THE INVESTIGATION OF A WAKE POTENTIAL OF NON-RELATIVISTIC PROTONS IN THE CASE OF THEIR ORIENTATION MOTION IN SOLID-STATE PLASMA OF IONIC CRYSTALS

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For non-relativistic protons channeled along the planes (axes) in ionic crystals the time-dependent induced electric dipole moments of the crystalline medium ions under the influence of protons electric fields are calculated. For this dipoles system the summary non-stationary scalar potential both in planar and axial cases is found. At this on the example of an axial orientation motion it is shown that this potential can be expanded on two components, one of which contributes to the interaction potential of the channeled protons with the crystal lattice, and the other, correspondingly, appears in the form of so-called “wake potential”, effecting only protons velocity.

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INTRODUCTION

It is well known that the motion of charged particles in arbitrary material environment is accompanied by a mutual influence of medium on a particle and a particle on medium which in a very complicated way depends on medium characteristics, the particle type and parameters.

It is also known that at ion motion at some velocity in a crystal medium there is an ion potential screening with some delay in space and time (see, e.g., [1]). It leads to the origin of so-called “wake potential” which quantitative theory for the charged particle moving in plasma was proposed and presented in [2]. Similar effects arise just in other cases (see, e.g., [3 – 5]). In [6] there is an idea on a possibility of a wake potential origin at relativistic electron channeling in ionic crystals. In particular, in [7] a wake potential was investigated in the case of orientation motion of weakly relativistic electrons in LiH crystal.

The given paper which is a continuation of [6], deals with the consideration of the orientation motion (both planar and axial) of non-relativistic protons in different ion crystals. Besides, it’s shown that in the result of delayed (in space and time) polarization of medium by a moving proton some non-stationary dipole moments are induced. Such effect leads to formation of scalar electric potential. A specific analysis shows that this potential is expanded on two components one of which just almost completely corresponds to the analogous expression got in [1].

1. THE POLARIZATION OF CRYSTAL IONS BY THE CHANNELING NON-RELATIVISTIC PROTONS

Let’s consider a non-relativistic proton moving in the regime of planar channeling along z axis in Cartesian coordinate system presented in Fig. 1.

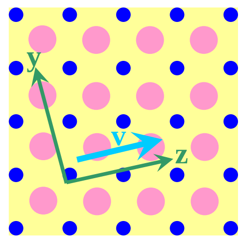

Fig. 1. Schematic presentation of electro-neutral plane of ionic crystal parallel to which in the direction of z axis of Cartesian coordinate system a non-relativistic proton moves at $\vec{v} = v\vec{e}_z$ velocity

Then, if it’s in the state $\psi_n(x)$ of a transverse quantum motion, an electric charge density may be written down in the standard form

$$\rho_n(\vec{r},t) = -e\psi_n^2(x)\,\delta(y)\,\delta(z-vt). \qquad (1)$$

In this case the electric field intensities is determined by the following expression (see, e.g., [8]):

$$\vec{E}_n(\vec{r},t) = e\int_{-\infty}^{\infty}\psi_n^2(\xi)\,\left[\vec{R}(\vec{r},t,\xi)\big/R^3(\vec{r},t,\xi)\right]d\xi, \qquad (2)$$

where $\vec{R}(\vec{r},t,\xi) = (x-\xi, y, z-vt)$ – the radius-vector from the channeling proton position to the point (x,y,z) of a field observation. Magnetic field on condition $v << c$ may not be taken into account.

An electric dipole moment $\vec{p}_n(\vec{r}_{\vec{n}},t)$ of a separate crystal ion which is in the point with the radius-vector $\vec{r}_{\vec{n}} = (n_x a_x, n_y a_y, n_z a_z)$ ($a_{x,y,z}$ – the periods of a crystal lattice, correspondingly, along x,y,z axes, $n_{x,y,z} = 0,\pm1,\pm2,...$), is calculated by the equation:

$$\ddot{\vec{p}}_n(\vec{r}_{\vec{n}},t) + g\dot{\vec{p}}_n(\vec{r}_{\vec{n}},t) + \omega_0^2\vec{p}_n(\vec{r}_{\vec{n}},t) = -\left(e^2/m\right)\int d\vec{r}\,\kappa\left(|\vec{r}-\vec{r}_{\vec{n}}|\right)\vec{E}_n(\vec{r},t), \qquad (3)$$

