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On the suprathermal distribution in an anisotropic phonon system in He II

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The equation that describes the suprathermal distribution of high-energy phonons (h phonons) created in anisotropic phonon systems in superfluid helium is obtained. The solution of this equation enables the derivation of the value of suprathermal ratio S as the ratio of the actual distribution to the Bose—Einstein one, its dependences on the momentum of the h phonons, the anisotropy parameters, and the temperature of the low-energy phonons from which the h phonons are created. We analyze this equation to obtain an estimate of the value of the ratio between the h-phonon number density in anisotropic and isotropic phonon systems and draw conclusions about the dependence of S on the relevant parameters.

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1. Introduction

For a phonon system in superfluid ^{4}He (He II) the rates of kinetic processes are determined by the unusual form of the phonon energy—momentum dependence. At zero pressure the phonon dispersion curve in He II bends upwards [1–3], and this causes spontaneous decay of phonons with energies $\varepsilon < \varepsilon_c = 10$ K [4,5]. For these phonons, the energy and momentum conservation laws allow processes in which the phonon numbers in the initial and final states do not equal one another. The fastest process among these is the three-phonon process ($3pp$), in which one phonon decays to two phonons or two phonons interact to create one phonon. The rate of these three-phonon processes was obtained in [6,7], in the two extreme limits; and the general case was calculated in [8].

At higher energies the phonon dispersion curve bends downwards, and at $\varepsilon > \varepsilon_c$ phonon spectrum becomes nondecaying. Here the most rapid process is the four-phonon process ($4pp$), in which there are two phonons in the initial and final states.

The rate of three-phonon processes ν_{3pp} is obtained using Landau's Hamiltonian in first-order perturbation theory, and the rate of four-phonon ν_{4pp} processes is determined by second-order perturbation theory [9–11]. This is quantitatively evaluated and confirmed by experiment [12]. The difference between the orders of perturbation theory results in the strong inequality $\nu_{3pp} >> \nu_{4pp}$. Thus the phonons of superfluid ^{4}He form two subsystems: one of low energy (l phonons) with $\varepsilon < \varepsilon_c$, which very quickly attains equilibrium; and the other of high-energy phonons (h phonons), which goes to equilibrium relatively slowly.

The angles between the phonons which take part in $3pp$ is small due to the smallness of the deviation of the energy—momentum dispersion from linearity, $\varepsilon = cp$. Thus in isotropic phonon systems, when all directions in momentum space are uniformly occupied, equilibrium is not attained isotropically in the short term because interactions involve only phonons within a limited solid angle. Thus, all thermodynamic parameters of the l-subsystem become functions of angle [11]. In the long term, four-phonon processes and diffusion in angular space bring about complete equilibrium. This quasidiffusion is determined by fast three-phonon interactions involving small angles [13–15].

The situation is quite different in highly anisotropic phonon systems. Here the momenta of all phonons are in a narrow cone with solid angle Ω_p, which has a value close to the typical angle for three-phonon processes. Such strongly anisotropic phonon systems have been created in liquid ^{4}He [16–20]. This pure and isotropic superfluid can have such a low temperature that one can neglect the existence of thermal excitations. Low-energy phonons are injected by a heater and then propagate along the direction normal to the surface of the heater. In momentum space all the phonons lie in a narrow cone of solid angle $\Omega_p = 0.125$ sr.

In such experiments [16–20] the unusual phenomenon of the creation of high-energy phonons was observed. These h phonons are created by a pulse of l phonons, which has a temperature an order of magnitude less than the energy of the high-energy phonons that it creates. The theory of this unique phenomenon was proposed in [21,22]. A further development of the theory [23–28], shows that in strongly anisotropic phonon systems there exists an asymmetry between processes of creation and decay for the h phonons. Such an asymmetry causes the distribution function of h phonons in the anisotropic phonon system to be S times greater than that in the Bose–Einstein distribution. Moreover, it is possible to have $S >> 1$. Using the notation of [23], we will call such an unusual distribution of h phonons a suprathermal distribution, and the parameter S, the suprathermal ratio.

