TWO DIMENSIONAL CODE FOR MODELING OF HIGH IONE CYCLOTRON HARMONIC FAST WAVE HEATING AND CURRENT DRIVE

D. Grekov[1], S. Kasilov[1,2], W. Kernbichler[2]

[1]Institute of Plasma Physics of the NSC KIPT, Kharkov, Ukraine;

[2]Institut fur Theoretische Physik–Computational Physics, TU Graz, Graz, Austria

E-mail: grekov@ipp.kharkov.ua

A two dimensional numerical code for computation of the electromagnetic field of a fast magnetosonic wave in a tokamak at high harmonics of the ion cyclotron frequency has been developed. The code computes the finite difference solution of Maxwell's equations for separate toroidal harmonics making use of the toroidal symmetry of tokamak plasmas. The proper boundary conditions are prescribed at the realistic tokamak vessel. The currents in the RF antenna are specified externally and then used in Ampere's law. The main poloidal tokamak magnetic field and the "kinetic" part of the dielectric permeability tensor are treated iteratively. The code has been verified against known analytical solutions and first calculations of current drive in the spherical torus are presented.

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INTRODUCTION

None-inductive current drive is essential for long-pulse or steady-state operation of spherical tori. Along with Electron Bernstein Waves, the High Harmonic Fast Waves (HHFW) are among the candidates for current drive in these devices. The basic mechanism of HHFW power absorption is electron transit time magnetic pumping (TTMP). Due to high beta values in spherical torus discharges, the absorption of HHFW is rather high and essential heating [1] and current drive [2] have been demonstrated. Up to now HHFW heating and current drive have been modeled with the ray tracing code GENRAY and the code AORSA (All-ORders Spectral Algorithm) [3]. However, the ray tracing approach has intrinsic drawbacks, e.g., undefined conditions of ray reflection at plasma boundaries and ignoring of wave field interference. In addition, the code AORSA can only be used on supercomputers. Within this investigation, HHFW power absorption and current drive in a spherical torus with a divertor are modeled numerically on a PC where the distribution of electromagnetic fields in the plasma is computed taking into account a real equilibrium magnetic field and plasma density distribution.

1. THE CODE STRUCTURE

The code is organized in the following way (Fig. 1). First, real space coordinates of the chamber are digitized and provided as input. Data such as main magnetic field and magnetic flux surfaces are provided using a given EFIT equilibrium or another equilibrium format. Then, the location and the poloidal structure of the HHFW antenna are specified. The density and temperature dependencies on the magnetic flux label ψ are assigned for the main plasma, the scrape-off layer, as well as for the divertor region. These values are then passed to the wave solver. The wave solver computes the electromagnetic field distributions of the fast magnetosonic wave (FMSW) in the minor cross-section of the torus as well as the absorbed power $P_{abs}(\psi)$. The wave magnetic field $\tilde{B}_{||}(\psi)$ parallel to main magnetic field $\vec{B}_0$ determines the quasilinear diffusion coefficient being the input for the code SYNCH [4] which then calculates the density profile of the driven current $j_{CD}(\psi)$.

2. THE WAVE SOLVER

The wave solver uses with the Maxwell equations in cylindrical coordinates (R, φ, Z). The Z-axis of this system coincides with torus axis and φ is the toroidal angle. The main magnetic field has normalized components b_R, b_φ, b_Z with prevailing component b_φ. Further, only harmonic oscillations $\sim\exp(-i\omega t)$, where ω is the wave frequency, are considered. The frequency band $\omega \sim n\omega_{ci} << \omega_{ce}$ is investigated. Here $n \sim 6...10$ and ω_{ci}, ω_{ce} are the ion and electron cyclotron frequencies, respectively. The harmonic dependence of wave fields on the toroidal angle φ is prescribed $\vec{E}(\vec{r}), \vec{B}(\vec{r}) = \left[\vec{E}(R,Z), \vec{B}(R,Z)\right]\cdot\exp(il\varphi)$ due to toroidal symmetry. Each toroidal harmonic l is processed separately.

