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Mathematical model of seismic signal, as a flow of physically non realizable single seismic waves

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Запропоновано нову концепцію аналізу сейсмічних даних. Вона базується на попередньому вивченні сейсмічного фону. Для його параметризації використовується математична модель з фізично нездійсненними сигналами. Також пропонується специфічна математична модель самого сейсмічного сигналу. Особливість моделі полягає в тому, що вона дозволяє симулювати сейсмічні хвилі різними класами сигналів, кожен з яких з'являється в потоці зі своєю тимчасовою затримкою. Цей процес розглядається на мікросейсмічному фоні. Природно моделювати потік фізично здійсненними сигналами. Мається на увазі, що сигнали не мають передісторії. Але таке уявлення сигналу неприйнятно з двох причин. Перша пов'язана з гладкістю сигналу в точці його появи на сейсмічному записі. Друга пов'язана з тим, що на фоні сейсмічного шуму не можливо чітко виділити час вступу сигналу. Остання обставина не дає можливості отримати точний детермінований час вступу сигналу. Тому час вступу сигналу представляється як випадкова величина з довірчим інтервалом, що залежить від рівня мікросейсмічних шуму. Поняття узагальненого сейсмічного сигналу представлено як функцію часу і вектора параметрів, які визначають його форму, енергію, місце в потоці інших сигналів, його спектральні характеристики і в цілому його поведінку в усій історії його існування. Будь-яка широко поширена модель сейсмічного сигналу є окремим випадком запропонованої моделі. Або, більш строго, будь-який інший клас широко відомих моделей сейсмічного сигналу є гіперплощиною в просторі параметрів запропонованої моделі.

Ключові слова: сейсмічний сигнал, випадковий потік, апостериорна вірогідність, сейсмічний шум, математична модель.

Introduction. For creation of a universal model of seismic signal we took into account the fundamental empirical research of seismic signal and mathematical models were used for their approximation [Berzon et al., 1962; Ricker, 1953]. Also we used the streaming nature of seismic process and we considered that seismic signal must be a wave as it was noticed in [Addison, 2002]. So far as the aim of discussed modeling is to use the model for estimating the real signal parameters, which is registered against the background microseismic noise, we use the next additional restrictions. The first one is connected with the accuracy of the signal parameter evaluation that is dependent on the background noise power. The second one is connected with the fact that such evaluation depends on the prehistory of signal energy. The prehistory we represent as a time of signal existence under the level of background noise. Key property of mathematical model of every seismic signal: is this signal model physically realizable or it isn't? We will set such a frame rule. If the mathematical model is different from physically realizable one in a selected metrics less then ε, we consider the model may be taken in as a physically realizable model. Of course the e is dependent from the power of background noise. Note that the physical reliability of the signal is to satisfy two conditions. These conditions have causality and stability [Robinson, 1967].

[1] С учетом комментария В. Н. Пилипенко (с. 170).

Mathematical model. The seismic process as a stochastic one has a dual nature. The coda of seismic waves consists of superposition of a set of single waves. Every of these elements of a set is a supplement for this composition in different time. These time moments are the structure of stochastic flow. The second stochastic component of seismic process is the stochastic vector that defines the form of every single wave from this set. In proposed mathematical model of seismic process we assembled the parameters that are defining a flow nature of process and the stochastic vector of wave shape into common stochastic vector. This vector defines the every single wave in the set but when vectors are combined into matrix this matrix represents the whole process.

Stochastic flow process. Taking into account the physical essence of Bernoulli flow as flow of points we stopped our attention at this one as a more suitable to model the real seismic process. Without loss of generality in mathematical modeling of seismic process we shall use Bernoulli one as one of possible flow process.

In definition of stochastic Bernoulli process we shall be followed by [Bolshakov, 1969]. We call Bernoulli flow such a stream in which the events are falling out independently, and their number in the Ω area is fixed or, in more general case, do not exceed the specified number K. Let into Ω area there are appeared K or less then K events, but no more than K, that occurs with known partial probability density $e_i(t, \mathbf{P}_i)$, $\overline{i=1, K}$. $\mathbf{P}_i$ is vector of free parameters of distribution. Moreover the events under number i might be not occurred because of the fact that for probability p_i of such event it is permitted to be less than unit. It depends from area Ω.

$$p_i(\Omega, \mathbf{P}_i) = \int_{\Omega} e_i(t, \mathbf{P}_i)\, dt \le 1, \quad \overline{i=1, K}. \tag{1}$$

From our point of view we stopped our attention at Bernoulli process as a more suitable to model the real seismic one. Without loss of generality in mathematical modeling of seismic process we shall use Bernoulli one as one of possible. For modeling seismic background we use Poisson process as a flow arrival time of a single microseism.