In this paper we derive an exact equation that allows us to calculate the suprathermal ratio and to determine its dependences on momentum, anisotropy parameter, and temperature. We analyze the contribution of all possible processes that lead to the formation of suprathermal distributions in anisotropic phonon systems. From this equation we derive an estimate of the average value of the suprathermal ratio S: the ratio of the number density of h phonons in the anisotropic and isotropic phonon systems.

2. The inequalities for determining of the stationary distribution function of phonons in an anisotropic phonon system

The attainment of equilibrium in the subsystem of h phonons can be described by the kinetic equation for the distribution functions, which can be written as

$$\frac{dn_1}{dt} = N_b(\mathbf{p}_1) - N_d(\mathbf{p}_1), \quad (1)$$

where

$$N_b(\mathbf{p}_1) = \int_{\Omega_b} W(\mathbf{p}_1, \mathbf{p}_2 | \mathbf{p}_3, \mathbf{p}_4) n_3 n_4 (1 + n_1)(1 + n_2) \times \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \times \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) d^3p_2 d^3p_3 d^3p_4 \quad (2)$$

is the number of phonons with momentum $\mathbf{p}_1$ created per unit time as the result of four-phonon ($4pp$) interactions;

$$N_d(\mathbf{p}_1) = \int_{\Omega_d} W(\mathbf{p}_1, \mathbf{p}_2 | \mathbf{p}_3, \mathbf{p}_4) n_1 n_2 (1 + n_3)(1 + n_4) \times \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \times \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) d^3p_2 d^3p_3 d^3p_4 \quad (3)$$

is the number of phonons that decay per unit time; $W(\mathbf{p}_1, \mathbf{p}_2 | \mathbf{p}_3, \mathbf{p}_4) = W(\mathbf{p}_3, \mathbf{p}_4 | \mathbf{p}_1, \mathbf{p}_2)$ defines the transition probability density for $4pp$ processes, which have been calculated [25]; Ω_b and Ω_d each represent a set of three solid angles, one for each momentum over which the function is integrated, i.e., Ω_{bi} and Ω_{di} ($i = 2,3,4$). These maximum angles are determined by the anisotropy of the phonon system and the angles in the $4pp$ interactions. In the isotropic case $\Omega_{bi} = \Omega_{di}$. In relations (1)–(3) and below we consider $p_1 \geq p_c = k_B \varepsilon_c / c$ (i.e., phonon «1» is the h_1 phonon), while the other three phonons can be l phonons or h phonons. The stationary distribution function is determined by the equality

$$N_b = N_d. \quad (4)$$

For isotropic phonon systems $\Omega_{bi} = \Omega_{di}$, and equality (4) gives:

$$n_1 n_2 (1 + n_3)(1 + n_4) = n_3 n_4 (1 + n_1)(1 + n_2). \quad (5)$$

The solution of this equation is the Bose–Einstein distribution

$$n_i^{(0)} = \left\{ \exp\left(\varepsilon_i / T\right) - 1 \right\}^{-1}. \quad (6)$$

In an anisotropic phonon system, when the initial phonons are inside a narrow cone with solid angle $\Omega_p \leq 4\pi$, the result differs from (5) and (6). In this case the limits of integration in (2) and (3) with respect to angular variables are defined by the inequalities

$$\Omega_i \leq \Omega_p. \quad (7)$$

Here $i = 3,4$ for creation processes, and $i = 2$ for decay processes. The angular variables for the final phonons have no such restrictions.

Such asymmetry for the initial and final states results in the inequality $\Omega_b \neq \Omega_d$. In this case in anisotropic phonon systems the equality (4) cannot be satisfied by solution (5), and the Bose–Einstein distribution is not a solution of equation (4).

