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Charged snowball in nonpolar liquid

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The problem of correct definition of the charge carrier effective mass in superfluid helium is revised. It is demonstrated that the effective mass M of a such quasiparticle can be introduced without use of the Atkins's idea concerning the solidification of liquid He in the close vicinity of ion. The two-liquid scenario of the «snowball» mass formation is investigated. The normal fluid contribution to the total snowball effective mass, the physical reasons of its singularity and the way of corresponding regularization procedure are discussed. Within of two-liquid model the existence of two different effective snowball radiuses: $R_{\rm id}$ for superfluid flow component and R_n for the normal one, $R_n > R_{\rm id}$ is demonstrated. Agreement of the theory with the available experimental data is found.

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72.20.Jv Charge carriers: generation, recombination, lifetime, and trapping.

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1. The ion–dipole interaction between the inserted charged particle and the induced electric dipoles of surrounded atoms is one of the interesting phenomena which take place in a nonpolar liquid. It is interaction which is generally responsible for the different salvations phenomena [1], while in nonpolar cryogenics liquids (like He, Ne, Ar, etc.) it leads to so-called «snowball effect». The latter consists in the formation of a nonuniformity $\delta\rho(r)$ in a density of a liquid around the inserted charged particle.

The ion–dipole interaction $U_{i-d}(r)$ between the inserted charged particle and solvent atoms in the simplest form can be written as

$$U_{i-d}(r) = -\frac{\alpha}{2}\frac{e^2}{r^4}, \qquad (1)$$

where α is the polarization of a solvent atom and r is the distance to the charged particle. The presence of attraction potential (1) in equilibrium has to be compensated by the growth of the solvent density $\delta\rho(r)$ in direction of the charged particle. The latter can be estimated, using the requirement of the chemical potential constancy:

$$\delta\mu(r) = P_s(r)v_s - \frac{\alpha}{2}\frac{e^2}{r^4} = 0,$$

$$\delta\rho(r) = \left(\frac{\partial\rho}{\partial P}\right)P_s(r) \equiv \frac{1}{s^2}P_s(r), \qquad (2)$$

where $P_s(r)$ is the local pressure around the ion, v_s is the volume of individual solvent atom and s is the sound velocity. The distance at which $P_s(r)$ reaches the value of pressure of a solvent solidification P_s^s

$$R_s^4 = \frac{\alpha}{2}\frac{e^2}{P_s^s v_s} \qquad (3)$$

corresponds to the radius of rigid sphere which is called «snowball». In the case of positive ions being in liquid helium, where the inter-atomic distance $a \approx 3$ Å and the pressure $P_s^s \approx 25$ atm, the snowball radius is estimated as $R_s^{\rm He} \approx 7$ Å.

Described above so-called Atkins's snowball model [2] is quite transparent and it was found useful for the various qualitative predictions. In particular, it provides by the natural definition and estimation for the effective mass M of a such quasi-particle as the sum of the extra mass caused by the presence of the density perturbation $\delta\rho(r)$

$$M^{(\mathrm{sol})} = 4\pi \int_{\alpha}^{\infty} r^2 \delta\rho(r) dr \quad (4)$$

and so-called hydrodynamic associated mass, related to the appearance of the velocity distribution around the sphere moving in liquid

$$M_0^{(\mathrm{ass})}(\rho, R_s) = \frac{2}{3}\pi\rho R_s^3. \quad (5)$$

Both of these contributions turn out to be of the same order. Calculated in this way snowball effective mass turns out to be $M \geq 50 m_4$, what roughly corresponds to the experimental data [3].

Careful analysis shows that both expressions (4) and (5) require more precise definitions and further development of the Atkins's model. Indeed, one can easily see that the value of $M^{(\mathrm{sol})}$ turns out to be critically sensitive to the lower limit of the integral. Atkins used the value of helium inter-atomic distance $a \approx 3$ Å [2] as a rough cut-off parameter only. The microscopic analysis shows (see Eq. (21) below) that the real lower limit turns out to be less that this value.

Principal revision requires the definition of the associated mass $M^{(\mathrm{ass})}$ for the two-fluid model. The matter of fact that its strong temperature dependence, which has been systematically observed experimentally [4–7], cannot be explained within the «ideal» flow picture. It is why below we propose the hydrodynamics scenario of the associated mass $M^{(\mathrm{ass})}$ formation that takes into account the viscous part of this problem. The straightforward accounting for the nonzero viscosity results in a dramatic growth (compared to the ideal case) of kinetic energy of the moving sphere and, consequently, its effective associated mass. This fact is caused by setting in motion of a spacious domain of viscous liquid around the moving particle. Nevertheless, the arising divergency can be cut off by accounting for the nonlinear effects in stationary flow.

