THE INFLUENCE OF THE EXTERNAL MAGNETIC FIELD ON ENERGY LOSSES OF A CHARGED PARTICLE IN AN ELECTRON GAS

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The influence of an external magnetic field and a temperature of an electron gas on energy losses of a charged massive particle is taken into account in the framework of quantum-field approach. The analytical expressions of real and imaginary parts of the dielectric susceptibility of the electron gas in the external magnetic field are obtained. Energy losses of the heavy charged particle in the electron gas in the magnetic field are numerical calculated. Results of the numerical calculations are verified by analytical expressions obtained in work [1].

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IINTRODUCTION

This article continues research of energy losses of heavy charged particles in an electron gas in an external magnetic field within quantum-field methods [1-5]. The influence of a temperature of the electron gas [2] and the external magnetic field [1] on energy losses of the charged massive particle is concidered in the framework of quantum-field methods from the first principles. These factors are crucial for experiments on electron cooling [6, 7] and nuclear fusion [8].

Energy losses of the heavy charged particle in the electron gas in the external magnetic field are calculated numerically in the research. In section 3, analytical expressions of real and imaginary parts of the dielectric susceptibility of the electron gas in the external magnetic field are obtained with the temperature of the electrons considered. In section 4, the results of numerical calculations of energy losses for the cases $\omega_H / \omega_P = 10^4 ... 10^5$ are presented. The results of the numerical calculation are compared to the analytical results obtained in work [1].

1. ENERGY LOSSES OF THE HEAVY CHARGED PARTICLE IN THE ELECTRON GAS

Energy losses of the heavy charged particle are determined by the properties of the medium of electrons [1, 2].

$$-\frac{dE}{dt}\frac{V_0}{e^2\omega_0^2} = S = \frac{1}{\pi^2}\int d^3k \frac{w}{k^2\left[1-\exp\left(-\frac{\omega}{T_e}\right)\right]}\times)$$

$$\frac{Im\kappa(\vec{k},\omega)}{\left(1+Re\kappa(\vec{k},\omega)\right)^2+\left(Im\kappa(\vec{k},\omega)\right)^2}, \quad (1)$$

where $\kappa(\vec{k},\omega)$ is a susceptibility of the electron gas. The susceptibility of the magnetized electron gas within the quantum-field method can be written as [1]

$$\kappa(\vec{k},\omega) = -\frac{2\pi e^2 m\omega_H}{(2\pi\hbar)^2}\frac{1}{k^2}\sum_{\nu,\nu'}\Lambda_{\nu\nu'}\left(\frac{\hbar k_t}{\sqrt{2m\hbar\omega_H}}\right)\times$$

$$\int_{-\infty}^{\infty} dp_z \frac{n_{\nu p_z} - n_{\nu', p_z-\hbar k_z}}{\varepsilon_{\nu p_z} - \varepsilon_{\nu', p_z-\hbar k_z} - \hbar\omega - i0}, \quad (2)$$

where $n_\alpha = n_{\nu p_z} = \left(e^{\beta(\varepsilon_{\nu p_z}-\mu)}+1\right)^{-1}$ is the Fermi distribution function; μ is the chemical potential; k is the wave vector, $\omega_H = eH/mc$ is the Larmor frequency of the electron in the magnetic field H;

$$\Lambda_{\nu,\nu'}(a) = \int_0^\infty ds\exp(-s)J_0(2a\sqrt{s})L_\nu(s)L_{\nu'}(s); \quad (3)$$

$$k_t = \sqrt{k_x^2+k_y^2}; \quad (4)$$

$$\varepsilon_\alpha = \varepsilon_{\nu p_z} = \eta(\nu+1/2)+p_z^2/2m \quad (5)$$

is the energy of electrons in the external magnetic field.

