

Intrasubband plasmons in a weakly disordered array of quantum wires

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Received August 16, 2002, revised February 4, 2003

A theoretical investigation is carried out for plasmons in a weakly disordered array of quantum wires, consisting of a finite number of quantum wires arranged at an equal distance from each other. The array of quantum wires is characterized by the fact that the density of electrons of one «defect» quantum wire was different from that of the other quantum wires. It is assumed that the defect quantum wire can be arranged at an arbitrary position in the array. The existence of a local plasmon mode, whose properties differ from those of usual modes, is found. It is pointed out that the local plasmon mode spectrum is slightly sensitive to the position of the defect quantum wire in the array. At the same time the spectrum of usual plasmon modes is shown to be very sensitive to the position of the defect quantum wire.

PACS: 73.20.Mf, 78.67.Lt

1. Introduction

Quasi-one-dimensional electron systems (1DESs) or quantum wires (QWs) are artificial structures in which the motion of charge carriers is confined in two transverse directions but is essentially free (in the effective mass sense) in the longitudinal direction [1–3]. Usually QWs are produced by adding an additional one-dimensional confinement of a two-dimensional electron system (2DES). This additional confinement is, in general, weaker than the strong confinement of the original 2DES [4]. One of the motivations for studying QWs is the fact that the charge-carrier mobility is higher than in the 2DESs on which they are built. The reason for this is that the impurity content and distribution around QWs can be selectively controlled, producing enhanced mobility [5].

Collective charge-density excitations, or plasmons, in QW are objects of physicists' great interest. Plasmons in QW have been investigated previously both theoretically [5–9] and experimentally [10–12]. In those papers it was shown that plasmons in QWs possess some new unusual dispersion properties. First, the plasmon spectrum depends strongly on the width of the QW. Second, 1D plasmons are free of Landau damping [6,9] in the whole range of wave vectors.

From the point of view of practical application the so-called weakly disordered arrays of low-dimensional systems are the objects of interest. Recently the plasmons in weakly disordered superlattices, formed of a finite number of equally spaced two-dimensional electron systems, have been investigated theoretically [13–16]. The weakly disordered superlattice is characterized by the fact that all 2DESs possess equal electron densities except one («defect») 2DES, whose electron density differs from that of the other 2DESs. It was found that the plasmon spectrum of such a superlattice contains a local plasmon mode, whose properties differ from those of other plasmon modes. The existence of a local plasmon mode is completely analogous to the existence of the local phonon mode, first obtained by Lifshitz in 1947 for the problem of the phonon modes in a regular crystal containing a single isotopic impurity [17]. Notice that practically all the electromagnetic energy flux of plasmons corresponding to the local mode is concentrated in the vicinity of the defect 2DES. At the same time, the opportunity of using the peculiarities of the plasmon spectrum to determine the parameters of defects in the superlattice was pointed out in Ref. 16.

This paper deals with the theoretical investigation of plasmons in a finite weakly disordered array of

QWs, containing one defect QW whose one-dimensional density of electrons differs from that of the other QWs. We suppose that the defect QW can occupy an arbitrary position in the array. We show that the plasmon spectrum in the weakly disordered array of QWs is characterized by the existence of a local plasmon mode whose electromagnetic field is localized in the region of defect QW. We find that the position of the defect QW in the array does not affect strongly the spectrum of the local plasmon mode but it exerts a significant influence on the spectrum of other plasmon modes. At the same time, when the defect QW is arranged inside the array, the plasmon spectrum contains modes whose dispersion properties do not depend on the value of the electron density in the defect QW.

This paper is organized as follows. In Sec. 2 we derive the dispersion relation for plasmons in a weakly disordered array of QWs. In Sec. 3 we present the results of a numerical solution of the dispersion relation and discuss the dispersion properties of plasmons in the weakly disordered array of QWs. We conclude the paper with a brief summary of results and possible applications (Sec. 4).