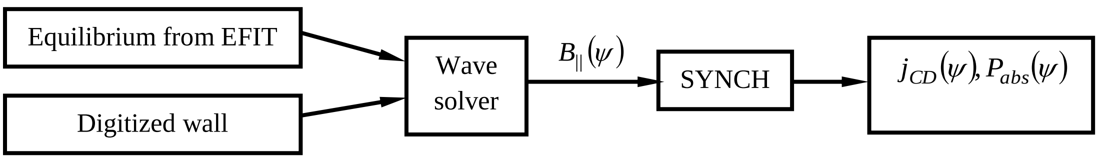

Fig. 1. The structure of the code

With this, the Maxwell equations take the form

$$\frac{il}{R}E_Z - \frac{\partial E_\varphi}{\partial Z} = i\frac{\omega}{c}B_R\,, \qquad (1)$$

$$\frac{\partial E_R}{\partial Z} - \frac{\partial E_Z}{\partial R} = i\frac{\omega}{c}B_\varphi\,, \qquad (2)$$

$$\frac{1}{R}\frac{\partial(RE_\varphi)}{\partial R} - \frac{il}{R}E_R = i\frac{\omega}{c}B_Z\,, \qquad (3)$$

$$\frac{il}{R}B_Z - \frac{\partial B_\varphi}{\partial Z} = -i\frac{\omega}{c}\left(\hat{\varepsilon}\vec{E}\right)_R + \frac{4\pi}{c}j_{AR}\,, \qquad (4)$$

$$\frac{\partial B_R}{\partial Z} - \frac{\partial B_Z}{\partial R} = -i\frac{\omega}{c}\left(\hat{\varepsilon}\vec{E}\right)_\varphi\,, \qquad (5)$$

$$\frac{1}{R}\frac{\partial(RB_\varphi)}{\partial R} - \frac{il}{R}B_R = -i\frac{\omega}{c}\left(\hat{\varepsilon}\vec{E}\right)_Z + \frac{4\pi}{c}j_{AZ}\,. \qquad (6)$$

Here, physical field components are used, $\hat{\varepsilon}$ is the tensor of the plasma dielectric permittivity, $\vec{j}_A$ is the antenna current ("external" current in Ampere's law). Because $\omega >> \omega_{ci}$ and $\omega << \omega_{ce}$, the MHD approximation is used for the tensor $\hat{\varepsilon}$. The input from transit time magnetic pumping is the only important kinetic component. It is taken into account using an iteration procedure. It is known, that for FMSW the parallel field component of the wave electric field $E_{||}$ is negligibly small $E_{||} \sim \frac{\varepsilon_1}{\varepsilon_3}E_\perp \sim \frac{m}{M}E_\perp$. Here ε_1 and ε_3 are the perpendicular and parallel diagonal components of the plasma dielectric permittivity tensor, respectively, m is electron mass and M is ion mass. So, $E_{||}$ is neglected in the code everywhere, apart from terms, which contain $\varepsilon_3 E_{||}$. Thus, the expression $E_\varphi = -\frac{b_R}{b_\varphi}E_R - \frac{b_Z}{b_\varphi}E_Z$ follows from $E_{||}=0$. Finally, the system of equations (1)-(6) is transformed to

$$-i\frac{\omega}{c}B_\varphi + \frac{\partial E_R}{\partial Z} - \frac{\partial E_Z}{\partial R} = 0\,, \qquad (7)$$

$$i\frac{\omega}{c}\frac{\partial B_\varphi}{\partial Z} + \frac{\omega^2}{c^2}\left(\varepsilon_1 - N_\varphi^2\right)E_R - i\frac{\omega^2}{c^2}\frac{\varepsilon_2}{b_\varphi}E_Z = \frac{4\pi}{c}j_{AR} + \tilde{V}_R + \frac{4\pi}{c}j_R^{TTMP}\,, \qquad (8)$$

$$-i\frac{\omega}{c}\frac{\partial(RB_\varphi)}{\partial R} + i\frac{\omega^2}{c^2}\frac{\varepsilon_2}{b_\varphi}E_R + \frac{\omega^2}{c^2}\left(\varepsilon_1 - N_\varphi^2\right)E_Z = \frac{4\pi}{c}j_{AZ} + \tilde{V}_Z + \frac{4\pi}{c}j_Z^{TTMP}\,. \qquad (9)$$