In highly anisotropic phonon systems, when Ω_p is less than the typical solid angle for four-phonon processes, the stationary distribution of h phonons will be essentially different from the Bose–Einstein distribution (6) in magnitude and in momentum dependence.

The integrals (2) and (3) can be written as a sum of five terms with definite ranges of integration. These terms correspond to the different processes, which are possible for the interactions of h phonons between themselves and with l phonons:

$$1)\ h_1 + l_2 \leftrightarrow l_3 + l_4;\quad 2)\ h_1 + l_2 \leftrightarrow h_3 + l_4;$$
$$3)\ h_1 + l_2 \leftrightarrow h_3 + h_4;\quad 4)\ h_1 + h_2 \leftrightarrow h_3 + l_4;\quad (8)$$
$$5)\ h_1 + h_2 \leftrightarrow h_3 + h_4$$

The arrow to the right indicates the decay of an h_1 phonon and to the left creation. We define the rates $\nu^{(n)}_{b,d}$ of creation (b) and decay (d) processes with distribution function n for h phonons by the equalities:

$$N_{b\alpha} = n_1^{(0)}\nu^{(n)}_{b\alpha};\ N_{d\alpha} = n_1\nu^{(n)}_{d\alpha};\ \ (\alpha = 1, 2, 3, 4, 5). \quad (9)$$

As N_b is the sum over all $N_{b\alpha}$, we rewrite relation (4) as follows:

$$n_1^{(0)}\sum_{\alpha=1}^{5}\nu^{(n)}_{b\alpha} = n_1\sum_{\alpha=1}^{5}\nu^{(n)}_{d\alpha}. \quad (10)$$

We take into account that 3*pp* processes instantaneously establish equilibrium (on the time scale of h-phonon creation and propagation) in the subsystem of l phonons, which occupy the solid angle Ω_{3pp} that is typical for 3*pp*. The phonon pulses in experiments [16–20] have Ω_p close to Ω_{3pp} [13–15]. Therefore in this case we can consider that the l phonons in the pulse have a Bose–Einstein distribution:

$$n(\mathbf{p}_l) = n_l^{(0)},\ \text{at}\ p_l < p_c. \quad (11)$$

For the stationary distribution of h phonons, the distribution function can be written in the form:

$$n(\mathbf{p}_h) = S(\mathbf{p}_h)n_h^{(0)},\ \ \text{at}\ p_h > p_c. \quad (12)$$

Starting from equalities (10)–(12) we have

$$\sum_{\alpha=1}^{5}\nu^{(n)}_{b\alpha} = S(\mathbf{p}_1)\sum_{\alpha=1}^{5}\nu^{(n)}_{d\alpha}. \quad (13)$$

This equation is an integral equation with respect to the unknown function $S(\mathbf{p}_1)$. For decay processes when h_1 combines with an l or h phonon, the rate is independent or a linear functional of S respectively. For creation processes if initially there are zero, one, or two h phonons then the rate is independent, a linear functional, or a quadratic functional, respectively. Therefore the desired function $S(\mathbf{p}_1)$ is absent in the rates $\nu^{(n)}_{b1}$, $\nu^{(n)}_{d1}$, $\nu^{(n)}_{d2}$, $\nu^{(n)}_{d3}$ if one takes into account that $1 + n_h \approx 1$. The rates $\nu^{(n)}_{b2}$, $\nu^{(n)}_{b4}$, $\nu^{(n)}_{d4}$, $\nu^{(n)}_{d5}$ include linear functionals, and $\nu^{(n)}_{b3}$, $\nu^{(n)}_{b5}$ include quadratic functionals. Using these facts, we present the rates from (9) as