2. Dispersion relation

We consider a weakly disordered array of QWs consisting of a finite number M of QWs, arranged at planes $z = ld$ ($l = 0, \dots, M - 1$ is the number of QW, d is the distance between adjacent QWs). We suppose that all QWs possess equal 1D electron densities N except one defect QW whose electron density is equal to N_d . So, the density of electrons in l th QW can be expressed as $N_l = (N_d - N)\delta_{pl} + N$. Here p is the number of defect QW arranged at the plane $z = pd$, and δ_{pl} is the Kronecker delta. The QWs are considered to be placed in a uniform dielectric medium with dielectric constant ϵ . We use such a simple model (in which the dielectric constants of the media inside and outside the array are equal) to avoid the appearance of a surface plasmon mode. We consider the motion of electrons to be free in the x direction and considerably confined in the directions y and z . At the same time we suppose that the width of all QWs is equal to a in the y direction and equal to zero in the z direction.

In other words, each QW can be represented as a square quantum well with infinite barriers at $y = -a/2$ and $y = a/2$ and a zero thickness in the z direction. Meanwhile we take into account only the lowest subband in each QW. In that case the single-particle wave function for the electron can be written in the form:

$$\psi_{k_x, l}(x, y, z) = |k_x, l\rangle = \exp(ik_x x)\varphi(y)[\delta(z - ld)]^{1/2}, \quad (1)$$

where $\varphi(y) = \sqrt{\frac{2}{a}} \cos \frac{\pi y}{a}$, and k_x is the one-dimensional wave vector describing the motion in the x direction. In that case the single-particle energy can be written as follows:

$$E_{k_x, l} = E_0 + \frac{\hbar^2 k_x^2}{2m^*}.$$

Here E_0 is the energy of the subband bottom (for simplicity we may put $E_0 = 0$), and m^* is the effective mass of the electron.

To obtain the spectrum of collective excitations we start with a standard linear-response theory in a random phase approximation. To obtain the collective excitation spectrum we consider $\delta n(\mathbf{r})$, which is the deviation of the electron density from its equilibrium value. After using the standard linear-response theory and the random phase approximation, $\delta n(\mathbf{r})$ can be related to the perturbation as

$$\delta n(x, y, z) = \sum_{\alpha, \alpha'} \frac{f_{\alpha'} - f_{\alpha}}{E_{\alpha'} - E_{\alpha} + \hbar\omega} V_{\alpha\alpha'} \psi_{\alpha'}^* \psi_{\alpha}, \quad (2)$$

where $\alpha = (k_x, l)$ is a composite index which is defined by (1), f_{α} is the Fermi distribution function, $V_{\alpha, \alpha'} = \langle \alpha | V | \alpha' \rangle$ are the matrix elements of the perturbing potential $V = V^{\text{ex}} + V^H$, and V^{ex} and V^H are the external and Hartree potentials, respectively.

For our system Eq. (2) can be rewritten in the form

$$\delta n(q_x, y, z) = \sum_l \Pi^l V_l \varphi^2(y) \delta(z - ld), \quad (3)$$

where

$$\Pi^l = \frac{1}{\pi} \int_{-\infty}^{\infty} dk_x \frac{f_{k_x + q_x, l'} - f_{k_x, l'}}{E_{k_x + q_x, l'} - E_{k_x, l'} + \hbar\omega}$$

is the noninteracting 1D polarizability («bare bubble») function, and $V_l \equiv \langle k_x, l' | V | k_x + q_x, l' \rangle$. At zero temperature the function Π^l can be written as

$$\Pi^l = \frac{m^*}{q_x \pi \hbar^2} \ln \frac{\omega^2 - \left(\frac{\hbar q_x k_F^l}{m^*} - \frac{\hbar q_x^2}{2m^*} \right)^2}{\omega^2 - \left(\frac{\hbar q_x k_F^l}{m^*} - \frac{\hbar q_x^2}{2m^*} \right)^2}. \quad (4)$$

Here $k_F^l = \frac{\pi N_l}{2}$ is the Fermi wave number in the l th QW. In the long-wavelength limit (where $q_x \rightarrow 0$) the function Π^l can be written as $\Pi^l = \frac{N_l}{m^*} \frac{q_x^2}{\omega^2}$.