Even more interesting is the flexible behavior of positive ion's dynamic responde versus of excitation conditions. It (responde) cannot be described neither by means of effective mass approximation, nor using some alternative, which are not sensitive to excitation conditions. Therefore below we propose several snowball motion scenario and try to overlap the details of its dynamic responde, when the excitation conditions are close one to another.

2. Let us start from the definition of the stationary effective associated mass within the snowball approximation and using the two-fluid model. In that case

$$M^{(\mathrm{ass})} = M_s^{(\mathrm{ass})} + M_n^{(\mathrm{ass})}. \quad (6)$$

The first one can be defined as follows:

$$\frac{M_s^{(\mathrm{ass})} V^2}{2} = \frac{\rho_s(T)}{2} \int_{R_{\mathrm{id}}}^{\infty} \mathbf{v}_s^2(\mathbf{r}) d^3\mathbf{r}, \quad (7)$$

where $\rho_s(T)$ is the superfluid density, V is the velocity of a «snow-cloud» center of mass forward motion, $\mathbf{v}_s(\mathbf{r})$ is the superfluid component local velocity distributions appearing in liquid due to charge carrier motion. Let us underline that some auxiliary parameter R_{id} instead of the snowball radius R_s^{He} (see Eq. (3)) here is used as the lower limit cut-off. Some physical reasons for the introduction of R_{id} beyond the Atkins model are presented below (see Eq. (21)).

The superfluid flow has potential character [8,9]

$$\mathbf{v}_s = \nabla\phi(\mathbf{r}), \quad \phi(\mathbf{r}) = -\frac{\mathbf{An}}{\mathbf{r}^2}, \quad \mathbf{A} = \mathbf{V} R_{\mathrm{id}}^3 / 2. \quad (8)$$

Correspondingly, the superfluid part of associated mass $M_s^{(\mathrm{ass})}$ (7) with the velocity distribution determined by the Eq. (8) is reduced to the expression (5) with the simple substitutions $R_s \to R_{\mathrm{id}}$ and $\rho \to \rho_s$.

As to the normal fluid part of the associated mass $M_n^{(\mathrm{ass})}$, its definition turns out to be more cumbersome. It would be natural to define it in the same way as the superfluid one, just substituting in Eq. (7) the subscript *s* by *n*. But this programme runs against the considerable obstacles in spite of the fact that the problem of rigid sphere motion in a viscous liquid was considered a long ago (see, for instance, Refs. 8, 9). Indeed, for small Reynolds numbers $\mathrm{Re} = \rho V R_n / \eta << 1$ and distances $R_n < r < R_n / \mathrm{Re}$ the gradient term $(\mathbf{v} \cdot \nabla)\mathbf{v}$ in corresponding Navier–Stokes equation can be ignored. The solution of such linearized equation in zero approximation turns out to be independent on the liquid viscosity η. In the frame related with the sphere center of mass it has the form:

$$v_r(r,\theta) = -V \cos\theta \left(\frac{3R_n}{r} - \frac{R_n^3}{2r^3} \right), \quad (9)$$

$$v_\theta(r,\theta) = V \sin\theta \left(\frac{3R_n}{4r} + \frac{R_n^3}{4r^3} \right). \quad (10)$$

Here θ is the polar angle counted from the x axes, which coincides with the ion velocity direction.

One can easily see, that corresponding contribution to the kinetic energy diverges at the upper limit of integration. To regularize this divergence it is necessary to use at large distances $r \gtrsim l_\eta = R_n / \mathrm{Re}$ ($l_\eta = \eta/\rho V$ is the characteristic viscous length) more precise, so-called Ossen, solution of the Navier–Stokes equation which was obtained taking into account the gradient term in Ref. 9. This solution shows that almost for all angles, besides the domain restricted by the narrow paraboloid $\theta(x) = \pi - \sqrt{\eta/\rho V|x|}$ behind the snowball (so-called «laminar trace») at large distances the velocity field decays exponentially. In latter the velocity decays by power law [9] and it gives logarithmically large contribution with respect to other domain of disturbed viscous liquid. Such specification permits to cut off formal divergency of the kinetic energy and to find with the logarithmic accuracy the value of the normal component associated mass for the stationary moving in viscous liquid charge carrier:

$$M_{n(\mathrm{st})}^{(\mathrm{ass})} \sim M_0^{(\mathrm{ass})}(\rho_n, R_n)\left(\frac{l_\eta}{R_n}\right)\ln\frac{l_\eta}{R_n} = R_n^2\frac{\eta}{V}\ln\frac{\eta}{\rho V R_n}. \tag{11}$$