where m – the rest mass of electron, $\kappa(r)$ – the density of electron distribution in an ion, g – a damping constant connected with electron and nucleus oscillations, $\omega_0 = (4\pi ne^2/m)^{1/2}$ – the plasma frequency. For the solution of the equation (3) we expand the field $\vec{E}_n(\vec{r},t)$, as it was done in [2], into the following Fourier integral:

$$\vec{E}_n(\vec{r},t) = (2\pi)^{-4}\int d\vec{k}d\omega \exp\left[i(\vec{k}\vec{r}-\omega t)\right]\vec{E}_n(\vec{k},\omega), \quad (4)$$

where Fourier-component of the expressions (2) with the help of [8] is presented in the form

$$\vec{E}_n(\vec{k},\omega) = -4\pi i e f_n(k_x)\frac{\vec{k}}{k^2}\delta(\omega-k_z v), \quad (5)$$

where $f_n(k_x) = \int_{-\infty}^{\infty}\psi_n^2(x)\exp(-ik_x x)dx$.

Further, making the transformations like in [7], we find a Fourier-component of an electric dipole moment $\vec{p}_n(\vec{r}_{\vec{n}},t)$:

$$\vec{p}_n(\vec{k},\omega) = -\frac{4\pi i e^3}{m}\frac{\vec{k}f_n(k_x)\kappa(k)\delta(\omega-k_z v)}{(\omega^2-\omega_0^2+i\omega g)k^2}. \quad (6)$$

Here $\kappa(k) = \int\kappa(r)\exp(i\vec{k}\vec{r})d\vec{r}$ – a Fourier-image of the density of electron distribution in ion (the integration is made over the crystal volume). Now let's stop on the calculation of this function both for positive and negative ions of the ionic crystal.

Let's write down one-particle potentials of crystal ions using the approximation in the form of a screening Coulomb potential [9], i.e.

$$\varphi_\pm(r) = e\left[\frac{Z_\pm \mp \alpha}{r}\exp\left(-\frac{r}{R_\pm}\right) \pm \frac{\alpha e}{r}\right], \quad (7)$$

where α – a degree of crystal atoms ionicity, $Z_\pm$, $R_\pm$ – atom numbers and radii of the screening, correspondingly, of positively and negatively charged ions (note that from the point of view of boundary conditions at $r\to 0$ and $r\to\infty$ the potentials $\varphi_\pm(r)$ are correct [10]). By means of Poisson equation [8], and also after the procedure of averaging by thermal oscillations $u_\pm$ ion crystals, we come to the following expression for the functions of radial densities of electron distribution, correspondingly, for positively and negatively charged ions:

$$\kappa_\pm(r) = \frac{Z_\pm \mp \alpha}{8\pi R_\pm^2 r}\exp\left(\frac{u_\pm^2}{2R_\pm^2}\right)\cdot$$

$$\cdot\left\{\exp\left(-\frac{r}{R_\pm}\right)\operatorname{erfc}\left[\tau_\pm^-(r)\right] - \exp\left(\frac{r}{R_\pm}\right)\operatorname{erfc}\left[\tau_\pm^+(r)\right]\right\}, \quad (8)$$

where $\tau_\pm^\mp(r) = u_\pm/R_\pm\sqrt{2} \mp r/u_\pm\sqrt{2}$. For further analytical trasformations it is expedient to use the functions $\kappa_\pm(r)$ in the form:

$$\kappa_\pm(r) = A_\pm\exp(-B_\pm r)(1+C_\pm r). \quad (8')$$

At last, making a Fourier-transformation of the expression (8′), we get

$$\kappa_\pm(k) = 8\pi A_\pm\left[\frac{4C_\pm B_\pm^2}{(k^2+B_\pm^2)^3} + \frac{B_\pm - C_\pm}{(k^2+B_\pm^2)^2}\right]. \quad (9)$$