Note that the Hartree potential can be expressed in terms of the perturbation [6] as

$$V^H(\mathbf{r}) = \int d\mathbf{r}' \frac{e^2}{\epsilon|\mathbf{r}-\mathbf{r}'|} \delta n(\mathbf{r}') \quad (5)$$

Using equations (3) and (5), we get the following expression for the matrix element $V_l^H = \langle k_x, l | V^H(x, y, z) | k_x + q_x, l \rangle$:

$$V_l^H = \sum_{l'} \Pi^{ll'} V_{l'} U_{l,l'} \quad (6)$$

where

$$U_{l,l'} = \frac{8e^2}{\epsilon a^2} \int_{-a/2}^{a/2} dy' \int_{-a/2}^{a/2} dy K_0 \{ q_x [(y-y')^2 + (l-l')^2 d^2]^{1/2} \} \cos^2\left(\frac{\pi y}{a}\right) \cos^2\left(\frac{\pi y'}{a}\right),$$

and $K_0(x)$ is the zeroth-order modified Bessel function of the second kind. Collective excitations of the QW array exist when Eq. (6) has a nonzero solution V^H in the case when the external perturbation $V^{\text{ex}} = 0$. Hence, the intrasubband plasmon dispersion relation takes the form

$$\det|\delta_{l,l'} - \Pi^{ll'} U_{l,l'}| = 0. \quad (7)$$

It should be noted that for $M = 2$ the dispersion relation (7) coincides with the dispersion relation for plasmons in the double-layer QW system obtained in [6].

3. Numerical results

Figure 1 shows the intrasubband plasmon spectrum (solid lines) in a weakly disordered array of QWs in the case $p = 0$. The y axis gives the dimensionless frequency ω/ω_0 ($\omega_0^2 = 2Ne^2/\epsilon m^* a^2$ is the plasma frequency), and the x axis gives the dimensionless wave vector $q_x a^*$ ($a^* = \epsilon \hbar^2 / m^* e^2$ is the effective Bohr radius). For comparison the dispersion curves for the plasmons in a single QW with electron densities N_d and N are depicted by dashed curves 1 and 2, respectively. As the model of QW we use a GaAs heterostructure with the effective mass of electrons $m^* = 0.067m_0$ (m_0 is the free electron mass) and the dielectric constant $\epsilon = 12.7$.

As is seen from Fig. 1, the intrasubband plasmon spectrum in the finite array of QWs contains M modes. Thus, the number of modes in the spectrum is equal to the number of QWs in the array [13]. Notice that with an increase of the wave number q_x the plasmon frequency ω increases likewise. At the same