Here the associated mass $M_0^{(\mathrm{ass})}(\rho_n, R_n)$ is defined by Eq. (5) with radius R_n and density ρ_n. Since in our assumptions $l_\eta >> R_*$ the value of $M_{n(\mathrm{st})}^{(\mathrm{ass})}$ turns out much larger than the value of the associated mass in the ideal liquid. Moreover, when velocity V such definition, in spite of the performed above regularization procedure, fails since Eq. (11) diverges.

3. The simplest alternative scenario for snowball dynamics is it periodic oscillation. Let us consider the situation, when a periodic electric field is applied to a charge carrier placed in normal liquid. The dynamic Stokes force appearing when the sphere oscillates in the viscous liquid with finite frequency has the form [8,9]:

$$F(\omega) = 6\pi\eta R_n\left(1+\frac{R_n}{\delta(\omega)}\right)V(\omega) + + 3\pi R_n^2\sqrt{\frac{2\eta\rho_n}{\omega}}\left(1+\frac{2R_n}{9\delta(\omega)}\right)i\omega V(\omega), \tag{12}$$

where $\delta(\omega) = (2\eta/\rho_n\omega)^{1/2}$ is so-called dynamic penetration depth. It is natural to identify the coefficient in front of the Fourier transform of acceleration $i\omega V(\omega)$ with the effective dynamic associated mass, that gives:

$$M_n^{(\mathrm{ass})}(\omega) = 3\pi R_n^2\sqrt{\frac{2\eta\rho_n}{\omega}}\left(1+\frac{2R_n}{9\delta(\omega)}\right). \tag{13}$$

One can see that for high frequencies ($\delta(\omega) << R_n$) the dynamic associated mass coincides with that one of a sphere moving stationary in an ideal liquid, while when $\omega \to 0$ $M_n^{(\mathrm{ass})}(\omega)$ diverges as $\omega^{-1/2}$. As we already have seen above this formal divergence is related to fall down of the linear approximation in the Navier–Stokes equation assumed in derivation of Eq. (12). It is clear that the definition (13) is valid for high enough frequencies, until $\delta(\omega) \lesssim l_\eta$ (i.e., $\omega \gtrsim \tilde{\omega} = \eta/\rho_n R_n^2$). When ω becomes lower than $\tilde{\omega}$ the penetration depth $\delta(\omega)$ in Eq. (13) has to be substituted by l_η and up to the accuracy of $\ln l_\eta/R$ the dynamic definition Eq. (13) matches with the static one (see Eq. (11)).

The additional possibility to formulate the beginning of nonlinear situation in $M_n(\omega)$ behavior is reorganization of requirement following from (11)

$$l_\eta/R \sim 1$$

to synonym

$$\omega\tau \sim 1, \tag{14}$$

where τ is express in terms η and V is presented as $V \sim \omega R$.

To finish the discussion (12)–(14) it is reasonably to add the following speculations. If instead of purely oscillation regime, we have for, e.g., the cyclotron snowball motion, in this case the overlap between (11), (13) has to be correct, including the logarithm correction (in CR picture there are oscillations, but the velocity never goes to zero; as a result the stationary $v(r)$ picture can be saved permanently, just the laminar trace maybe tilted with respect to its conventional position).

Thus, the following qualitative picture arises. The general expression for the normal effective mass definition could be presented in a form, similarly (7). But any time the velocity distribution in this integral has to be written for the real dynamic scenario, e,g., stationary motion, CR-motion, simple oscillations, relaxation phenomena etc. There is no general $M_n^{(\mathrm{ass})}(T)$ definition, which is indicated in all experimental papers [4–7]. Even, there is no $M_n^{(\mathrm{ass})}(T,\omega)$-presentation without indexation who is responsible for the motion excitation. And this many picture situation is not the fables of the theory. It corresponds to the nature of $V(t)$ snowball motion in viscose liquid.