For the final numerical calculation is more convenient to use dimensionless variables so we introduce some notation:

• dimensionless momenta of an electron and an ion

$$r = \frac{p}{mV_0}; \quad s_i = \frac{P_i}{mV_0}; \quad (6)$$

• a dimensionless wave vector

$$q = k\frac{V_0}{\omega_0}; \quad (7)$$

• dimensionless frequencies ω_H and ω

$$h = \frac{\omega_H}{\omega_0}; \quad w = \frac{\omega}{\omega_0}; \quad (8)$$

• a dimensionless temperature

$$\tau = \frac{T_e}{mV_0^2}. \quad (9)$$

The susceptibility in dimensionless variables is

$$\kappa(\vec{k},\omega) = -\pi\frac{4\pi e^2}{m\omega_0^2}\frac{(mV_0)^3}{(2\pi\hbar)^3}\frac{h}{q^2}\sum_{\nu,\nu'}\Lambda_{\nu\nu'}\left(q_t\sqrt{\delta_0 h^{-1}}\right)\times$$

$$\int_{-\infty}^{\infty} dr_z \frac{n_{\nu r_z} - n_{\nu', r_z-q_z}}{q_z r_z - \delta_0 q_z^2 + h(\nu-\nu') - w - i0}, \quad (10)$$

where (2)

$$\delta_0 = \frac{\hbar\omega_0}{2mV_0^2}. \quad (11)$$

Values of V_0 and ω_0 are determined according to parameters of the task.

1.1. DISTRIBUTION FUNCTION n_{ν,p_z}

A $0z$ axis is directed along the external magnetic field. The movement of the electron beam also coincides with the axis $0z$. Taking into account eq. (5) one can write distribution function in a form

$$n_{\nu,p_z} = \exp\left(-\frac{\hbar\eta(\nu+1/2)}{T_e} - \frac{p_z^2}{2mT_e}\right). \quad (12)$$

In dimensionless variables n_{ν,p_z} is written as:

$$n_{\nu',r_z} = \exp\left(-\frac{4h(\nu'+1/2)\delta_0 + r_z^2}{2\tau}\right). \quad (13)$$

The electron energy distribution function has to satisfy the normalization conditions [1]

$$n = \pi\frac{2m^2V_0\hbar\omega_0}{(2\pi\hbar)^3}h\sum_{\nu=0}^{\infty}\int dr_z\, n_{\nu,r_z}. \quad (14)$$

Substituting eq.(13) into eq.(14) and carrying out integration and summation, we obtain

$$n = \pi\frac{2m^2V_0\hbar\omega_0}{(2\pi\hbar)^3}h\sqrt{2\pi\tau}\exp\left(-\frac{h\delta_0}{\tau}\right)\times \left(1+\frac{\exp\left(-\frac{2h\delta_0}{\tau}\right)}{1-\exp\left(-\frac{2h\delta_0}{\tau}\right)}\right). \quad (15)$$

In a weak magnetic field limit $\omega_H \to 0$ the equation (15) can be simplified to the nonmagnetic form [2, 4].

$$n = \frac{(mV_0)^3}{(2\pi\hbar)^3}(2\pi\tau)^{3/2}. \quad (16)$$

2. THE SUSCEPTIBILITY OF THE ELECTRON GAS IN THE EXTERNAL MAGNETIC FIELD

2.1. INTEGRATION OVER p_z

Integration over p_z in the expression for the dielectric susceptibility eq.(10) occurs as in the case without a magnetic field [4, 5]. A final formula is written as

$$\kappa(\vec{k},\omega) = -\frac{\pi}{4}\frac{\omega_P^2}{\omega_0^2}\frac{1}{\delta_0}\frac{1}{\sqrt{2\pi\tau}}\frac{1}{q^2q_z}\left[1-\exp\left(-\frac{2h\delta_0}{\tau}\right)\right]\times$$
$$\times\sum_{\nu,\nu'}\Lambda_{\nu\nu'}(a)\left[\exp(-\beta\nu)\exp\left(-\xi_{1h}^2\right)\{i-erfi(\xi_{1h})\}-\right.$$
$$\left.\exp\left(-\beta\nu'\right)\exp\left(-\xi_{2h}^2\right)\{i-erfi(\xi_{2h})\}\right], \quad (17)$$

where

$$\xi_{jh} = \xi_j - \xi_h = \frac{\left(w+(-1)^{j-1}\delta_0q_z^2\right)}{q_z\sqrt{2\tau}} - \frac{h(\nu-\nu')}{q_z\sqrt{2\tau}},\ j=1,2; \quad (18)$$

$$\beta = \frac{2h\delta_0}{\tau}; \quad (19)$$

$$a = q_t\sqrt{\delta_0h^{-1}}. \quad (20)$$

2.2. $\Lambda_{\nu,\nu'}(a)$ FUNCTION

A general solution of the $\Lambda_{\nu,\nu'}(a)$ function eq.(3) with using recurrence relations of the Laguerre polynomials [9]

$$(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x) \quad (21)$$

is

$$\Lambda_{\nu,s}(a) = \frac{\nu!}{(\nu+s)!}a^{2s}\exp\left(-a^2\right)L_\nu^s(a^2)L_\nu^s(a^2). \quad (22)$$

