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Landau parameter of elasticity

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Abstract. Based on the consideration given by the Ginzburg-Landau (GL) theory according to the variational principle, we assume that the microscopic Gibbs function density given by [1] $\int_V G_S dV = \int_V \left(F_S - \frac{1}{4\pi} \vec{B} \cdot \vec{H} \right) dV$ must be stationary at the thermodynamical equilibrium. To describe the universal propagation of the order parameter, we express order phases and amplitudes as dealing with tensor elements. In addition to the variation of the order parameter and the vector potential limited by the condition $\vec{\nabla} \times \vec{A}(x) = \vec{B}(x)$, we introduce here the concept of elasticity to describe the propagation of the superconducting state as "the little waves borning on smooth Superconductor Sea [2]". The coherence concept transits to the asymptotic behaviour, we shall say that equivalence concept is its limit, this must transgress the propagation laws of superconductivity to be replaced by the increasing of superconductivity. Superconductivity will be viewed as second order extensive value, propagation seems to be so quick to avoid the stability, the increasing of superconductivity requires more time, and more time will be equivalent to a second and added measurement process eliminating the degeneracy of the first integral during the cooling process. It may deal with the first approximated stability of Superconductor State. The uncertainly in quantum mechanics is limited as scale length relations for the dimension coherence of the order parameter and temperatures.

Keywords: superconductivity, order parameter, elasticity.

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1. Introduction

We introduce relations between order parameter features and elasticity qualities. Elasticity expression of the second order, as an effective language translated to make a new equivalence between quantum mechanics description and thermodynamical one which unfortunately is severely limited by the fundamental requirements of reversibility of processes and irreversibility of time. This will be considered as a very little step to express the asymptotic coherence of classical equations.

2. Order phases and amplitude partial derivative equations

The superconducting state, viewed as perfect mixing wave functions regulated by a distribution of potentials, seems to be a result depending on temperature. As giving by classical equations, the free energy density and the order parameter expressed by equality meaning equivalence.

These limitations imposed to this equivalence by the requirements presented by the ratio between the lengthscales of temperatures, just near the transition points. This fact was mentioned by Kenneth Wilson in 1972. The temperature acts as a deformation causing a phase transition.

The order parameter given by [3]

$$\Psi(r) = \eta(r) e^{i\phi(r)} \tag{1}$$

is replaced by

$$\Psi(r_i - r_k) = \eta(r_i - r_k) e^{i\phi(r_i - r_k)}. \tag{2}$$

The adjacent in levels distance is given by [3],

$$D(E) = \Delta E \cdot e^{-S(E)} \quad (3)$$

and $D(E') = \Delta E \cdot e^{-S(E')}$. (4)

E and E' are two centres of two intervals of the energy. The distance D is universal for macroscopic bodies.

The elasticity means the existence of $D(F)$ given by

$$D(F) = D(E) - D(TS(E)),$$

$$D(F) = e^{-S(E)}\left[\Delta E - (T\Delta S + S\Delta T)\, e^{-k_B T}\right]. \quad (5)$$

The expression $\Delta E - (T\Delta S + S\Delta T)\, e^{-k_B T}$ must be written as $\Delta\Omega$, where Ω is a function of S

$$\Delta\Omega = \Delta E - (T\Delta S + S\Delta T)\, e^{-k_B T},$$

$$\Delta\Omega = \Delta E - (\Delta Q + S\Delta T)\, e^{-k_B T}. \quad (6)$$

$\Delta\Omega$ acts as a potential for $T = T_c$, revealing the power of statistical weights.

The electrons having a mean role obey to

$$\ln \frac{\Delta E}{\Delta\Omega\ (\Delta Q + S\Delta T)} \sim k_B T_c . \quad (7)$$

The expression $-(T\Delta S + S\Delta T)\, e^{-TS(E)}$ is equivalent to $2u_{ik}dr_i dr_k$. u_{ik} is an effective deformation tensor measuring the macroscopicity scale of the body, its form is as $\frac{\partial^2 \Psi(r_i - r_k)}{\partial r_i \partial r_k}$.

We have:

$$-(T\Delta S + S\Delta T)\, e^{-TS(E)} \sim$$

$$e^{i\phi(r)} \left\{ \frac{\partial^2 \eta(r_i - r_k)}{\partial r_i \partial r_k} + \eta(r) \frac{\partial^2 \phi(r_i - r_k)}{\partial r_i \partial r_k} \right\}. \quad (8a)$$

When $|\phi(r)|^2 = T^2 S^2(E)$, (8b)

$$-(T\Delta S + S\Delta T) = \frac{\partial^2 \eta(r_i - r_k)}{\partial r_i \partial r_k} + \eta(r) \frac{\partial^2 \phi(r_i - r_k)}{\partial r_i \partial r_k} . \quad (9)$$

Eq. (8b) expresses the equivalence between the phase and entropy. Eq. (8a) expresses the partial derivatives combination of the phase and amplitude, as generated by the thermodynamical quantity $-(T\Delta S + S\Delta T)$.

We write (8a)

$$-(T\Delta S + S\Delta T)\, e^{-TS(E)} \sim$$

$$2\, e^{i\phi(r)} \left\{ \frac{\partial^2 \eta(r_i - r_k)}{\partial r_i \partial r_k} + \eta(r) \frac{\partial^2 \phi(r_i - r_k)}{\partial r_i \partial r_k} \right\}, \quad (10)$$

also $\left[1 + \frac{T}{S}\left(\frac{\Delta S}{\Delta T}\right)\right] e^{-TS(E)}$ with $\frac{T}{S}\left(\frac{\Delta S}{\Delta T}\right) << 1$ that expresses the limit of the second member of Eq. (10).