4. The next question comes, how sensitive are the introduced above definitions of the associated mass Eqs. (13), (7) to the real shape of the liquid density perturbation in the vicinity of the charge carrier? The examples are already known when the snowball Atkins's model and more realistic snow-cloud model lead to qualitatively different predictions for the value of positive ion mobility in liquid ^{4}He (see Refs. 10–12). In order to clear up this problem let us consider the hydrodynamic picture of a snow-cloud motion. The latter we assume as the density compression which decays with distance by power law:

$$\rho(r) = \rho + \delta\rho(r) = \rho + \frac{C}{r^4} \tag{15}$$

(constant C is expressed in terms of Eqs. (2)). The continuity equation is read as

$$\rho(r)\,\mathrm{div}\,\mathbf{v} + \mathbf{v}\cdot\nabla\rho(r) = 0. \tag{16}$$

For an ideal (superfluid) liquid the velocity field is potential and it can be presented in the form Eq. (8). Substituting Eqs. (8) and (15) to Eq. (16) one finds

$$\Delta\phi(\mathbf{r}) - \frac{4C}{r^5}\frac{\nabla\phi(\mathbf{r})}{\rho+\delta\rho} = 0 . \qquad (17)$$

The Eq. (17) has to be solved with the additional requirement

$$\phi(r \to \infty) \to rV\cos\theta . \qquad (18)$$

In Born approximation Eq. (17) is reduced to the Poisson equation. Supposing that $\phi(r) = rV\cos\theta + \phi_1(r)$, it can be rewritten in the form

$$\Delta\phi_1 - \frac{4C}{r^5}\frac{V\cos\theta}{\rho+\delta\rho} = 0, \quad \phi_1(r\to\infty)\to 0, \qquad (19)$$

which solution can be written down as

$$\phi_1(\mathbf{x}) = \frac{\mathbf{d}\cdot\mathbf{x}}{r^3}, \;\; \mathbf{d} = \frac{C}{\pi}\int \frac{\mathbf{r}}{r^6}\frac{(\mathbf{V}\cdot\mathbf{r})}{\rho+\delta\rho(r)}\, d^3\mathbf{r}. \qquad (20)$$

The Eq. (20) shows, that far enough from a snow-cloud center the potential of velocity distribution looks like a dipole one, exactly in the same way as for the flow around the rigid sphere of the radius $R_{\rm id}$. Using Eqs. (8) and (20) one finds

$$R_{\rm id}^3 = 2\pi C\int_0^\infty \frac{dr}{r^2(\rho+\delta\rho)} . \qquad (21)$$

It is easy to see that in spite of presence of r^2 in denominator of the integrand this integral converges at the lower limit.

The above analysis demonstrates that the problem of the associated mass definition in the snowball and the snow-cloud models for an ideal liquid are qualitatively identical. It is just enough to renormalize the effective radius R_* (8) to the hydrodynamic dipole radius $R_{\rm id}$ (20).

The same analysis has to be done basing on the Navier–Stokes formalism for the motion of the snow-cloud in the viscous liquid. The problem is to solve the continuity equation, similar (16)

$$\rho(r)\,\mathrm{div}\,\mathbf{v} + \mathbf{v}\cdot\nabla\rho(r) = 0 \qquad (22)$$

and a stationary Navier–Stokes equation

$$\eta\Delta\mathbf{v} = \nabla p(r) , \qquad (23)$$

here η is the first liquid viscosity, $p(r)$ is the pressure distribution.

Acting (23) by operator div and using (22), we have finally

$$p(r) = -\eta\,\frac{\mathbf{v}\cdot\nabla\rho(r)}{\rho(r)} . \qquad (24)$$

Using this definition (and the suitable Born simplification: $\mathbf{v}\to V_s$) as right part of (23), we have the possibility to describe $\mathbf{v}$ distribution. First, it is evident such a distribution is not sensitive to η. Secondly, it is convenient to build the velocity components in cartesian coordinates

$$\Delta v_x = \frac{V_x}{\rho(r)}\frac{\partial^2\rho}{\partial x^2} , \qquad (25)$$

$$\Delta v_y = \frac{V_x}{\rho(r)}\frac{\partial^2\rho}{\partial x\partial y}, \;\; \Delta v_z = \frac{V_x}{\rho(r)}\frac{\partial^2\rho}{\partial x\partial z} . \qquad (26)$$

