sigma\sigma'}(-p_x))$; for the case $\varepsilon << |\Delta|$ the antisymmetric $\hat{\tau}_a$ and symmetric $\hat{\tau}_s$ matrices (which match the normal and anomalous components of the Green's function $\hat{g}^{\alpha}(b; p_x)$ are as follows:

$$\hat{\tau}_a = i \operatorname{sign} p_x \begin{pmatrix} 0 & t_a \exp(i\varphi) & 0 & r_a \exp(i\varphi) \\ t_a \exp(-i\varphi) & 0 & -r_a \exp(-i\varphi) & 0 \\ 0 & -r_a^* \exp(i\varphi) & 0 & -t_a \exp(-i\varphi) \\ r_a^* \exp(-i\varphi) & 0 & -t_a \exp(-i\varphi) & 0 \end{pmatrix} \tag{17}$$

and

$$\hat{\tau}_s = \begin{pmatrix} 1 & t_s \exp(i\varphi) & 0 & r_s \exp(i\varphi) \\ -t_s \exp(-i\varphi) & 1 & r_s \exp(-i\varphi) & 0 \\ 0 & -r_s^* \exp(i\varphi) & 1 & t_s \exp(i\varphi) \\ -r_s^* \exp(-i\varphi) & 0 & -t_s \exp(-i\varphi) & 1 \end{pmatrix} . \tag{18}$$

$t_{sf} = it_a + t_s$ and $r_{sf} = ir_a + r_s$ are the probability amplitudes for an electron incident on the magnetically inhomogeneous region to be reflected back as a hole with the same and with the opposite direction of its spin, respectively ($|t_{sf}|^2 + |r_{sf}|^2 = 1$; see Fig. 1).

For the domains with collinear magnetization (case 1) one has

$$t_a \approx 1 - \frac{1}{2}\left(\frac{\pi L_{DW}}{L_h n_x}\right)^2 , \quad r_a \approx 0 ; \; t_s \approx 0 , \; r_s = \frac{\pi L_{DW}}{L_h |n_x|} . \tag{19}$$

For the domains with noncollinear magnetization (case 2) one has

$$t_a = -\cos\frac{2L_D}{|n_x|L_h}\ , \quad r_a \simeq 0\ ; \quad t_s = \frac{h_z^{(0)}}{I_0}\sin\frac{2L_D}{|n_x|L_h}\ ,$$

$$r_s = -\frac{h_+^{0*}}{I_0}\sin\frac{2L_D}{|n_x|\,L_h} \qquad (20)$$

(Eqs. (19) and (20) are written for $1 \geq |n_x| >> \pi L_{DW}/L_h$).

Equation (16) together with Eqs. (19), (20) show that the spin-flip scattering at the magnetization inhomogeneity accompanied by the Andreev reflection at the F/S interface produces a new triplet order parameter Eq. (1) and a new normal singlet correlation function $G_{\sigma,-\sigma}$ proportional to the spin-flip probability amplitude r_{sf} . In contrast to the conventional singlet order parameter, this triplet order parameter does not decay exponentially at distances from the F/S interface greater than the magnetic length L_{I_0} in the diffusive ferromagnet. This fact can be proved for two cases: 1) $l_0 << L_h$, and 2) $l_0 >> L_h$. In case 1) one neglects the terms quadratically small in $|r_{sf}| << 1$ in the Usadel equation (12) (that is, if one neglects the terms quadratic in $F_{\sigma,\sigma}$ and $G_{\sigma,-\sigma}$ and their derivatives). In this case, for the «usual» components $F^{\alpha}_{\sigma,-\sigma}$ and $G^{\alpha}_{\sigma,\sigma}$, one gets the conventional Usadel equation that shows exponential decay of $F^{\alpha}_{\sigma,-\sigma}$ at distances greater than L_{I_0} [2]. As for the $F^{\alpha}_{\sigma,\sigma}$ components, the equations for them show a slow variation of these components at such distances because these equations have no terms proportional to I_0 . The latter is a mathematical manifestation of the fact that these correlation functions are associated with such a scattering process under which the incident electron is transformed into a hole without changing the direction of the spin, and hence this electron–hole transformation requires no change in the magnetic energy (I_0) of the quasiparticles, as is qualitatively explained in the Introduction.