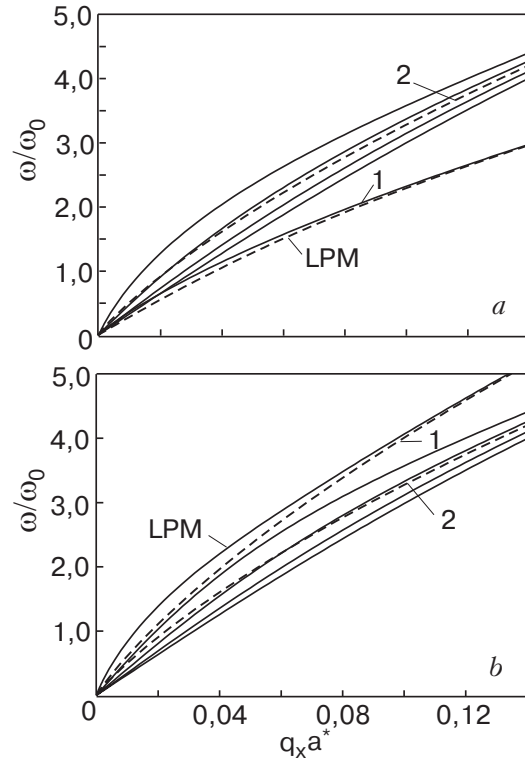


Fig. 1. Dispersion curves of plasmons (solid curves) in a weakly disordered array of QW with parameters $M = 5$, $d = 15 a^*$, $a = 20 a^*$, $p = 0$ for two values of the electron density in the defect QW: $N_d/N = 0.5$ (a), $N_d/N = 1.5$ (b).

time, the propagation of plasmons in the weakly disordered array of QWs is characterized by the presence of a local plasmon mode (LPM). In the case where the electron density in the defect QW is less than the electron density in the other QWs ($N_d < N$), the LPM lies in a lower-frequency region in comparison with the usual plasmon modes (Fig. 1,a). Accordingly, if $N_d > N$, the LPM lies in a higher-frequency region in comparison with the usual modes (Fig. 1,b) [13]. It should be emphasized that in the limit $q_x d \rightarrow \infty$, when the Coulomb interaction between electrons in adjacent QWs is negligible, the LPM dispersion curve is close to the dispersion curve for the plasmons in a single QW with electron density N_d . In this case the dispersion curves for usual plasmon modes in the limit $q_x d \rightarrow \infty$ are gradually drawn together and are close to the dispersion curve for the plasmon in a single QW with electron density N .

Now we consider the dependence of the plasmon spectrum on the value of the 1D electron density in the defect QW. Figure 2 presents the dependence of the plasmon frequency on the ratio N_d/N in the case of a fixed value of the wave vector q_x and for different positions of the defect QW in the array. As is seen from Fig. 2, the frequency of the LPM increases when the value of ratio N_d/N is increased. At the same time the spectrum of the usual plasmon modes is character-

ized by these features. For $p = 0$ (Fig. 2,*a*) as the value of the ratio N_d/N is increased, the frequency of all the usual plasmon modes increases as well. In other words, for $N_d < N$ all of the dispersion curves lie in a lower-frequency region as compared with the dispersion curves of plasmons in the QW array with equal electron densities in all the QWs (ordered array of QWs). Correspondingly, in the case $N_d > N$, all the dispersion curves of plasmons in the weakly disordered array of QWs lie in a higher-frequency region in comparison with the plasmons in the ordered array of QWs. However, when $p = 1$ (Fig. 2,*b*) the frequency of one of the usual plasmon modes (curve 2) becomes practically independent of the ratio N_d/N . In the case $p = 2$ (Fig. 2,*c*) there are already two plasmon modes