The Eqs. (25), (26) show the reasons for r^{-1} anomaly in velocity (9), (10) distributions. Both (25) and (26) definitions correspond to Poison equations. If, like in Eq. (25),

$$\int d^3r\,\frac{\partial^2\rho}{\partial x^2} \neq 0 ,$$

in this case the solution for $v_x(r)$ looks like the potential distribution around point-like charge (i.e., $v_x(r)\propto r^{-1}$). In two other equations (26)

$$\int d^3r\,\frac{\partial^2\rho}{\partial x\partial y} = 0, \quad \int d^3r\,\frac{\partial^2\rho}{\partial x\partial z} = 0 .$$

Therefore the solution of these equations cannot have r^{-1} behavior. They could excite the distributions with the asimptotes $v_y(r)\propto v_z(r)\propto r^{-3}$.

Go back to Eq. (25). Its solution has the form

$$v_x(xyz) = \frac{V_x}{4\pi}\int_0^{+\infty} dx'\int_0^{+\infty} dy'\int_0^{+\infty}\frac{dz'}{R\rho(x'y'z')}\frac{\partial^2\rho(x'y'z')}{\partial(x')^2} , \qquad (27)$$

$$R^2 = (x-x')^2 + (y-y')^2 + (z-z')^2.$$

At the big distances $r >> R_n$

$$v_x(xyz) \simeq \frac{V_x}{4\pi}\frac{R_n}{r}, \qquad (28)$$

$$R_n \simeq \int_0^{+\infty} dx'\int_0^{+\infty} dy'\int_0^{+\infty}\frac{dz'}{\rho(x'y'z')}\frac{\partial^2\rho(x'y'z')}{\partial(x')^2} . \qquad (29)$$

As in $R_{\rm id}$ definition (21), the main contribution in R_n (29) follows from the small distances $r' << R_n$. For these r' the divergency (29) looks much stronger, than the same one in (21). Such a qualitative difference leads to conclusion

$$R_n > R_{\rm id} . \qquad (30)$$

Therefore the discussing model of snowball effective mass is not only two-liquid one. It contains also two radiuses: $R_{\rm id}$ (21) and R_n (29) with the qualitative prediction (30).

5. Let us pass to discussion of the experimental situation. First of all it is necessary to stress that the effective mass temperature dependence for positive charge carriers in liquid ^{4}He has been observed in the interval from 20 mK up to 2 K (see [4–7] and Fig. 1). The original data [4] for $M^{(ass)}(T)$ in interval 1 K $< T <$ 2 K have been extracted from the impedance behavior of 2D-ion pool. In addition to measurements this paper contains a reasonable scenario for $M^{(ass)}(T)$ interpretation. The following publications: [5–7]-deal with the 2D-plasma resonances, which are sensitive to M. These measurements show $M^{(ass)}(T)$ dependence until $\simeq$ 20 mK. Below 20 mK the temperature influence in $M^{(ass)}$-structure is negligible. Limiting value

$$M(T \to 0) \simeq 30 m_4, \quad R_{\rm id} < 6 \text{ Å} \tag{31}$$

corresponds to modern level of total snowball effective mass limitation, caused by helium compression and superfluid flow association contribution in low-temperature regime. Estimation (31) for $R_{\rm id}$ corresponds to requirement, that the total mass $M(T)$ has a finite contribution from $M^{(sol)}$ (4).

From other side dc-mobility data in «Stokes area» from Ref. 4, when radius R_n is important, lead to estimation

$$R_n \simeq 7 \text{ Å} > R_{\rm id}$$

which is consistent with (30).

It is interesting to note that all authors [4–7] indicate the temperature influence in snowball effective mass behavior. And nobody discuss $M^{(ass)}(\omega)$ dependence. But all information above leads to the conclusion: both $M^{(ass)}(T)$ and $M^{(ass)}(\omega)$ are coupled. To demonstrate at least qualitatively $M^{(ass)}(\omega)$ sensitivity, we can use the experiments with 2D-plasma resonance $\omega(l,m)$ excitations [5]. Plasma excitations in an infinite 2D-ion system have the dispersion law $\omega(q)$

$$\omega^2(q) = \frac{2\pi e^2 n_s}{M(\omega)} |\mathbf{q}| . \tag{33}$$

Here n_s is the 2D ion density; $\omega, \mathbf{q}$ are the frequency and the wave vector, respectively; $M(\omega)$ is the ion effective mass which generally depends on ω. The knowledge of the spectrum $\omega(q)$ obviously allows one to extract the ion effective mass.