In case 2) ($l_0 >> L_h$) at distances $x >> l_0$ the Green's functions behave in the same way. In order to see this we start with solving the Eilenberger equation at distances $L_D << x << l_0$, where the equation is linear with constant coefficients. It is straightforward to see that the Green functions $F^{\alpha}_{\sigma,-\sigma} \propto \exp\,(ix/(L_h n_x))$, $G^{\alpha}_{\sigma,-\sigma} \propto \exp\,(ix/(L_h n_x))$, and hence they are rapidly oscillating functions of the momentum direction n_x at distances $x >> L_h$. This means that the averaging with respect to the momentum direction results in their decay proportional to $L_h/x << 1$ [2]. As to the Green's functions with the same direction of the spin variables ($G^{\alpha}_{\sigma,\sigma}$ and $F^{\alpha}_{\sigma,\sigma}$), they are slowly varying functions of the momentum direction, and they survive at such distances. Neglecting the rapidly oscillating components of the Green's function $\langle \hat{g}^{(\alpha)} \rangle$ averaged over the momentum direction (which are small in parameter $L_h/x << 1$), one finds that only the triplet components are nonzero:

$$\langle \hat{g}^{(\alpha,\,\mathrm{tr})} \rangle \equiv \hat{G}^{(\alpha,\,\mathrm{tr})} = \begin{pmatrix} G^{(\alpha)}_{\uparrow\uparrow} & 0 & 0 & F^{(\alpha)}_{\downarrow\downarrow} \\ 0 & \overline{G}^{(\alpha)}_{\downarrow\downarrow} & \overline{F}^{(\alpha)}_{\downarrow\downarrow} & 0 \\ 0 & F^{(\alpha)}_{\downarrow\downarrow} & G^{(\alpha)}_{\downarrow\downarrow} & 0 \\ \overline{F}^{(\alpha)}_{\uparrow\uparrow} & 0 & 0 & \overline{G}^{(\alpha)}_{\uparrow\uparrow} \end{pmatrix} . \qquad (21)$$

Using Eq. (21), one sees that nonlinear Eilenberger equation (8) splits into two sets of equations for the slowly varying components of the Green's function and the rapidly oscillating ones in the region $L_h/x << 1$. The latter are not of interest to us as at distances $x \geq l_0$ they decay exponentially; the matrix Eilenberger equation for the slowly varying components is as follows:

$$iv_F\mathbf{n}\,\frac{\partial}{\partial \mathbf{R}}\,\check{g}^{\mathrm{tr}} + \left[\varepsilon\check{\tau}_3 + \frac{i}{t_0}\,\langle \check{g}^{\mathrm{tr}} \rangle, \check{g}^{\mathrm{tr}}\right] = 0\ , \qquad (22)$$

where the reduced matrix Green function (see Eq. (7)) is

$$\check{g}^{(\alpha,\,\mathrm{tr})} = \begin{pmatrix} g^{(\alpha)}_{\uparrow\uparrow} & 0 & 0 & f^{(\alpha)}_{\downarrow\downarrow} \\ 0 & \overline{g}^{(\alpha)}_{\downarrow\downarrow} & \overline{f}^{(\alpha)}_{\uparrow\uparrow} & 0 \\ 0 & f^{(\alpha)}_{\downarrow\downarrow} & g^{(\alpha)}_{\downarrow\downarrow} & 0 \\ \overline{f}^{(\alpha)}_{\uparrow\uparrow} & 0 & 0 & \overline{g}^{(\alpha)}_{\uparrow\uparrow} \end{pmatrix} \qquad (23)$$

In the dirty limit, Eq. (22) reduces to a set of Usadel equations for the Green's functions $\check{G}^{(\alpha,\mathrm{tr})}$ (see Eq. (21)) that reads

$$D\,\frac{d}{dx}\left(\check{G}^{(\mathrm{tr})}\,\frac{d\check{G}^{(\mathrm{tr})}}{dx}\right) + [\varepsilon\check{\tau}_3,\ \check{G}^{(\mathrm{tr})}] = 0\ . \qquad (24)$$

From the above considerations it follows that Eq. (22) and Eq. (24) are valid for the both cases $l_0 << L_h$ and $l_0 >> L_h$.