Now we return to the formula (6). Subsequently substitute into it the expressions (9), and then Fourier-component $\vec{p}_n(\vec{k},\omega)$ to Fourier integral $\vec{p}_n(\vec{r}_{\vec{n}},t) = \int\frac{d\vec{k}d\omega}{(2\pi)^4}\exp\left[i(\vec{k}\vec{r}_{\vec{n}}-\omega t)\right]\vec{p}_n(\vec{k},\omega)$, we come to the following expression for the value $\vec{p}_n(\vec{r}_{\vec{n}_\pm},t)$:

$$\vec{p}_n(\vec{r}_{\vec{n}_\pm},t) = \frac{8e^3 A_\pm}{\pi m v^2}\left(\vec{e}_x\int_{-\infty}^{\infty}\psi_n^2(x')(x_{\vec{n}_\pm}-x')dx'\cdot\right.$$

$$\cdot\int_0^\infty F_{k_z}^\pm(x',x_{\vec{n}_\pm},y_{\vec{n}_\pm})f_{t\pm}^{k_z}T^{k_z}dk_z + \quad (10)$$

$$+\vec{e}_y y_{\vec{n}_\pm}\int_{-\infty}^{\infty}\psi_n^2(x')dx'\int_0^\infty F_{k_z}^\pm(\xi,x_{\vec{n}_\pm},y_{\vec{n}_\pm})f_{t\pm}^{k_z}T^{k_z}dk_z -$$

$$\left.-\vec{e}_z\int_{-\infty}^{\infty}\psi_n^2(x')dx'\int_0^\infty G_{k_z}^\pm(\xi,x_{\vec{n}_\pm},y_{\vec{n}_\pm})g_{t\pm}^{k_z}T^{k_z}k_z dk\right),$$

where

$$T^{k_z} = \left[\left(k_z^2-\omega_0^2/v^2\right)^2 + g^2k_z^2/v^2\right]^{-1},$$

$$f_{t\pm}^{k_z} = \frac{k_z g}{v}\sin\left[k_z\lambda_\pm(t)\right] - \left(\frac{\omega_0^2}{v^2}-k_z^2\right)\cos\left[k_z\lambda_\pm(t)\right],$$

$$g_{t\pm}^{k_z} = \frac{k_z g}{v}\cos\left[k_z\lambda_\pm(t)\right] + \left(\frac{\omega_0^2}{v^2}-k_z^2\right)\sin\left[k_z\lambda_\pm(t)\right],$$

$$F_{k_z}^\pm(\xi,x_{\vec{n}_\pm},y_{\vec{n}_\pm}) = \left[\frac{k_z K_1(k_z\rho_\pm^\xi)}{B_\pm^2\rho_\pm^\xi} - \frac{K_0(k_{z\pm}\rho_\pm^\xi)}{2}\right]\cdot$$

$$\cdot\frac{D_\pm}{B_\pm^2} - K_1(k_{z\pm}\rho_\pm^\xi)\left[\frac{D_\pm}{B_\pm^4\rho_\pm^\xi} + \frac{C_\pm\rho_\pm^{\xi 2}}{2k_{z\pm}}\right],$$

$$G_{k_z}^\pm(\xi,x_{\vec{n}_\pm},y_{\vec{n}_\pm}) = \frac{D_\pm}{B_\pm^4}\left(K_0(k_z\rho_\pm^\xi) - K_0(k_{z\pm}\rho_\pm^\xi)\right)\cdot$$

$$\cdot\left[\frac{C_\pm\rho_\pm^{\xi 2}}{2k_{z\pm}^2} + \frac{D_\pm}{B_\pm^4}\right] - \frac{\rho_\pm^\xi}{k_{z\pm}}K_1(k_{z\pm}\rho_\pm^\xi)\left[\frac{D_\pm}{2B_\pm^4} + \frac{C_\pm\rho_\pm^{\xi 2}}{2k_{z\pm}}\right].$$

Here $D_\pm = 3C_\pm + B_\pm$, $k_{z\pm}^2 = k_z^2 + B_\pm^2$, $\lambda_\pm(t) = z_{\vec{n}_\pm} - vt$, $\rho_\pm^\xi = \left[(\xi - x_{\vec{n}_\pm})^2 + y_{\vec{n}_\pm}^2\right]^{1/2}$, $K_{0,1}(x)$ – MacDonald functions.

Now let's calculate an electric potential formed by the system of such non-stationary electric dipoles.