Mathematical model of isolated single generalized seismic impulse. In composing the mathematical model of seismic signal we are basing on the results of fundamental empirical researches of seismic signal. The big part of these outcomes are reflected in [Berzon et al., 1962]. To follow this result we have a possibility to formulate the main requirement to mathematical model of seismic signal. It should be compatible with the physical principles of mechanics on the one hand. And the same time model should satisfy the requirements of a particular mathematical model in estimating the parameters in the optimization procedure with the other hand. But these principles are included partly in mutual contradiction. And we have to smooth these differences due to the approximation approach to solving the problem. Moreover, it is unavoidable in the conditions of the presence of natural microseismic background.

We have to taking into account seismic signal has to satisfy the three main properties [Berzon et al., 1962]. The specifics of these properties in are not discussed in rigorous manner but rather verbal one. Into mentioned investigation there is assumed that the signal is physically realizable one and is considered it is appearing at the moment $t = 0$. The prehistory of seismic process before the signal appearance is not discussed. The set of successful examples of the mathematical models of signal are given. The requirements are as the following.

1. The duration of impulse has to be approximately not less than duration of several the dominant periods.
2. The seismic impulse has to have the smooth pulse envelope function. The beginning of velocity equals zero. Its smoothing degree has to be not less than degree of smoothing acceleration.
3. The wave front of seismic impulse has to be rather smooth one. Its smoothing degree has to be not less than degree of smoothing acceleration. It means at beginning of ground motion the speed is equal zero. The first derivative, which has discontinuity might be starting not before than with acceleration.

We complete the requirements to a single seismic signal in more rigorous manner following for [Robinson, 1967; Addison, 2002]. In order to be classified as a single seismic signal $S(t)$, this function must satisfy certain mathematical criteria. The first one is stability. It means to have finite energy E in L_2 metric:

$$E = \int_{-\infty}^{\infty} (S(t))^2 \, dt < \infty. \tag{2}$$

The second one have to be a causal function. It means to satisfy such requirement:

$$\int_{-\infty}^{\infty} (S(t))^2 \, dt = \int_{\tau}^{\infty} (S(t-\tau))^2 \, dt. \tag{3}$$

The last property means the signal has a prehistory in which it does not exist and appears only after moment τ.

Together, these mentioned conditions are known as physically realizable requirements.

And the last condition for the signal is to be a wave. It means to satisfy such requirement as it does not contain constant constituent in Fourier transform. If $\tilde{S}(f)$ is the Fourier transform of $S(t)$, i. e.

$$\tilde{S}(f) = \int_{-\infty}^{\infty} S(t) \, e^{-i(2\pi f)t} dt. \tag{4}$$

Than such a condition must be fulfilled

$$\int_{0}^{\infty} \frac{\left| \hat{\Psi}(f) \right|^2}{f} \, df < \infty. \tag{5}$$

But working with such a kind of model for the flow of signals are associating with the difficulties with using of the variation approach [Kirkpatrick et al., 1983] to the problem of estimating the parameters of the flow. To say more accurate to calculate analytically the derivatives of parameters associated with the appearance of signals in the stream. Taking into consideration that the process is accompanied by microseismic background [Mostovoy et al., 2008], we cannot accurately estimate the parameters of the signal in the stream. This caused the need to consider the model physically unrealizable signals, but other than realizable ones is not more than the amount of the background power in metric L_2.

Heaviside function is used in mathematical model of seismic signal in [Mostovoy, Mostovyi, 2014]. The cumulative probability function (1) as a function of t (upper limit of integral in (1)) is a probability of arrival time of the signal number i. Moreover vector $\mathbf{P}_i$ has components P_{i1} — mathematical expectation and P_{i2} — a dispersion. For instance when we use $p_i(t, \mathbf{P}_i)$ normal distribution in our mathematical model we use it as an approximation of Heaviside function. If we approximate a cumulative probability function of the normal distribution the degree of approximation quality is determined by the dispersion of the distribution. The smaller the variance P_{i2} more accurate approximation of the Heaviside function. But this approximation is infinitely differentiable at each point in contrast to the Heaviside function. The same parameter is the variance of the distribution of the signal appearance. Naturally, this dispersion depends on the power of the background. Hereinafter we'll use instead of Heaviside function for ranging signals not a real Heaviside function, but a mollified approximation $H(t, \tau, \sigma)$, which is infinitely differentiable. The last condition is necessary for optimization approach to the solution [Evans, 1998]. $H(t, \tau, \sigma) = F(t, \tau, \sigma)$, where $F(t, \tau, \sigma)$ is cumulative normal distribution function with dispersion y that is depending on background power.