We obtain the boundary conditions for Eq. (24) at distances from the F/S interface of the order of l_0 ($l_0 >> L_D$) for the case $\hbar v_F/\varepsilon >> l_0$ that permits us to neglect the term proportional to ε in the Eilenberger equation (22) and rewrite it as the following equation [20]:

$$v_F t_0 \mathbf{n} \frac{\partial}{\partial \mathbf{R}} \check{g}_a^{\rm tr} = \check{g}_a^{\rm tr} \langle \check{g}_s^{\rm tr} \rangle - \check{g}_s^{\rm tr} \langle \check{g}_a^{\rm tr} \rangle \quad , \qquad (25)$$

where

$$\check{g}_{s(a)}^{\rm tr} = \frac{1}{2} (\check{g}^{\rm tr}(p_x) \pm \check{g}^{\rm tr}(-p_x)) \ .$$

Averaging Eq. (25) over the momentum direction one sees $C = \langle n_x \check{g}^{\rm tr} \rangle$ does not depend on x [20]. Using this fact, Eqs. (16), (18), and (21), and the relation $\check{g}^{\rm tr} = l_0 \check{G}^{(\rm tr)} \nabla \check{G}^{(\rm tr)}$ which couples isotropic Usadel functions and the anisotropic one at distances greater than l_0 , one gets the desired effective boundary conditions for the Usadel equation (24) at $x \approx 0$ (that is in the vicinity of the magnetically inhomogeneous region adjacent the F/S interface) as follows:

$$\left. \frac{d}{dx} F_{\sigma,\sigma}^{(\alpha,\rm tr)} \right|_{x=0} = \sigma \frac{v_F}{D} e^{i\varphi} \langle |n_x| \, r^s \rangle \ . \qquad (26)$$

The exchange energy I_0 does not appear in Eqs. (22), (24) as these equations contain only triplet normal and anomalous Green's functions: formally this set of equations is the same as for a nonmagnetic conductor–superconductor diffusive structure if one changes the triplet anomalous Green's functions to the singlet ones and uses the boundary conditions (26). From here it obviously follows that the spin-flip scattering due to magnetic inhomogeneity accompanied by Andreev reflection produces a new (triplet) order parameter (1) (see also Eq. (7)) that decays at distances from F/S interface of the same order of magnitude as in nonmagnetic metal–superconductor structures $L_\varepsilon = \sqrt{\hbar D/\varepsilon}$ producing a long-range proximity effect in the ferromagnet. In the next Section we solve the Usadel equation (24) using boundary conditions (26) and find the conductance of such a structure.

4. Solution of the effective Usadel equation and the conductance of the structure

The current flowing through the structure under consideration can be written as follows (see, e.g., [22,23])

$$j = \frac{\sigma_N}{2e} \int \mathrm{Tr}\, \hat{\hat{\sigma}}_z \left(\hat{G}^{(R,\rm tr)} \frac{d\hat{G}^{(K,\rm tr)}}{dx} + \hat{G}^{(K,\rm tr)} \frac{d\hat{G}^{(A,\rm tr)}}{dx} \right) d\varepsilon \qquad (27)$$

where σ_N is the conductivity of the normal metal; $\hat{\hat{\sigma}}$ is a 4×4 matrix $\hat{\sigma}_0 \otimes \hat{\sigma}_z$.

According to the relationship [17,21]

$$\check{G}^{(\rm tr)} \check{G}^{(\rm tr)} = \check{1} \qquad (28)$$

the Keldysh function $\hat{G}^{(K,\rm tr)}$ reads

$$\hat{G}^{(K,\rm tr)} = \hat{G}^{(R,\rm tr)} \hat{q} - \hat{q} \hat{G}^{(A,\rm tr)} \qquad (29)$$

where $\hat{q}$ is a diagonal matrix

$$\hat{q} = \begin{pmatrix} q_{\uparrow 1} & 0 & 0 & 0 \\ 0 & q_{\downarrow 2} & 0 & 0 \\ 0 & 0 & q_{\downarrow 3} & 0 \\ 0 & 0 & 0 & q_{\uparrow 4} \end{pmatrix}, \qquad (30)$$