Let moving on from the summation over (ν,ν') to the summation over $(\nu, s=\nu-\nu')$. Imaginary and real parts of susceptibility are written as

$$Im\kappa(\vec{k},\omega)_{\nu,s} = \frac{\pi}{2}\frac{\omega_P^2}{\omega_0^2}\frac{1}{\delta_0}\frac{1}{\sqrt{2\pi\tau}}\frac{1}{q^2q_z}\left[1-\exp(-\beta)\right]\times$$
$$\times\exp\left\{-\xi_2^2\right\}\left[1-\exp\left\{-\frac{2w\delta_0}{\tau}\right\}\right]\times$$
$$\sum_{\nu=0}^{\infty}\sum_{s=0}^{\infty}\Lambda_{\nu,s}(a)\exp\{-\beta\nu\}\exp\left\{-s\frac{\beta}{2}\right\}\times$$
$$\exp\left\{-\xi_{hs}^2\right\}\cosh\left\{s\frac{hw}{q_z^2\tau}\right\}. \quad (23)$$

$$Re\kappa(\vec{k},\omega)_{\nu,s} = \frac{\pi}{4}\frac{\omega_P^2}{\omega_0^2}\frac{1}{\delta_0}\frac{1}{\sqrt{2\pi\tau}}\frac{1}{q^2q_z}\left[1-\exp(-\beta)\right]\times$$
$$\times\sum_{\nu=0}^{\infty}\sum_{s=0}^{\infty}\Lambda_{\nu,s}(a)\exp(-\beta\nu)\{\exp(-s\beta)\times$$
$$\left[e^{-(\xi_1-\xi_{hs})^2}erfi(\xi_1-\xi_{hs}) - e^{-(\xi_2+\xi_{hs})^2}erfi(\xi_2+\xi_{hs})\right]+$$
$$\left.\left[e^{-(\xi_1+\xi_{hs})^2}erfi(\xi_1+\xi_{hs}) - e^{-(\xi_2-\xi_{hs})^2}erfi(\xi_2-\xi_{hs})\right]\right\}. \quad (24)$$

Let use equation from [10] to sum expressions (23) and (24) over ν

$$\sum_{n=0}^{\infty}\frac{n!}{\Gamma(n+\alpha+1)}L_n^\alpha(x)L_n^\alpha(y)z^n =$$
$$= \frac{(xyz)^{-\frac{1}{2}\alpha}}{1-z}\exp\left(-z\frac{x+y}{1-z}\right)I_\alpha\left(2\frac{\sqrt{xyz}}{1-z}\right),$$
$$|z|<1,\quad \alpha>-1. \quad (25)$$

In this case $z=\exp(-\beta)$, $\alpha = s$, $y=x=a^2$. Hence,

$$\times\left[1-\exp\left\{-\frac{2w\delta_0}{\tau}\right\}\right]\exp\left(-a^2\frac{1+\exp(-\beta)}{1-\exp(-\beta)}\right)\times$$
$$\sum_{s=0}^{\infty}I_s\left(2a^2\frac{\exp\left(-\frac{\beta}{2}\right)}{1-\exp(-\beta)}\right)\exp\left\{-\xi_{hs}^2\right\}\cosh\left\{s\frac{hw}{q_z^2\tau}\right\}. \quad (26)$$

$$Im\kappa(\vec{k},\omega) = \frac{\pi}{2}\frac{\omega_P^2}{\omega_0^2}\frac{1}{\delta_0}\frac{1}{\sqrt{2\pi\tau}}\frac{1}{q^2q_z}\exp\left\{-\xi_2^2\right\}\times$$