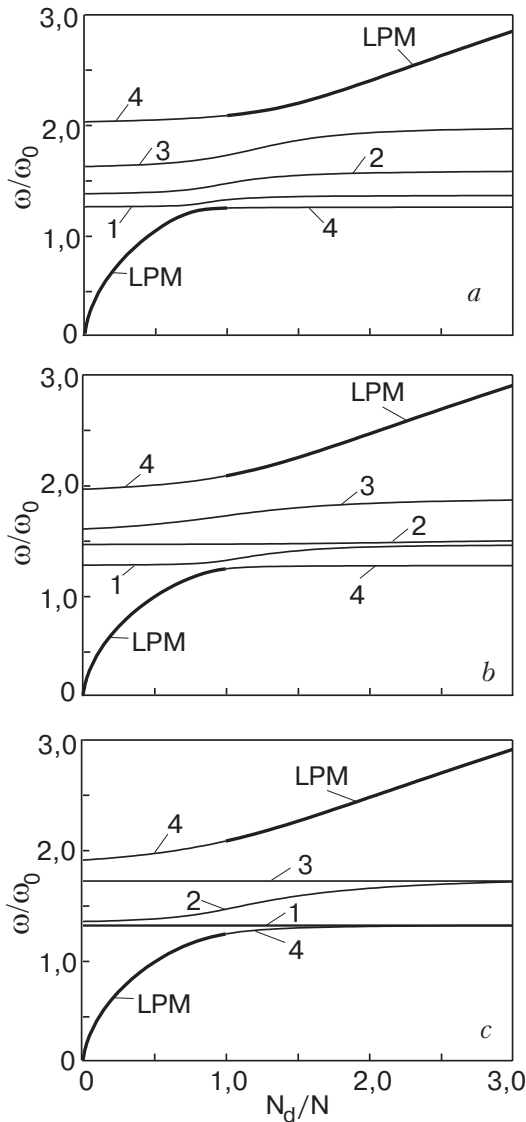


Fig. 2. Dependence of the plasmon frequency upon the ratio N_d/N when $q_x a^* = 0.04$, $M = 5$, $d = 15 a^*$, $a = 20 a^*$ and for three cases of the defect QW position in the array: $p = 0$ (a), $p = 1$ (b), $p = 2$ (c).

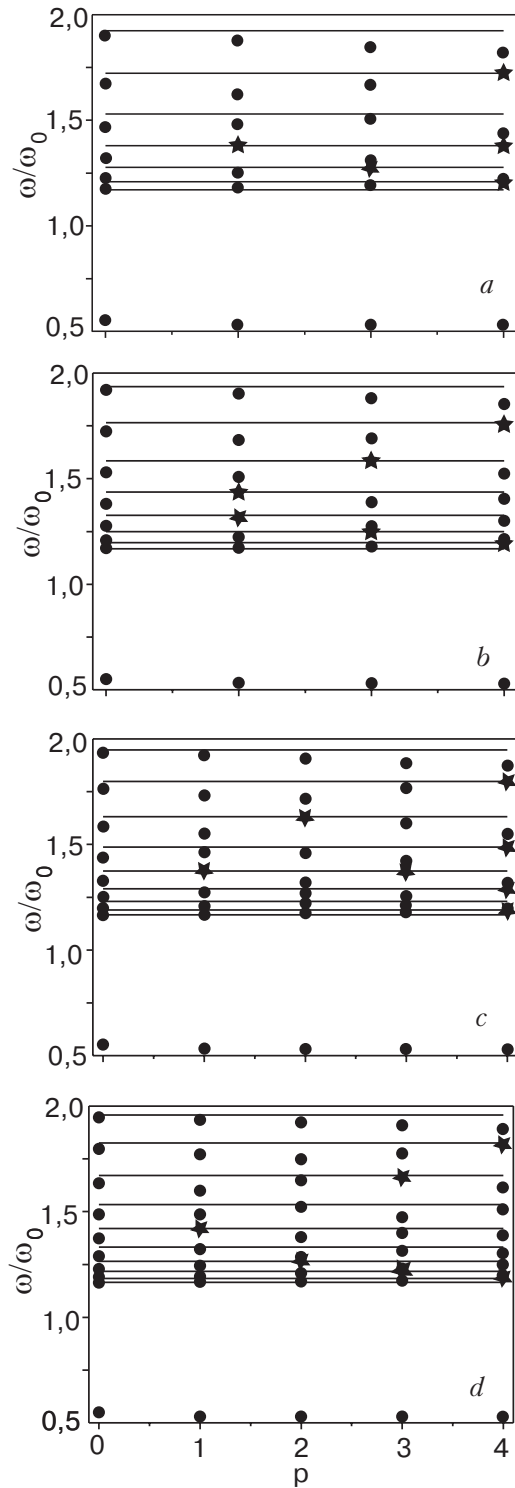


Fig. 3. Dependence of the plasmon frequency upon the position of the «defect» QW in the array p , when $q_x a^* = 0.05$, $N_d/N = 0.15$, $d = 15 a^*$, $a = 20 a^*$ and for different values of the amount of QWs in the array: $M = 7$ (a), $M = 8$ (b), $M = 9$ (c), $M = 10$ (d).