$$\nu^{(n)}_{b1} = \nu^{(0)}_{b1};\quad \nu^{(n)}_{d1} = \nu^{(0)}_{d1};$$
$$\nu^{(n)}_{d2} = \nu^{(0)}_{d2};\quad \nu^{(n)}_{d3} = \nu^{(0)}_{d3}; \quad (14)$$
$$\nu^{(n)}_{b2} = S_{b2}(\mathbf{p}_1)\nu^{(0)}_{b2};\quad \nu^{(n)}_{b4} = S_{b4}(\mathbf{p}_1)\nu^{(0)}_{b4};$$
$$\nu^{(n)}_{d4} = S_{d4}(\mathbf{p}_1)\nu^{(0)}_{d4};\quad \nu^{(n)}_{d5} = S_{d5}(\mathbf{p}_1)\nu^{(0)}_{d5};$$

and

$$\nu^{(n)}_{b3} = S^2_{b3}(\mathbf{p}_1)\nu^{(0)}_{b3};\quad \nu^{(n)}_{b5} = S^2_{b5}(\mathbf{p}_1)\nu^{(0)}_{b5}; \quad (15)$$

where

$$\nu^{(0)}_{b,d\alpha} = \nu^{(n_0)}_{b,d\alpha}(S = 1) \quad (16)$$

are the rates calculated with distribution function (6). The above equations define $S_{b,d\alpha}$ which are functionals of the function $S(\mathbf{p}_h)$.

Using (14)–(16) the relation (13) can be written as

$$\left\{SS_{d5}\nu^{(0)}_{d5} - S^2_{d5}\nu^{(0)}_{b5}\right\} + \left\{SS_{d4}\nu^{(0)}_{d4} - S^2_{b3}\nu^{(0)}_{b3}\right\} +$$
$$+\left\{S\nu^{(0)}_{d3} - S_{b4}\nu^{(0)}_{b4}\right\} + \left\{S\nu^{(0)}_{d2} - S^2_{b2}\nu^{(0)}_{b2}\right\} =$$
$$= \nu^{(0)}_{b1} - S\nu^{(0)}_{d1}. \quad (17)$$

For isotropic phonon systems, when $\Omega_{bi} = \Omega_{di}$, relations (2), (3) give $\nu^{(0)}_{b\alpha} = \nu^{(0)}_{d\alpha} = \nu^{(\text{isotr})}_{\alpha}$. In this case Eq. (17) has the solution $S(\mathbf{p}_h) \equiv 1$, and, according to (12), we get the Bose–Einstein distribution (6) for all phonons in an isotropic system.

3. Asymmetry of processes of *h*-phonon creation and decay, resulting in a suprathermal distribution on an anisotropic phonon system

In anisotropic phonon systems, when $\Omega_{bi} \neq \Omega_{di}$ the creation rate $\nu^{(0)}_{b\alpha}$ can be significantly different from the decay rate $\nu^{(0)}_{d\alpha}$. In Refs. [25,26] the rates of all five processes of creation and decay are calculated for phonons with momentum $\mathbf{p}_1$ directed along the axis of symmetry of the pulse, chosen as the Z axis, so $\theta_1 = 0$. These rates we denote $\nu_{b,d}$, where the superscript (0) is understood.

The main role in (17) is played by a type-1 process that describes the exchange of phonons between the l and h systems. For a pulse typically used in the experiments [16–20] the values are: anisotropy parameter $\Omega_p = 0.125$ sr and temperature $T = 1$ K. Then the minimum value of the ratio $\nu_{b1}/\nu_{d1} = 30$ at $p_1 = p_c$; it grows quickly with increasing p_1 and becomes equal to infinity at $p_1 \geq p_0$ (see Fig. 1). The limiting momentum p_0 is determined by the solid angle Ω_p and