$$Re\,\kappa(\vec{k},\omega)=\frac{\pi}{4}\frac{\omega_P^2}{\omega_0^2}\frac{1}{\delta_0}\frac{1}{\sqrt{2\pi\tau}}\frac{1}{q^2 q_z}\exp\left(-a^2\frac{1+\exp(-\beta)}{1-\exp(-\beta)}\right)\times$$

$$\sum_{s=0}^{\infty} I_s\left(2a^2\frac{\exp\left(-\frac{\beta}{2}\right)}{1-\exp(-\beta)}\right)\left\{\exp\left(-s\frac{\beta}{2}\right)\times\right.$$

$$\left[e^{-(\xi_1-\xi_{hs})^2} erfi(\xi_1-\xi_{hs})-e^{-(\xi_2+\xi_{hs})^2} erfi(\xi_2+\xi_{hs})\right]+$$

$$\exp\left(s\frac{\beta}{2}\right)\times$$

$$\left.\left[e^{-(\xi_1+\xi_{hs})^2} erfi(\xi_1+\xi_{hs})-e^{-(\xi_2-\xi_{hs})^2} erfi(\xi_2-\xi_{hs})\right]\right\}. \quad (27)$$

3. NUMERICAL CALCULATIONS

Numerical calculations were performed for the following parameters: the electron Langmuir frequency is $\omega_P = 2.9\cdot 10^8 s^{-1}$, the electron temperature is $T_e = 10^{-3} eV$, the ratio of the cyclotron and plasma frequencies $h=\omega_H/\omega_P = 10^4...10^5$. It is a case of the strong magnetized electron gas.

The depending of dimensionless energy losses S on the velocity of the projectile particle V_i is presented on Figs. 1, 2 in the case $\vec{V_i}\otimes\vec{H}$. The temperature of electron gas T_e is isotropic. Energy losses $-dE/dt$ is normalized to $e^2\omega_P^2/V_0$, the velocity of the projectile particle V_i is in units of V_0.

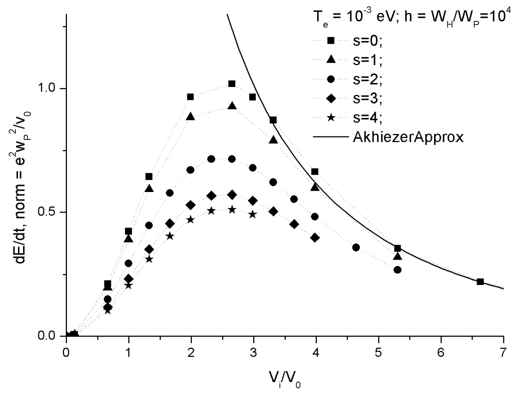

Fig. 1. Dependency of energy losses $-dE/dt$ on the velocity of the projectile particle V_i. Number of terms taken into account to sum: dotted line with squares corresponds index of summation s = 0; triangles corresponds s = 1; cirles corresponds s = 2; with rombuses corresponds s = 3; with stars corresponds s = 4. Electron gas temperature is $T_e = 10^{-3} eV$. A solid curve: the magnetized electron gas without temperature, after [1]. Energy losses are in units $e^2\omega_P^2/V_0$. Velocity V_i is in units $V_0 = 10^6$ cm/s

Fig. 1 exhibits the dependency of energy losses on velocity of the projectile particle with parameters: $h=10^4$, $\beta<1$. Dotted lines with squares, triangles, circles, rhombuses and stars correspond to sum of the zero ($s = 0$), the zero and the first ($s=1$), the zero, the first and the second ($s = 2$), the zero, the first , the second and the third ($s = 3$) terms in eq.(23), (24), respectively. The solid curve corresponds to the analytic dependence of energy losses of the heavy charged particle in the approximation of a cold electron plasma ($T_e = 0$), obtained in [1].

Fig. 1 shows that it is sufficient to sum first few terms in the dielectric susceptibility to obtain energy losses of the heavy charged particle in a magnetized electron gas for parameters $h=10^4$, $T_e = 10^{-3} eV$. Results of numerical calculations confirms the analytical curve (see solid curve in Fig. 1), obtained in [1] for the case of high velocities of the incident particle, $v_e^2/V_i^2 = 1$.