Here ε_2 is the off-diagonal perpendicular component of the dielectric permittivity tensor, and

$$\tilde{V}_R = i\frac{l}{R}\frac{1}{R}\frac{\partial(RE_\varphi)}{\partial R} - \frac{b_R}{b_\varphi}\left\{\frac{\partial}{\partial R}\left(\frac{il}{R}E_R\right) + \frac{il}{R}\frac{\partial E_Z}{\partial Z} - \frac{\partial}{\partial R}\left[\frac{1}{R}\frac{\partial(RE_\varphi)}{\partial R}\right] - \frac{\partial^2 E_\varphi}{\partial Z^2} - \frac{\omega^2}{c^2}\varepsilon_1 E_\varphi\right\}, \qquad (10)$$

$$\tilde{V}_Z = i\frac{l}{R}\frac{1}{R}\frac{\partial E_\varphi}{\partial Z} - \frac{b_Z}{b_\varphi}\left\{\frac{\partial}{\partial R}\left(\frac{il}{R}E_R\right) + \frac{il}{R}\frac{\partial E_Z}{\partial Z} - \frac{\partial}{\partial R}\left[\frac{1}{R}\frac{\partial(RE_\varphi)}{\partial R}\right] - \frac{\partial^2 E_\varphi}{\partial Z^2} - \frac{\omega^2}{c^2}\varepsilon_1 E_\varphi\right\}, \qquad (11)$$

are the terms proportional to the main poloidal magnetic field, and

$$\vec{j}^{\,TTMP} = \nabla\times U^{TTMP}\vec{B}_0\,,\; U^{TTMP} = c\frac{\beta_e}{2\pi}i\sqrt{\pi}z_e W(z_e)\frac{\tilde{B}_{||}}{B_0}\,, \qquad (12)$$

where c is speed of light, $\beta_e = \frac{4\pi n_e T_e}{B_0^2}$, n_e is electron density, T_e is electron temperature, $z_e = \frac{\omega}{\sqrt{2}k_{||}v_{Te}}$, $k_{||} = l/R$, $v_{Te} = \sqrt{T_e/m}$, $W(z_e)$ is the Kramp function. To resolve the system of equations (7)-(9), the finite difference method is used. The code uses an equidistant rectangular mesh with a choice of the step-sizes ΔR and ΔZ for the wave magnetic field B_φ. Following the logic of Eq. (7), other grids for the wave electric fields are introduced in the computational domain shifted by $\Delta Z/2$ for E_R and $\Delta R/2$ for E_Z (Fig. 2).

The effect of the poloidal tokamak magnetic field ($\tilde{V}_R$ and $\tilde{V}_Z$ in Eqs. (8, 9)) is taken into account with help of a preconditioned iteration technique and kinetic part of dielectric permittivity tensor is included in the same way.

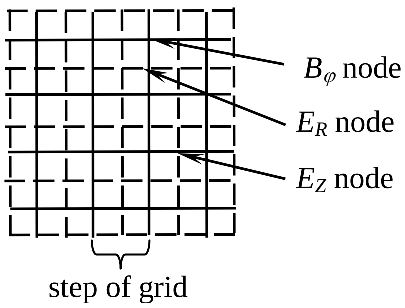

Fig. 2. Computational grid for B_φ consists of the nodes, located in the intersections of the solid lines. Grid for E_R consists of the nodes, located in the intersections of the vertical solid lines and horizontal dashed lines. Grid for E_Z consists of the nodes, located in the intersections of the vertical dashed lines and horizontal solid lines

3. VERIFICATION OF THE WAVE SOLVER

To verify the wave solver it is necessary to compare the results with an analytical solution.

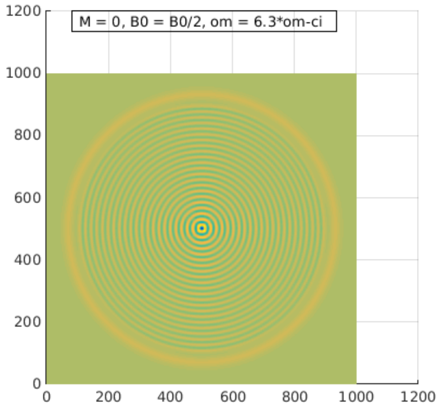

Fig. 3. Distribution of fast wave magnetic field in the circular minor cross-section for a poloidally symmetric antenna

It is possible to do for the case of homogeneous plasma cylinder with a constant axial main magnetic field. In order to do so, the minor cross-section of the plasma was chosen to be a circle, plasma density was set to a constant, aspect ratio was set to 10^5, and the main poloidal magnetic field was set to zero. In addition, the antenna current was set to a single poloidal harmonic. In this case, the Bessel function is the analytical solution for the wave magnetic field component in the plasma. In Fig. 3 the component B_φ pattern is shown for $\omega = 1.8 \cdot 10^9 s^{-1}$, $k_{\|} = 0.03\,cm^{-1}$, $n_e = 1.5 \cdot 10^{13} cm^{-3}$, $B_0 = 3kG$ and a poloidally symmetric antenna (poloidal number M = 0). As seen in this figure, the result has quite symmetric azimuthal distribution. In addition, the radial structure of the wave magnetic field is shown in Fig. 4.