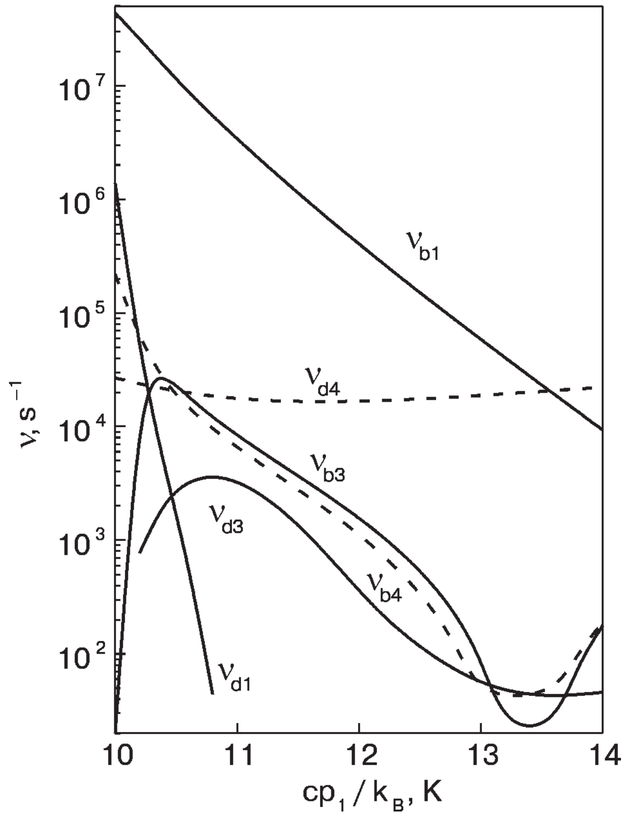

Fig. 1. The momentum dependences of the creation rates ν_b and decay ν_d rates, at T = 1 K and Ω_p = 0.125 sr, for all the processes which exchange phonons between the l- and h-phonon systems.

the conservation laws of energy and momentum, which govern the interaction of l phonons with such h_1 phonons. The corresponding analytical expressions and detailed discussion of the rates and their dependences on momentum shown in Fig. 1, are given in Refs. 25, 26.

An infinite lifetime coupled with a finite creation rate of h phonons with $p_1 \geq p_0$ means that in anisotropic phonon systems, type-1 processes cannot effect a dynamic equilibrium between the h and l subsystems. However such an equilibrium can be provided by type-4 processes, because $\nu_{d4} > \nu_{b4}$ in the momentum region where $\nu_{d1} = 0$ (see Fig. 1). Using the numerical values for rates calculated for the anisotropic phonon system (see Fig. 1), equation (17) can be satisfied for $S >> 1$. As a result, in anisotropic phonon systems, the stationary distribution function of such h phonons is many times greater than the Bose—Einstein one (6) and has a different energy dependence which is determined by the momentum-dependent rates shown in Fig. 1.

In the left-hand side of (17) the rates that have the same power of S and describe mutually compensating processes are separated in curved braces. These compensating processes in (17) are of two fundamentally different types. The second and the third braces describe processes which exchange phonons between the l and h systems. At the same time, type-4 decay processes are partly compensated by type-3 creation processes (but not by type-4 creation processes), and type-3 decay processes partly compensate type-4 creation process (but not type-3 creation process). The first and fourth curved braces describe processes that conserve the number of h phonons. Here decay is compensated by creation in processes of the same type.

The presence in (17) of processes that conserve and do not conserve the number of h phonons makes it useful to consider the expression obtained from (17) by averaging the anisotropic phonon system over $\mathbf{p}_1$. We define the average as

$$\langle A \rangle = \frac{\int\limits_{\Omega_p} A n_1^{(0)} d^3 p_1}{\int\limits_{\Omega_p} n_1^{(0)} d^3 p_1} . \tag{18}$$