Let us define the single seismic signal $S(t, \mathbf{P})$ as a function of argument t and of a vector $\mathbf{P}$ that consists of eight free parameters of model. The transposed vector $\mathbf{P}^T$ looks like as following:

$$\mathbf{P}^T = (\tau, a, \alpha, \omega, \gamma 1, \gamma 2, \psi, \sigma), \tag{6}$$

where: τ stands as an arrival time of this signal in a wave train; a is an amplitude of signal; α is a damper in form characteristic of signal; ω is an angle frequency in carrier function; $\gamma 1$ is a damper power in form characteristic of signal; $\gamma 2$ is a growing power of signal front in form characteristic; ψ is a carrier function phase shift; σ is dispersion of Heaviside approximation. We choose such a model of single seismic signal $S(t, \mathbf{P})$:

$$S : \mathrm{R} \times \mathrm{R} \times [-A, A] \times \mathrm{R}_+ \times [\Omega_1, \Omega_2] \times \mathrm{N} \cup \{0\} \times \mathrm{N} \cup \{0\} \times \mathrm{R}_+ \times \mathrm{R}_+ \cup \{0\} \to \mathrm{R},$$

$$S(t, \mathbf{P}) = H(t, \tau, \sigma)\, a\,(t-\tau)^{\gamma 1} \exp\left\{-\left(\alpha\,(t-\tau)^{\gamma 2}\right)\right\} \sin\left(\omega\left((t-\tau)-\psi\right)\right),$$

$$\mathbf{P}^T = (\tau, a, \alpha, \omega, \gamma 1, \gamma 2, \psi, \sigma). \tag{7}$$

We can see this function is the point into eight dimension space of free model parameters and time τ

$$S(t, \mathbf{P}) = Cr(t, \omega, \psi)\, En(t, \tau, a, \alpha, \gamma 1, \gamma 2, \sigma). \tag{8}$$

This chosen function as mathematical model of generalized seismic signal (8) is convenient to represent by the product of two independent factors: $Cr(t, \omega, \psi)$ and $En(t, \tau, a, \alpha, \gamma 1, \gamma 2, \sigma)$. The first one of the product in (8) is the carrier frequency function. In signal model it is responsible only for carrier frequency parameters

$$Cr(t, \omega, \psi) = \sin(\omega(t-\tau)+\psi). \tag{9}$$

The second factor in (8) is enveloping function $En(t, \tau, a, \alpha, \gamma 1, \gamma 2, \sigma)$ which is responsible for the shape of this single signal and for the placement of this signal in the streaming process. By the way the function $H(t, \tau, \sigma)$ in envelope function is responsible for signal shape in period before its appearance up to the time of overcoming the noise threshold. Another part of (10) controls the signal shape after the time of overcoming mentioned threshold.

$$En(t, \tau, a, \alpha, \gamma 1, \gamma 2, \sigma) = H(t, \tau, \sigma)\, a\,(t-\tau)^{\gamma 1} \exp\left\{-\left(\alpha\,(t-\tau)^{\gamma 2}\right)\right\}. \tag{10}$$

Particular cases. The universal impulse model gives us ability to get different models which are spread used in practice. For this aim we have to choose the vector of free parameters in full space or in hyperplane of this space or in a crossing some of hyper-planes. For instance if we choose the vector $\mathbf{P}$ when $\gamma 1 = 1$ and $\gamma 2 = 1$ it is a crossing of two hype-planes. We get the well known Berlage impulse [Berzon et al., 1962]. So vector $\mathbf{P}$ looks like as following: $\mathbf{P}^T(\tau, a, \alpha, \omega, 1, 2, \psi, \sigma)$. In such case free parameters give us dot in ($n-2$) dimension space ($n = 8$) and result looks like as (11).

$$S(t, \mathbf{P}) = H(t, \tau, \sigma)\, a\,(t-\tau) \exp\{-(\alpha\,(t-\tau))\} \sin\left(\omega\left((t-\tau)-\psi\right)\right). \tag{11}$$

Another example when the vector $\mathbf{P}^T(\tau, a, \alpha, \omega, 0, 2, \psi, \sigma)$. when $\gamma 1 = 0$ and $\gamma 2 = 2$ it is a crossing of two hypeplanes as well. We get the well known Puzirov impulse [Berzon et al., 1962]. In such case all free parameters give us dot in ($n-2$) dimension space and result is given as (12).

$$S(t, \mathbf{P}) = H(t, \tau, \sigma)\, a \exp\left\{-\left(\alpha\,(t-\tau)^2\right)\right\} \sin\left(\omega\left((t-\tau)-\psi\right)\right). \tag{12}$$