while the components of retarded and advanced Green's functions (21) satisfy the relations

$$G_{\sigma,\sigma}^{(\gamma,\rm tr)} G_{\sigma,\sigma}^{(\gamma,\rm tr)} + F_{\sigma,\sigma}^{(\gamma,\rm tr)} \overline{F}_{\sigma,\sigma}^{(\gamma,\rm tr)} = 1 \ ; \qquad G_{\sigma,\sigma}^{(\gamma,\rm tr)} = - \overline{G}_{\sigma,\sigma}^{(\gamma,\rm tr)} \qquad (31)$$

($\gamma = R,A$). Matrix $\hat{q}$ together with $\hat{G}^{(R,\rm tr)}$ satisfies matrix equation (24).

Following the reasoning of Ref. [1] and using Eqs. (27)–(30), one can rewrite Eq. (27) for the current as follows:

$$j = \frac{\sigma_N}{32} \int d\varepsilon \sum_\sigma (f_\sigma(L) - f_\sigma(0)) \frac{1}{m_\sigma(L)} \ , \qquad (32)$$

where $f_\sigma(x)$ is the distribution function for electrons with the spin $\sigma = (\uparrow, \downarrow)$, and

$$m_\sigma(\varepsilon) = \int_0^L \frac{dx}{1 - G_\sigma^{(R,\rm tr)} G_\sigma^{(A,\rm tr)} - F_\sigma^{(R,\rm tr)} F_\sigma^{(A,\rm tr)}} \ . \qquad (33)$$

Therefore, the current is determined by $\hat{G}^{(R,\rm tr)}$, the triplet Green's functions of which for normal pairing $G_{\sigma,\sigma}^R$, $\overline{G}_{\sigma,\sigma}^R$ and anomalous pairing $F_{\sigma,\sigma}^R$, $\overline{F}_{\sigma,\sigma}^R$ can be parameterized (see Eq. (31)) in the standard way as follows:

$$\hat{G}_{\sigma,\sigma}^R = \begin{pmatrix} \cosh(\Theta_\sigma) & \sinh(\Theta_\sigma) \exp(i\chi_\sigma) \\ -\sinh(\Theta_\sigma) \exp(-i\chi_\sigma) & -\cosh(\Theta_\sigma) \end{pmatrix}, \qquad (34)$$

where Θ_σ and χ_σ are complex functions; the function χ_σ does not contribute to the conductance (see Eq. (37)) below).

Using the parameterization (34) and Eqs. (24) and (26), one gets both the Usadel equation and its boundary conditions as

$$\hbar D \frac{d^2\Theta_\sigma}{dx^2} - 2i\varepsilon\Theta_\sigma = 0 , \qquad (35)$$

$$\left.\frac{d}{dx}\Theta_\sigma\right|_{x=0} = \sigma \frac{v_F}{D} |\langle|n_x|r_s\rangle| . \qquad (36)$$

Here r_s is the magnetic spin-flip scattering amplitude (see Eq. (19), (20). Equations (35) and (36) are linear due to the smallness of the amplitude for magnetic spin-flip scattering, $|r_s| << 1$, and are valid in the temperature interval $k_BT << \Delta$ which includes the Thouless energy $k_BT_{\text{Th}} = \hbar D/L^2 << \Delta$.

In order to calculate the conductance we follow Ref. 1 and find that the excess conductance can be written as

$$\frac{\delta G}{G_0} = -\frac{1}{16T}\sum_{\sigma=-1}^{1}\int_{-\infty}^{\infty} d\varepsilon \frac{\partial f_0}{\partial \varepsilon}\left[\frac{1}{L}\int_0^L dx\,(\text{Re}\,\Theta_\sigma)^2\right], \qquad (37)$$

where G_0 is the conductance of the ferromagnetic part of the structure; $f_0(\varepsilon)$ is Fermi distribution.