2. THE CALCULATION OF THE POTENTIAL OF ELECTRIC DIPOLE SYSTEM AT PLANAR CHANNELING OF NON-RELATIVISTIC PROTONS

Based on the principle of superposition we find a summary electric potential of the system of electric dipoles induced by a proton moving in the regime of planar channeling according to the following formula [8]:

$$\Phi(\vec{r},t) = \sum_{n,\vec{n}_\pm}\left[\vec{p}_n(\vec{r}_{\vec{n}_\pm},t)\cdot(\vec{r}-\vec{r}_{\vec{n}_\pm})\Big/\left|\vec{r}-\vec{r}_{\vec{n}_\pm}\right|^3\right]w_n\,, \quad (11)$$

where w_n – the probability of proton capture to the state $\psi_n(x)$ of the channeling motion. In particular, if confine oneself to two nearest crystallographic planes (at $x_{\vec{n}_\pm} = a_x/2$ and $x_{\vec{n}_\pm} = -a_x/2$), and also to the case of one quantum state of $\psi_0(x)$, then a non-stationary potential accounting the expressions of (10) and (11) is determined by the formula

$$\Phi_0(\vec{r},t) = \frac{32e^3 w_0}{\pi m v^2 S}\sum_{j=0}^{1}\int_{-\infty}^{\infty}\psi_0^2(x')\,dx'\int_0^{\infty}k_z dk_z T^{k_z} f^{k_z}(t)\cdot$$
$$\cdot\int_{-\infty}^{\infty}d\eta\left[(y-\eta)\eta+(a_{xj}-x')(x-a_{xj})\right]\cdot$$
$$\cdot\left[A_+F_{k_z}^{+}(x',a_{xj},\eta)+A_-F_{k_z}^{-}(x',a_{xj},\eta)\right]\cdot \quad (12)$$
$$\cdot\left[K_1(k_z\rho_\pm^{\xi\eta})\Big/\rho_\pm^{\xi\eta}-k_zK_0(k_z\rho_\pm^{\xi\eta})\right]\cdot$$
$$\cdot\left[A_+G_{k_z}^{+}(x',a_{xj},\eta)+A_-G_{k_z}^{-}(x',a_{xj},\eta)\right],$$

where $S = a_y a_z$ – a square of two-dimensional elementary cell in the channeling planes, $a_{xj} = (-1)^j 2^{-1}a_x$, $\rho_\pm^{\xi\eta} = \left[(\xi - x_{\vec{n}_\pm})^2 + (\eta - y_{\vec{n}_\pm})^2\right]^{1/2}$ (the expression $f_t^{k_z}$ is got from the expression $f_{t\pm}^{k_z}$ by the substitution of $\lambda_\pm(t)$ for $\lambda(t) = z - vt$).

Note that the situation with one energy level arises in the case of protons channeling in ionic crystals in high-index electro-neutral planes. In other cases it is necessary to calculate the super positional potential $\Phi(\vec{r},t)$ in accordance with the formula (11).

One may show that the potential (11) is expanded on two components, one of which definitely contributes to the potential of proton interaction with a crystal lattice, and the second one is a wake potential effecting only the protons longitudinal velocity. More clearly it may be demonstrated on the example of orientation motion of protons along the crystal axes to the consideration of which we proceed.

3. THE WAKE POTENTIAL AT THE AXIAL CHANNELING OF PROTONS IN THE IONIC CRYSTALS

The calculation of the induced dipole moments and further getting of a non-stationary scalar potential at an axial channeling is done in analogy to a planar case (for such replacement it is necessary only to use $\rho_n(\vec{r},t) = -e\psi_n^2(x,y)\delta(z-vt)$ instead of (1)). For more simple case when the wave function $\psi_0(x,y)$ takes a constant value in the ranges of two-dimensional cell (being on an isolated electro-neutral axis) we get the following formula for a scalar potential:

$$\Phi(\rho,z,t) = -\left(32e^3\big/mv^2V\right)\int_0^{\infty}k_z^2K_0(k_z\rho)f_t^{k_z}T_g^{k_z}\cdot$$
$$\cdot\left(A_+W_+^{k_z}+A_-W_-^{k_z}\right)dk_z. \quad (13)$$

Here V – the volume of an elementary cell of a crystal, $W_\pm^{k_z} = D_\pm\Big/B_\pm^4k_z^2 - D_\pm\left(B_\pm^{-2}+k_{z\pm}^{-2}\right)\Big/B_\pm^2k_{z\pm}^2 - 4C_\pm\Big/k_{z\pm}^6$. From the formula (13) follows that at $g\to 0$ the function $gT_g^{k_z}$ may be approximated as $\left(\pi v^3\big/4\omega_0^2\right)\delta(k_z-\omega_0/v)$. In the result one of the components tend to the function

$$\Phi(z,\rho,t) = -\Phi_0\sin\left[k_z\lambda(t)\right]K_0(k_z\rho)\,, \quad (14)$$

where $\Phi_0 = 8\pi e^3\omega_0\left(A_+W_+^{\omega_0/v}+A_-W_-^{\omega_0/v}\right)\Big/mv^3V$.