The third important particular case is discussed in [Berzon et al., 1962] it is damping sinusoid. In this case vector $\mathbf{P}$ will be as $\mathbf{P}^T(\tau, a, \alpha, \omega, 0, 1, \psi, \sigma)$. Result will be

$$S(t, \mathbf{P}) = H(t, \tau, \sigma)\, a \exp\{-(\alpha\,(t-\tau))\} \sin\left(\omega\left((t-\tau)-\psi\right)\right). \tag{13}$$

Numerical simulation. Numerical experiment was directed at simulation all discussed aspect of proposed mathematical model of seismic data analysis when the data is a wave flow against background. To check up a possibility to get the information about signal behaviour under background by using specific unrealizable seismic signals models. Under such condition estimation of wave parameters might be only in probability sense. Background noise was simulated as Poisson flow (Fig. 1). Waves flow was simulated as Bernoulli stream of discussed signal models (Fig. 2). The mo-

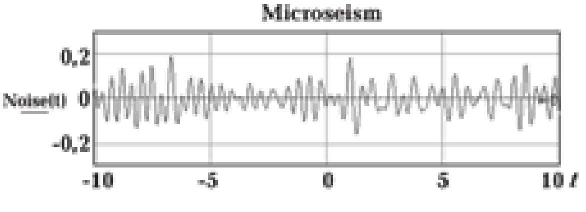

Fig. 1. Here is represented simulation of microseismic background noise as Poisson flow of microseismic signals with intensity one per one second. In metric C power is at level 2.

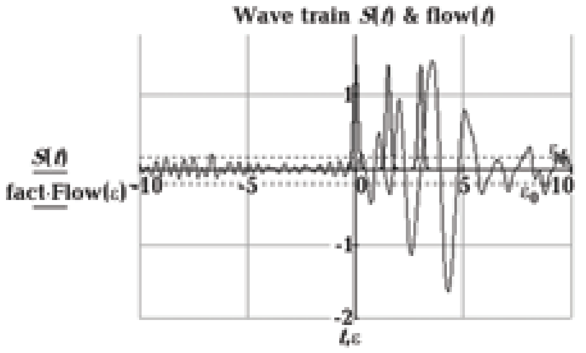

Fig. 2. Here is represented a simulation of Bernoulli flow of seismic waves against microseismic background noise. The wave train is consisted from three unresolved signals. Three sharp curves are evaluated a posteriori probability density of parameters τ. The result was got by variation method by Levenberg—Marquardt algorithm [Pujol, 2007].

del of wave train is determined by matrix $\mathbf{M} = \left\{ \mathbf{P}^{(1)},\ \mathbf{P}^{(2)},\ \mathbf{P}^{(3)} \right\}$.

Conclusions. A new conception of seismic data analysis is proposed. It is based on preliminary studying seismic background. Its characteristic is a base for using mathematical models of non realizable seismic signals. The specific mathematical model of the seismic signal is proposed as well. If we take into account the seismic background, the timing of the seismic wave packet can be estimated only as a posteriori characteristics in the form of a random vector. These estimates are based on the power of this background, proposed as a functional vector probability density functions or confidence intervals matrices.

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The new conception of seismic data analysis is proposed. It is based on preliminary studying of seismic background. Its characteristics are a base for using mathematical models of non realizable seismic signals. The specific mathematical model of the seismic signal is proposed as well. The peculiarity of the model is that it allows you to simulate the flow of seismic waves of different classes each of them appears in the stream with specific time delay. This process takes place against the micro-seismic background noise. It is natural to model the flow of signals by the physically realizable signal. It means those signals which do not have a trace in prehistory. But this representation of the signal is unacceptable for two reasons. The first one is related to the smoothness of the signal at the time of its appearance on the seismic record. The second one is related to the fact that the fade of the signal in the noise does not allow us to determine the time of its appearance on the record accurately. The latter circumstance does not leave us the possibility to simulate the time of the signal occurrence by using the determined value. Therefore, the time of occurrence of the signal is simulated by random variable with variance depending on the level of micro-seismic background. We introduce the notion of generalized seismic signal as a function of time and of the vector of parameters, which determine its shape, the energy, the place in flow of the other signals, spectral characteristics, and in general behaviour in the entire history of its existence. Any widely spread seismic signal models used in practice are a particular case of this one. Or in a more rigorous approach to the definition the different particular cases of the signals classes are transformed into the different hyper-planes into space of parameters.

Key words: seismic signal, stochastic flow, a posterior probability, seismic background noise, mathematical model.

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