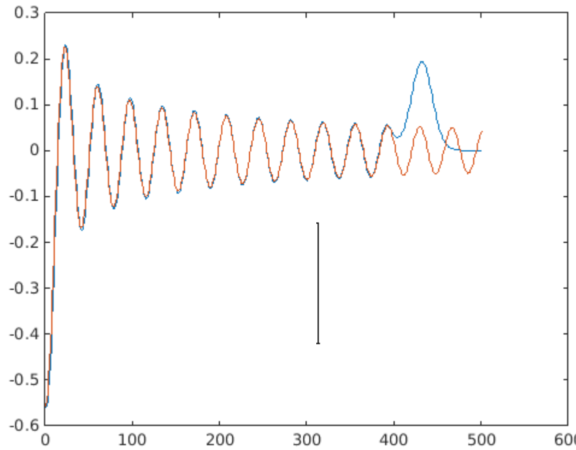

Fig. 4. Radial dependence of fast wave magnetic field (M = 0, blue) in comparison with the Bessel function of the order 0 (red)

The radial structure of the computed wave magnetic field coincides perfectly with the analytical solution within the plasma region (see Fig. 4). The discrepancy between the curves in the outer region is due to non-applicability of the analytical solution there. A check for high poloidal harmonics was also been performed. As seen in Fig. 5, the computed wave magnetic field also perfectly matches the analytical solution for M = 9.

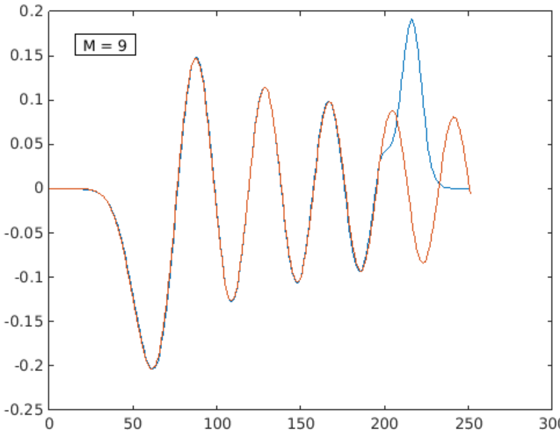

Fig. 5. Radial dependence of fast wave magnetic field (M = 9, blue) in comparison with the Bessel function of the order 9 (red)

Another crucial point of computations is the compliance with the energy conservation law

$$div\left(\vec{S}\right) + \frac{1}{2}\mathrm{Re}\left(\vec{j}\vec{E}^*\right) = 0\,, \qquad (13)$$

where $\vec{S}$ is the Poynting vector. As it shown in Fig. 6, Eq. (13) is valid to the order of $\delta \sim 10^{-17}$,

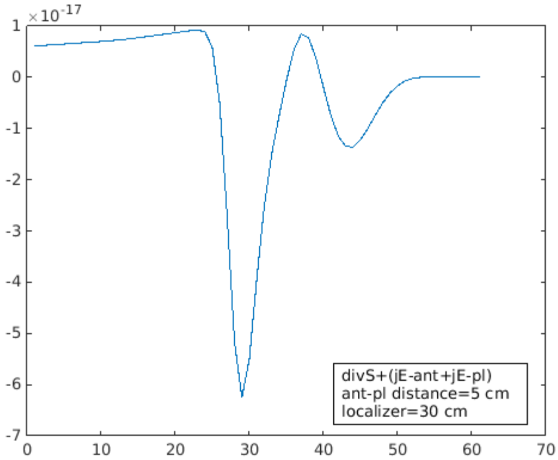

Fig. 6. Radial dependence of discrepancy in the energy conservation low δ

whereas, each term in (14) is of the order of 10^{-3}. The value of δ is slightly improved with decrease in grid spacing. In summary one can state that the wave solver is verified successfully.

4. CALCULATIONS OF CD

Test calculations of current drive efficiency were done for spherical torus with NSTX-like parameters [3]. The fast wave TTMP current drive efficiency was calculated by the SYNCH code [4] using the local approximation for the wave-particle interaction (Fig. 7).