Then one should take into account the following equalities, which are obtained by reindexing the variables of integration:

$$\int\limits_{\Omega_p} N_{d2}(\mathbf{p}_1) d^3 p_1 = \int\limits_{\Omega_{d2}^{(3)}} N_{b2}(\mathbf{p}_1) d^3 p_1 , \tag{19}$$

$$\int\limits_{\Omega_p} N_{d5}(\mathbf{p}_1) d^3 p_1 = \int\limits_{\Omega_{d5}^{(3)}} N_{b5}(\mathbf{p}_1) d^3 p_1 , \tag{20}$$

$$\int\limits_{\Omega_p} N_{d3}(\mathbf{p}_1) d^3 p_1 = \int\limits_{\Omega_{d3}^{(3)}} N_{b4}(\mathbf{p}_1) d^3 p_1 , \tag{21}$$

$$\int\limits_{\Omega_p} N_{d4}(\mathbf{p}_1) d^3 p_1 = \int\limits_{\Omega_{d4}^{(3)}} N_{b3}(\mathbf{p}_1) d^3 p_1 , \tag{22}$$

where $\Omega_{d\alpha}^{(3)}$ is the solid angle of the created $\mathbf{p}_3$ phonon in decay processes of the type α.

From the conservation laws of energy and momentum it follows that at the typical momenta of the phonons taking part in the decay processes, the created $\mathbf{p}_3$ phonon remains inside the initial phonon pulse. This fact allows us to consider approximately that $\Omega_{d\alpha}^{(3)} = \Omega_p$. In this case the averaging of (17) taking into account (19)–(22) gives:

$$\left\langle S S_{d4} \nu_{d4}^{(0)} \right\rangle - \left\langle S_{b4} \nu_{b4}^{(0)} \right\rangle = 2 \left\langle \nu_{b1}^{(0)} \right\rangle - 2 \left\langle S \nu_{d1}^{(0)} \right\rangle . \tag{23}$$

This result can be rewritten in the form

$$\left\langle S^2\right\rangle\tilde{\nu}^{(0)}_{d4} - \langle S\rangle\tilde{\nu}^{(0)}_{b4} = 2\overline{\nu^{(0)}_{b1}} - 2\langle S\rangle\overline{\nu^{(0)}_{d1}}, \qquad (24)$$

where $\overline{\nu^{(0)}_{b1}} = \left\langle \nu^{(0)}_{b1}\right\rangle$, and the other average values of the rates $\tilde{\nu}^{(0)}_{b1,d4}$ are determined by the obvious equalities of the respective terms in Eqs. (23) and (24). The solution of Eq. (24) is

$$\langle S\rangle = \frac{4\overline{\nu^{(0)}_{b1}}}{\sqrt{(2\tilde{\nu}^{(0)}_{d1} - \tilde{\nu}^{(0)}_{b4})^2 + 8\tilde{\nu}^{(0)}_{d4}\overline{\nu^{(0)}_{b1}}} + 2\tilde{\nu}^{(0)}_{d1} - \tilde{\nu}^{(0)}_{b4}}. \qquad (25)$$

Using this solution we can estimate the value of $\langle S\rangle$ if we substitute the rates in (25) (shown in Fig. 1) with their average values calculated over the range $10\text{ K} \le cp_1/k_B < 20\text{ K}$, at $\theta_1 = 0$. These rates we denote $\overline{\nu_{b,d}}$. In this case

$$2\overline{\nu_{d1}} - \overline{\nu_{b4}} << \sqrt{8\overline{\nu_{b1}}\,\overline{\nu_{d4}}} \qquad (26)$$

and from (25) we have

$$\langle S\rangle \approx \sqrt{2\frac{\overline{\nu^{(0)}_{b1}}}{\nu^{(0)}_{d4}}} = 30. \qquad (27)$$

This relation has a simple physical meaning. The value of $\langle S\rangle$ is defined as the square root of the ratio of the rate of growth of the number of h phonons by type-1 processes to the rate of decrease of the number of h phonons by type-4 processes, which are partly compensated by type-3 processes.