(curves 1 and 3) which possess such a distinctive feature.

Figure 3 presents the dependence of the plasmon frequency ω/ω_0 upon the number of the defect QW p ,

for different values of M . This dependence is depicted by separate dots. Solid horizontal lines denote the plasmon mode frequencies in the ordered array of QWs. As can be seen from Fig. 3, the LPM spectrum is weakly dependent upon the position of the defect QW in the array. However, the spectrum of usual plasmon modes is more sensitive to the position of the defect QW in the array. Notice that at every value of M , when the defect QW is arranged inside the array of QWs ($1 \leq p \leq M - 2$), the usual plasmon mode spectrum contains modes (shown by five-point stars), whose frequency does not depend upon the value of the electron density in the defect QW. At the same time, the maximum quantity of such modes is observed in the case where the defect QW lies at the very center of the array.

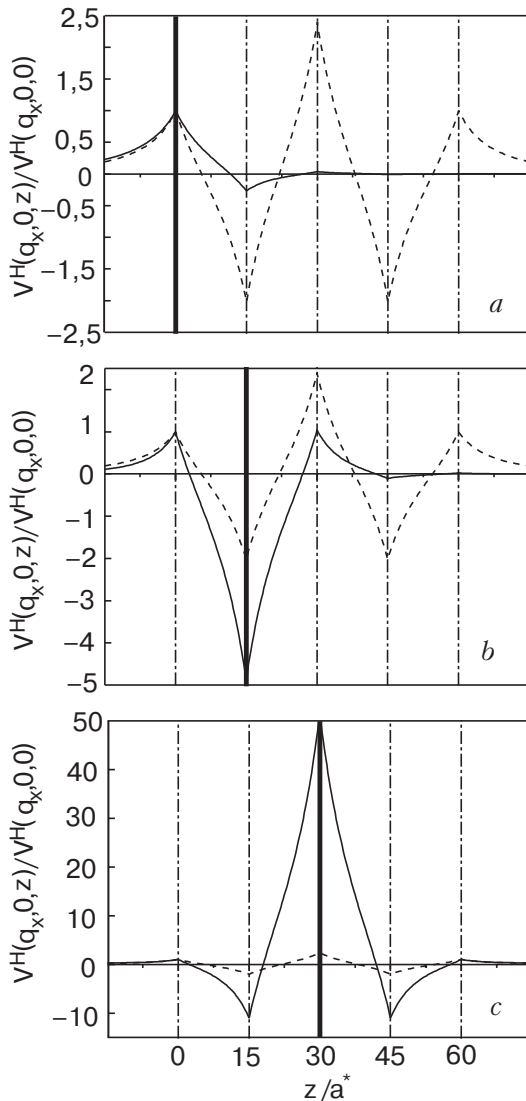


Fig. 4. Spatial distribution of the Hartree potential $V^H(q_x, 0, z)$ over the z coordinate for the local plasmon mode in the case when $q_x a^* = 0.05$, $N_d/N = 0.5$, $M = 5$, $d = 15 a^*$, $a = 20 a^*$ and for different positions of defect QW in the array: $p = 0$ (a), $p = 1$ (b), $p = 2$ (c).