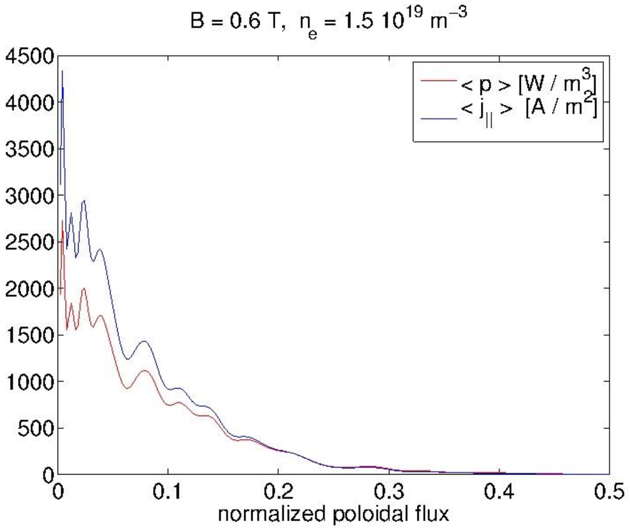

Fig. 7. CD density and absorbed power of fast wave vs magnetic flux label ψ

CONCLUSIONS

The two dimensional numerical code for calculation of HHFW power absorption and current drive has been developed. The code was successfully verified against known analytical solutions. The first calculations of current drive in the spherical torus using the local approximation for the wave-particle interaction have been performed. In the future the code will be modified to take into account non-local wave-particle interaction.

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REFERENCES

1. S. Kaye et al. // *Phys. Plasmas*. 2001, v. 8, № 5, p. 1977-1987.
2. D.A. Gates et al. // *29th EPS Conference on Plasma Phys. and Contr. Fusion* (Montreux, 17-21 June 2002), ECA 2002, v. 26B, O-2.09.
3. J. Hosea et al. // *Phys. Plasmas.* 2008, v. 15, p. 056104.
4. S.V. Kasilov, W. Kernbichler // *Phys. Plasmas*. 1996, v. 3, p. 4115-4127.

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ДВУМЕРНЫЙ КОД ДЛЯ МОДЕЛИРОВАНИЯ НАГРЕВА ПЛАЗМЫ И СОЗДАНИЯ ТОКОВ УВЛЕЧЕНИЯ БЫСТРЫМИ ВОЛНАМИ НА ВЫСОКИХ ГАРМОНИКАХ ЦИКЛОТРОННОЙ ЧАСТОТЫ ИОНОВ

Д. Греков, С. Касилов, В. Кернбихлер

Разработан двумерный код для расчёта электромагнитного поля быстрой магнитозвуковой волны в токамаках в диапазоне высоких гармоник ионной циклотронной частоты. Используя тороидальную симметрию плазмы токамака, методом конечных разностей код находит решение уравнений Максвелла для отдельной тороидальной гармоники. Граничные условия задаются на реальной стенке камеры токамака. ВЧ-антенна моделируется введением «внешнего» тока в закон Ампера. Полоидальное удерживающее магнитное поле и вклад «тепловой» части тензора диэлектрической проницаемости плазмы включены в расчёты с помощью итераций. Код был верифицирован путём сравнения расчётов с известным аналитическим решением. Представлены первые расчёты тока увлечения в сферическом торе.

ДВОВИМІРНИЙ КОД ДЛЯ МОДЕЛЮВАННЯ НАГРІВАННЯ ПЛАЗМИ ТА СТВОРЕННЯ СТРУМІВ ЗАХОПЛЕННЯ ШВИДКИМИ ХВИЛЯМИ НА ВИСОКИХ ГАРМОНІКАХ ЦИКЛОТРОННОЇ ЧАСТОТИ ІОНІВ

Д. Греков, С. Касілов, В. Кернбіхлер

Розроблено двовимірний код для розрахунку електромагнітного поля швидкої магнітозвукової хвилі в токамаках у діапазоні високих гармонік циклотронної частоти іонів. Використовуючи тороїдальну симетрію плазми токамаків, код знаходить рішення рівнянь Максвела методом скінчених різниць для окремої тороїдальної гармоніки. Граничні умови задаються на реальній стінці камери токамака. ВЧ-антена моделюється введенням «зовнішнього» струму в закон Ампера. Полоїдальне магнітне поле, що утримує, і внесок від «теплової» частини тензора діелектричної проникливості плазми включені в розрахунки за допомогою ітерацій. Код було перевірено шляхом порівняння розрахунків з відомим аналітичним рішенням. Наведено перші розрахунки струму захоплення в сферичному торі.