The use of such consideration is an essay to describe the features of the order parameter as given by $|\Psi(r)|^2$, meaning the phase density as a result of the ordinary phase submitted to the potential Ω, we can consider it as a distribution of phases, or to be called the phase order.

3. Constraint tensor

Having the variation principle that limits the causes of appearance of the order parameter through the whole sample,

$$\int_V G_S\, dV = \int_V \left(F_S - \frac{1}{4\pi} \vec{B} \cdot \vec{H} \right) dV .$$

We introduce the following notation,

$$F_S - \frac{1}{4\pi} \vec{B} \cdot \vec{H} \sim \frac{\partial}{\partial r_i} \sigma_{ik} \frac{\partial}{\partial r_{ik}} \sigma_{ik} .$$

We write

$$\int_V G_S\, dV = \int_V \left(\frac{\partial \sigma_{ik}}{\partial r_i} \frac{\partial \sigma_{ik}}{\partial r_k} \right) dV = \oint_A \sigma_{ik}^2\, dA . \quad (11)$$

A is an extremal area supporting the maximum order constraints.

The momentum of forces causing the transition over the whole sample is given by

$$M_{ik} = \int_V \left(\frac{\partial \sigma_{ik}}{\partial r_i} r_k - \frac{\partial \sigma_{ik}}{\partial r_k} r_i \right) dV . \quad (12)$$

dV is an extremal volume where these order constraints are observed.

The fundamental thermodynamical relation can be written as follows [4]:

$$dE = TdS + \sigma_{ik} du_{ik} .$$

For a weak deformation, the u_{ik} tensor is a linear function of the constraint tensor σ_{ik}, which is the Hooke law for the elasticity of the density free energy.

The general law [4] is

$$u_{ik} = \frac{1}{9K}\delta_{ik}\,\sigma_{11} + \frac{1}{2\mu}\left(\sigma_{ik} - \frac{1}{3}\delta_{ik}\,\sigma_{11}\right).$$

Where K is a compression amplitude, and μ is the Lame constant.

σ_{ik} is the solution of the equation

$$e^{i\phi(r)}\left\{\frac{\partial^2\eta(r_i - r_k)}{\partial r_i \partial r_k} + \eta(r)\frac{\partial^2\phi(r_i - r_k)}{\partial r_i \partial r_k}\right\} =$$

$$= \frac{1}{9K}\delta_{ik}\,\sigma_{11} + \frac{1}{2\mu}\left(\sigma_{ik} - \frac{1}{3}\delta_{ik}\,\sigma_{11}\right). \qquad (13)$$

σ_{ik} will be function of η and ϕ. K and μ are functions of Landau phenomenological constants a and b.

4. Thermodynamical function expressed by tensors

$$dF = -SdT + \sigma_{ik}du_{ik}.$$

The thermodynamical potential

$$\Phi = E - TS =$$

$$-\sigma_{ik}\; e^{i\phi(r)} \times \left\{\frac{\partial^2\eta\,(r_i - r_k)}{\partial r_i \partial r_k} + \eta\,(r)\frac{\partial^2\phi\,(r_i - r_k)}{\partial r_i \partial r_k}\right\}.$$

The expression of Φ means the creation or annihilation of levels, or a set of levels. This fact is interpreted as the interaction of scale lengths, when the phase order is regulated by the Dirac function.

In such way $\int |\phi(r)|^2 dr = \delta(r - r_0)$, r_0 is the mean radius over which the superconductor state propagates.

$\delta(r - r_0) = 1$, the appearance of a set of levels containing an extremum number of levels.

$\delta(r - r_0) = 0$, the disappearance of a set containing the extremum number of levels. (Extremum means a minimum number.)

The use of such consideration concerning the effective elasticity of the order parameter leads to draw the propagation of this parameter as “peaceful sea with little waves against the beach”.

Those little waves characterised by elasticity of the amplitude and phase, seems to be so weak to preserve the stability of Superconductor State when temperature increases.

5. Interpretation of the phase order equivalence equation

It determines “the levels of macroscopicity of the sample”. The coherence scale between two macroscopic levels is as T^2. The permitted levels are given by

$$T^2 \int S(E)dE = \min .$$

The macroscopic levels are different ways to express the recombination of entropies, every time the macroscopic level fluctuates.

The partial derivative equation amplitude, makes a meaning of an effective free mean path of the second order, in a polar manifold which corresponds to the maximum interval ΔE, where the elasticity of free energy density is undeterministically collected as to give locally u_{ik}, having the components excluding each other (we can reduce this interval ΔE of maximum number of levels, to a minimum number of levels under u_{ik} tensor, the σ_{ik} tensor will cause the reverse production of levels to reach the initial maximum number of levels).

6. Conclusion

The concept of elasticity introduced above is a little tendency to express the non-absolutism of propagation of the order parameter and show that the universality of undetermination considered in quantum mechanics for macroscopic bodies must be reformulated as a limitation imposed to the scale coherence by Landau and temperature, in such a way to evaluate a constant governing the precision under which the scale coherence and the temperature must be measured.

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