To explain the above-mentioned features of the plasmon modes, we consider the spatial distribution of the Hartree potential V^H . Figure 4 shows the dependence of the Hartree potential V^H upon the z coordinate for the LPM (solid curves). This dependence is presented for different positions of the defect QW in the array. The y axis gives the dimensionless Hartree potential $V^H(q_x, 0, z)/V^H(q_x, 0, 0)$, and the x axis gives the dimensionless z coordinate z/a^* . For comparison the spatial distribution of the Hartree potential for the lowest-frequency plasmon mode in the ordered array of QWs is depicted by dashed curves. Vertical dash-and-dot lines denote the positions of QWs in the array. Here the vertical solid bold line corresponds to the position of the defect QW in the array. As is evident from Fig. 4, a, b, c the absolute value

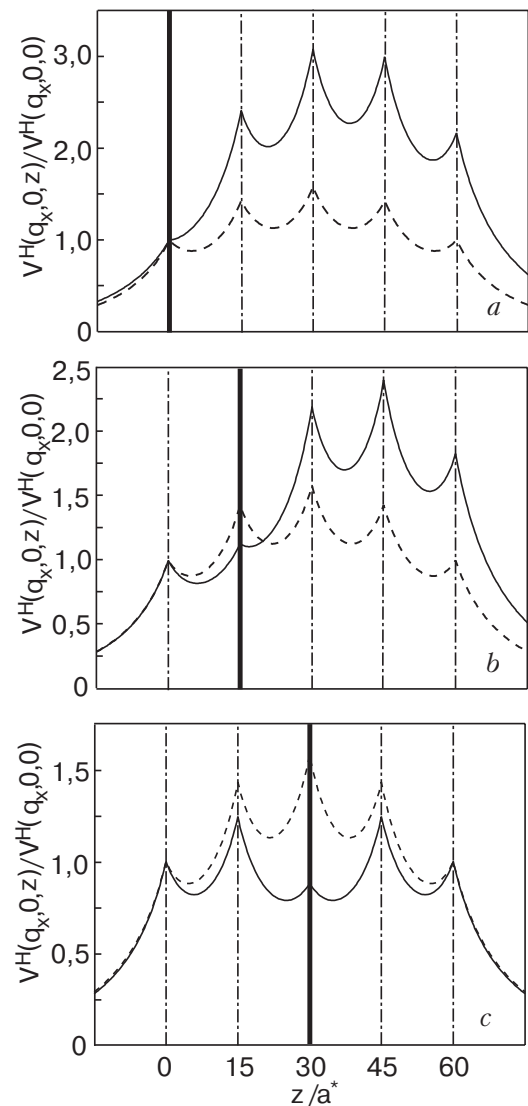


Fig. 5. Spatial distribution of the Hartree potential $V^H(q_x, 0, z)$ over z coordinate for mode 4 (see Fig. 2) in the case when $q_x a^* = 0.05$, $N_d/N = 0.5$, $M = 5$, $d = 15 a^*$, $a = 20 a^*$ and for different positions of the defect QW in the array: $p = 0$ (a), $p = 1$ (b), $p = 2$ (c).

of the Hartree potential in the vicinity of the defect QW exceeds considerably the absolute values of the Hartree potentials in the vicinity of the other QWs. This implies that practically the whole flux of LPM electromagnetic field energy is concentrated in the vicinity of the defect QW. The weak dependence of the LPM spectrum on the position of the defect QW can be accounted for by that peculiarity.

To explain the fact that usual plasmon mode spectra are sensitive to the position of the defect QW in the array, let us consider, for example, the spatial distribution of the Hartree potential for mode 4 (see Fig. 2). This dependence is represented in Fig. 5 for different positions of the defect QW in the array. The dashed curves present the distribution of the Hartree potential for mode 4 in the case of the ordered array of QWs. As one can see from a comparison of Fig. 5,*a,b,c*, the spatial distribution of the Hartree potential changes when the position of the defect QW in the array is varied. Thus, the changing of the Hartree potential causes the variation of the plasmon frequency.