On Fig. 2 results of numerical calculations with parameters $h=10^4$ (a line with empty figures) and $h=10^5$ (lines with filled figures) are compared.

Fig. 2 shows that only the term with the number $s = 0$ (a line with filled squares) gives a contribution to energy losses for parameters $h=10^5$ ($\beta>1$). The next term with the number $s=1$ (a line with filled rombuses) does not change results of calculation. The further increasing of the magnetic field does not affect the change in energy losses.

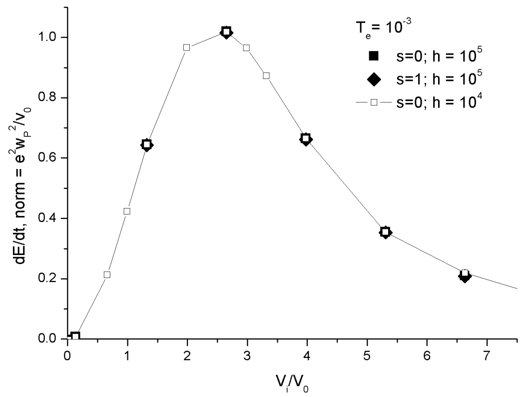

Fig. 2. Dependency of energy losses $-dE/dt$ on the velocity of the projectile particle V_i for different values of h: $h=10^5$ filled squares correspond index of summation s = 0, filled rombuses correspond s = 1; $h=10^4$ empty squars correspond s = 0. Electron gas temperature is $T_e = 10^{-3} eV$. Energy losses are in units $e^2\omega_P^2/V_0$. Velocity V_i is in units $V_0 = 10^6$ cm/s

CONCLUSIONS

The effect of the electron gas temperature and of the external magnetic field on the energy loss of heavy charged particles is calculated within of quantum field theory. Analytical expressions of the real and imaginary parts of the dielectric susceptibility of the electron gas are obtained. Dependencies of energy losses on the velocity of the projectile particle are numerical calculated for parameters of the electron gas $\omega_P = 10^8 s^{-1}$ and $\omega_H = 10^{12}...10^{13} s^{-1}$.

A transverse motion of electrons is suppressed by the external magnetic field for $\omega_H = 10^{13} s^{-1}$.

Energy transmitted by the projectile particle to the electron gas is not sufficient to ensure that electrons could move to another Landau level. Therefore, the contribution to energy losses gives only the term with $s = \nu - \nu' = 0$. The result of numerical computation at high speeds limits of the projectile particle, $v_e^2/V_i^2 = 1$, is in good agreement with the analytical results obtained in the research [1].

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ВЛИЯНИЕ ВНЕШНЕГО МАГНИТНОГО ПОЛЯ НА ЭНЕРГЕТИЧЕСКИЕ ПОТЕРИ ЗАРЯЖЕННОЙ ЧАСТИЦЫ В ЭЛЕКТРОННОМ ГАЗЕ

А.В. Хелемеля, Р.И. Холодов

В рамках квантово-полевого подхода учтены одновременно влияние и магнитного поля, и температуры электронного газа на энергетические потери тяжёлой заряженной частицы. Получены аналитические выражения для мнимой и реальной частей диэлектрической восприимчивости. Численно посчитаны энергетические потери тяжёлой заряженной частицы в электронном газе во внешнем магнитном поле. Результаты численного счёта верифицированы аналитическими результатами работы [1].

ВПЛИВ ЗОВНІШНЬОГО МАГНІТНОГО ПОЛЯ НА ЕНЕРГЕТИЧНІ ВТРАТИ ЗАРЯДЖЕНОЇ ЧАСТИНКИ В ЕЛЕКТРОННОМУ ГАЗІ

О.В. Хелемеля, Р.І. Холодов

У рамках квантово-польового підходу одночасно врахований вплив як зовнішнього магнітного поля, так і температури електронного газу на енергетичні втрати важкої зарядженої частинки. Отримано аналітичні вирази для уявної та дійсної частин діелектричної сприйнятливості. Чисельно підраховані енергетичні втрати важкої зарядженої частинки в електронному газі в зовнішньому магнітному полі. Результати чисельного розрахунку добре узгоджуються із аналітичними результатами, отриманими в роботі [1].