According to (17) and results obtained for $\nu_{b2,d2}$ and $\nu_{b5,d5}$ [25,26], it follows that S depends strongly on the angle θ_1 between the direction of the phonon $\mathbf{p}_1(p_1,\theta_1,\varphi_1)$ and the Z axis of symmetry of the anisotropic phonon system. Therefore, according to [25,26], over a relatively wide region of momentum $\mathbf{p}_1$ the creation rate of h phonons with $\theta_1 = 0$, for the second and the fifth processes, is greater than decay rate. Since the total number of h phonons is separately conserved in processes 2 and 5, the h phonons will concentrate in momentum space near the Z axis. With increasing θ_1, the number of h phonons decreases, so that there will be relatively few h_1 phonons at large $\theta_1 < \theta_p$ in the pulse. This result agrees with the results of experiments [17,19], where the h-phonon cone is found to be narrower than the l-phonon cone.

The suprathermal ratio S is also a function of the temperature of the l phonons, because according to [25,26] the rates of creation and decay of all five processes become smaller, at different rates, with decreasing temperature. Thus, at $\Omega_p = 0.125$ sr and $p_1 \approx p_c$, according to [26], with decreasing temperature from 1 K to 0.7 K the rates ν_{b1} and ν_{d1} becomes smaller by ≈ 5 and ≈ 6 times, respectively, the rates ν_{b2} and ν_{d2} by ≈ 9 and ≈ 6 times, the rates ν_{b3} and ν_{d3} by ≈ 70 and ≈ 65 times, the rates ν_{b4} and ν_{d4} by ≈ 80 and ≈ 95 times, and the rates ν_{b5} and ν_{d5} by ≈ 85 and ≈ 100 times, respectively. In general, the suprathermal ratio increases as the temperature decreases.

Although in this paper we are concerned with the value of S averaged over momentum, we do expect that S is strongly peaked just above p_c, where $\nu_{d4} - \nu_{d3}$ is small and ν_{b3} has a maximum [26]. However, the situation is complicated, as S is also expected to vary with angle within the beam.

4. Conclusion

In this paper we have shown that the asymmetry between the processes of decay and creation of high-energy phonons in long enough phonon pulses created in experiments [16–20] in superfluid helium results in a suprathermal distribution. Then the quasi-equilibrium distribution function of the h phonons differs from the Bose–Einstein distribution by a factor $S(p)$.

We have obtained an equation (17) whose solution determines the value of suprathermal ratio S and its dependence on momentum $\mathbf{p}_1$, anisotropy parameter Ω_p, and temperature T. Expressions that describe mutually compensating processes are separated in (17) by curved braces. These compensated processes have two different principal types: the first type describes processes that exchange phonons between the l and h systems, and the second type conserves the number of h phonons. That is why we consider the expressions (23), obtained from (17) by averaging with respect to all $\mathbf{p}_1$ of the anisotropic phonon system.

Starting from relation (23) and the available results for the rates of creation and decay of phonons with momentum $\mathbf{p}_1(p_1,\theta_1 = 0,\varphi_1)$ directed along the symmetry axis Z (see Fig.1), an estimate is made of the average value $\langle S\rangle$ of the suprathermal ratio. The full evaluation of the suprathermal ratio S and its dependence on the parameters $\mathbf{p}_1$, Ω_p, and T will only be possible after calculation of all the rates in (17) at arbitrary angles θ_1. We plan to carry out this calculation. At present we have only the values of all the rates $\nu_{b,d}$ for the case $\theta_1 = 0$. This estimation of and the analysis of Eqs. (17) and (23) indicates that the distribution function of h phonons can exceed the Bose–Einstein energy distribution by two orders of magnitude in anisotropic phonon systems. We find that $\langle S\rangle$ depends strongly on the parameters $\mathbf{p}_1$, Ω_p, and T. Besides the creation $\langle S\rangle$ of a more complete theory of the suprathermal distribution, we plan to carry out experiments to observe this very unusual phenomenon occuring in phonon pulses in He II.

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