Figure 6 presents the spatial distribution of the Hartree potential for plasmon modes whose frequencies are slightly sensitive to the value N_d . As is seen from Fig. 6,*a,b,c*, these plasmon modes possess one particular feature. Thus the spatial distribution of the Hartree potential corresponding to the spectra of these modes in a weakly disordered array of QWs either differs insignificantly from that in the ordered array of QWs (Fig. 6,*a*) or coincides with it exactly (Fig. 6,*b,c*). At the same time, the absolute value of the Hartree potential in the vicinity of the defect QW is negligible. Thus, in that case the value of the electron density in the defect QW does not exert a substantial effect on the plasmon spectrum.

4. Conclusion

In conclusion, we have calculated the plasmon spectrum of a finite, weakly disordered array of QWs which contains one defect QW. It is found that a local plasmon mode whose properties differ from those of other modes exists in the plasmon spectrum. We point out that the LPM spectrum is slightly sensitive to the position of the defect QW in array. That phenomenon can be explained by the fact that practically the whole flux of the LPM electromagnetic energy is localized in the vicinity of the defect QW. At the same time, the position of the defect QW exerts an influence on the spectrum of usual plasmon modes. It is shown that under certain conditions there can exist plasmon modes whose spectrum does not depend upon the density of electrons of the defect QW. The spatial distribution of the Hartree potential for those modes has the feature that the absolute value of the Hartree potential in the

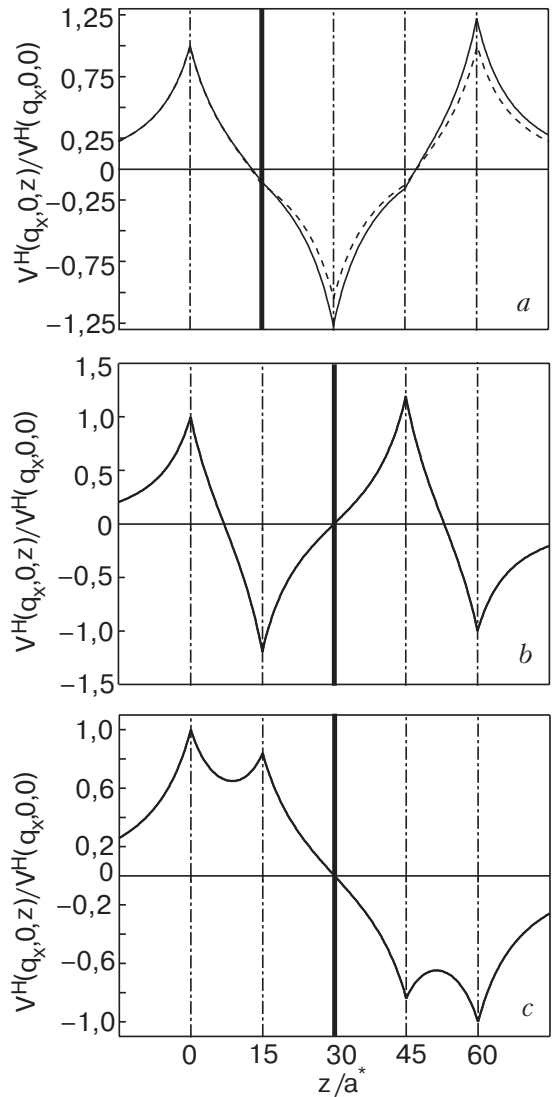


Fig. 6. Spatial distribution of the Hartree potential $V^H(q_x, 0, z)$ over z coordinate for the following plasmon modes: $p = 1$, mode 2 (see Fig. 2,*b*) (*a*), $p = 2$, mode 1 (see Fig. 2,*c*) (*b*), $p = 2$, mode 3 (see Fig. 2,*c*) (*c*).

vicinity of the defect QW is negligible. Therefore, the defect QW does not exert a significant influence on the dispersion properties of the plasmon modes.

To conclude, it should be emphasized that the above-mentioned features of the plasmon spectra can be used for the diagnostics of defects in QW structures. Hence, the LPM properties can be used for determination of the electron density in the defect QW. At the same time, the properties of the usual plasmon modes can be used to determine the position of the defect QW in the array.

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