From the other side, in [1] it is considered the same expression, but with the amplitude equals to

$$\Psi_0 = 2e\omega_0\left[(\varepsilon_\infty-1)/(\varepsilon_\infty+1)\right]^{1/2}\Big/v\,, \quad (15)$$

where ε_∞ – the optical dielectric permeability of the ionic crystal [12].

Concrete calculations for all ionic crystals showed that numerical values of the amplitudes Φ_0 and Ψ_0 in (15) and (16), correspondingly, have the values of the same orders (in the Table there are these data for sodium halides received at proton velocity $v\approx 10^8$ cm/s).

The amplitudes Φ_0 and Ψ_0 for sodium halides

Crystal	NaF	NaCl	NaBr	NaI
Φ_0	0.01	0.02	0.02	0.02
Ψ_0	0.08	0.07	0.07	0.07

Thus, these results testify to the correctness of the calculations of non-stationary potentials generated by the systems of electric dipoles both in planar and axial cases.

CONCLUSIONS

Thus, the paper deals with the calculation of non-stationary dipole moments induced in a crystal lattice by the electric fields generated by orientationally moving non-relativistic protons.

In the case of planar channeling of the protons for the system of electric dipoles, induced by these protons, there were calculated some non-stationary potentials having a strong inverse effect on protons motion.

On the example of axial channeling of protons it is demonstrated clearly that one of the components of formed non-stationary potential ion in a limited case is transforming to a classic wake potential.

Action of wake potential, running in the direction of channeling synchronously with the particle, can lead to some important physical effects. In particular, to pulsed formation of correlated coherent states of particles, localized in adjacent channels (see, e.g., [12, 13]).

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ИССЛЕДОВАНИЕ КИЛЬВАТЕРНОГО ПОТЕНЦИАЛА НЕРЕЛЯТИВИСТСКИХ ПРОТОНОВ В СЛУЧАЕ ИХ ОРИЕНТАЦИОННОГО ДВИЖЕНИЯ В ТВЁРДОТЕЛЬНОЙ ПЛАЗМЕ ИОННЫХ КРИСТАЛЛОВ

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Для каналируемых вдоль плоскостей (осей) в ионных кристаллах нерелятивистских протонов рассчитываются зависящие от времени наведенные электрические дипольные моменты ионов кристаллической среды, формируемые под воздействием электрических полей протонов. Для этой системы диполей находится суммарный нестационарный скалярный потенциал как в плоскостном, так и в осевом случаях. При этом на примере ориентационного движения вдоль осей кристалла показывается, что такой потенциал можно разложить на два слагаемых, одно из которых вносит вклад в потенциал взаимодействия каналируемых протонов с кристаллической решёткой, а другое, соответственно, проявляется в виде так называемого "кильватерного потенциала", влияющего только на скорость протонов.

ДОСЛІДЖЕННЯ КІЛЬВАТЕРНОГО ПОТЕНЦІАЛУ НЕРЕЛЯТИВІСТСЬКИХ ПРОТОНІВ У ВИПАДКУ ЇХ ОРІЄНТАЦІЙНОГО РУХУ В ТВЕРДОТІЛЬНІЙ ПЛАЗМІ ІОННИХ КРИСТАЛІВ

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Для канальованих уздовж площин (осей) в іонних кристалах нерелятивістських протонів розраховуються залежні від часу наведені електричні дипольні моменти іонів кристалічного середовища, які формуються під впливом електричних полів протонів. Для цієї системи диполів знаходиться сумарний нестаціонарний скалярний потенціал як у площинному, так і в осьовому випадках. При цьому на прикладі орієнтаційного руху уздовж осей кристала показується, що такий потенціал можна розкласти на два доданка, один з яких вносить вклад у потенціал взаємодії канальованих протонів з кристалічною граткою, а інший, відповідно, проявляється у вигляді так званого "кільватерного ефекту", впливаючого лише на швидкість протонів.