The solution of the Usadel equation (35) with the boundary condition given by Eq. (36) at $x = 0$ and by $\Theta_\sigma = 0$ at $x = L$, is

$$\Theta_\sigma = \sigma \frac{v_F|\langle|n_x|r_s\rangle|}{D\,k(\varepsilon)} \frac{\sinh[k(\varepsilon)(x-L)]}{\cosh(k(\varepsilon)\,L)} , \qquad (38)$$

$$k(\varepsilon) = (1+i)\sqrt{\varepsilon/\hbar D} .$$

Equation (38) shows that the superconducting correlations due to the spin-splitting processes in the magnetic inhomogeneous region decay exponentially in the ferromagnet and vanish at distances of order $L_T = \sqrt{\hbar D/k_BT}$ (for energies $\varepsilon \sim k_BT$) corresponding to the superconducting correlation length in nonmagnetic materials.

Inserting Eq. (38) into Eq. (37), one obtains an excess conductance that can be expressed as

$$\delta G/G_0 = \gamma f(T/T_{\text{Th}}) , \qquad (39)$$

where

$$\gamma = |\langle|n_x|r_s\rangle|^2 (L/l_0)^2$$

and $f(T/T_{\text{Th}})$ is a dimensionless function, the temperature dependence of which is presented in Fig. 3,

$$f(x) = \frac{1}{x}\int_0^\infty dt\,\cosh^{-2}(t^2/2x) \times$$

$$\times\left(\text{Re}\,\frac{\sinh[2(1+i)t] - 2(1+i)t}{4(i-1)t^2\cosh^2[(1+i)t]} + \frac{\sinh 2t - \sin 2t}{4t^2\,|\cosh(1+i)t|^2}\right). \qquad (40)$$

Using experimental values of the parameters taken from Ref. 10, $D = 100$ cm^2/s and $T/T_{\text{Th}} = 50$, and with the reasonable assumption that $r_{sf} \sim 0.1$ our result for the excess resistance, $\delta R \approx -10\ \Omega$, is in agreement with the experiment. The temperature dependence of the excess conductance in the range $T \sim T_{\text{Th}}$ is shown in Fig. 3. For higher temperatures, $T \sim \Delta/k_B >> T_{\text{Th}} = \hbar D/(k_BL^2)$, our theory is not valid, and contributions of order $k_BT/\Delta \sim 1$ can modify the temperature dependence of the resistance. Additional measurements around the Thouless temperature (where the proximity effect is most pronounced) would permit a comparison with the temperature dependence coming from the long-range proximity effect described by our theory. However, additional investigations of the magnetic structure of the F/S interface are needed to carry out a complete comparison with the theory. Multi-domain ferromagnets suitable for these studies can be created in various ways. It was recently demonstrated [24] that grain boundaries, magnetic inhomogeneities (including domains with nonparallel magnetization) can be introduced in a predetermined position in a ferromagnet film by controlling the epitaxial growth. Experiments in which such magnetic inhomogeneities are intentionally created would permit long-range proximity effects to be studied in well characterized ferromagnet—superconductor structures.

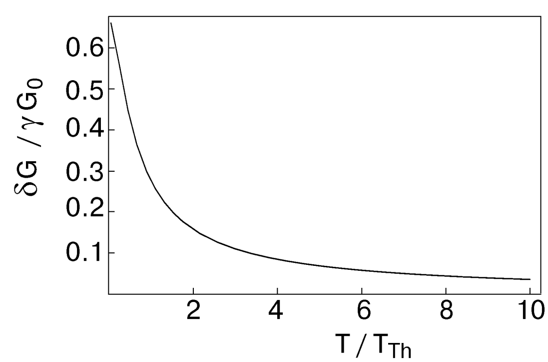

Fig. 3. Temperature dependence of the normalized excess conductance (see Eqs. (39) and (40)).

In conclusion, we have shown that spin-splitting scattering related to magnetic inhomogeneities modifies the spin-selection rule governing Andreev reflections at a ferromagnetic normal metal–superconductor interface. As a result a long-range proximity effect, due to correlations between spin-aligned electrons and holes, appears (a spin-triplet proximity effect). Estimations of the value of the excess conductance are consistent with experiments [8–11]. For applications, an important consequence of this phenomenon is that the proximity effect can be stimulated by orders of magnitude by intentionally produced magnetic inhomogeneity in the sample.

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