In the experiments of Ref. 6 the 2D-ion pool had the shape of a disk of radius R. In that case the continuous dispersion law $\omega(q)$ (33) is replaced by a set of discrete eigenfrequencies $\omega(l,m)$ where the integers l and m label the radial and azimuthal wave numbers

$$q^2 \to q^2(l,m) = q^2(l) + q^2(m) .$$

The eigenfrequencies observed in Ref. 6 correspond to the following pairs: (0,1); (0,2); (0,3). Their measured relative values are (information from Fig. 1 in Ref. 6)

$$\frac{\omega^2(0,3)}{\omega^2(0,1)} \simeq 5.849, \qquad \frac{\omega^2(0,3)}{\omega^2(0,2)} \simeq 1.878 \tag{34}$$

and depend on both the ratio of the wave numbers $q(l,m)$ and the ratio of the effective masses $M(\omega(l,m)) \to M(l,m)$. Thus, for example,

$$\frac{\omega^2(0,3)}{\omega^2(0,1)} = \frac{q(0,3)}{M(0,3)} \frac{M(0,1)}{q(0,1)}, \quad M(0,1) > M(0,3).$$

Further, one can calculate the same frequencies (0,1); (0,2); (0,3) for the cell geometry of Ref. 6 and find their ratios assuming $M = \text{const}$. In that case, e.g.,

$$\frac{\omega^2(0,3)}{\omega^2(0,1)_{|\rm theor}} = \frac{q(0,3)}{q(0,1)}$$

yielding

$$\frac{\omega^2(0,3)}{\omega^2(0,1)_{|\rm theor}} \simeq 5.115, \qquad \frac{\omega^2(0,3)}{\omega^2(0,2)_{|\rm theor}} \simeq 1.768. \tag{35}$$

If the mass M were frequency independent, the estimates (34) and (35) would give identical figures. However, actually the ratios (34) exceed the corresponding numbers in (35) suggesting $M(0,1) > M(0,3)$, in full agreement with (13).

Here several words should be said concerning the method used in Ref. 4 to determine $M^{(ass)}(T,\omega)$. In that paper (relevant to the viscous scenario of the $M^{(ass)}(T,\omega)$ description) the authors declare the possibility to measure both dc and two ac components for ion mobility $\mu(\omega)$ with the same 2D-ion density distribution. In this case within the Drude approximation we have the reasons for the following definitions:

$$\mu(0) = e\tau/M, \tag{36}$$

$$\text{Im } \mu(\omega,\tau) = \frac{e\tau}{M} \frac{\omega\tau}{1+\omega^2\tau^2}, \tag{37}$$

$$\text{Re } \mu(\omega,\tau) = \frac{e\tau}{M} \frac{1}{1+\omega^2\tau^2}, \tag{38}$$

where τ is the corresponding relaxation time, μ is the ac-ion mobility.

The ratio

$$\frac{\text{Im } \mu(\omega,\tau)}{\text{Re } \mu(\omega,\tau)} = \omega\tau \tag{39}$$

has to be linear function of ω. If so, we have from (39) the τ definition. Finally, using τ (39) and information (36) for

$\mu(0)$ the way is open for M estimations (see (19) from Ref. 4):

$$M(T) = \frac{e}{\omega\mu(0)}\omega\tau \equiv \frac{e}{\omega\mu(0)} \frac{\text{Im } \mu(\omega,\tau)}{\text{Re } \mu(\omega,\tau)} . \quad (40)$$

Drude–Stokes modification of the ion-motion equation with an additional information on $M_n(\omega)$ (13) leads to evident reorganization of the definitions (36)–(39). Nevertheless, the authors [4] believe the relationship (40) is still valid when $\omega \to 0$. The corresponding $M(T)$ data are presented in Fig. 1.

On the other side, using (13), we have in the limit $\omega \to 0$

$$\frac{\text{Im } \mu(\omega,\tau)}{\text{Re } \mu(\omega,\tau)} = \frac{\omega M_n(\omega)}{6\pi\eta R(1+R/\delta)} \propto \sqrt{\omega} . \quad (41)$$

Within this scenario, the combination (40) cannot be finite when ω, and therefore, the definition of $M_n(\omega)$ (22) alone cannot be used for interpretation of experimental data presented in Fig. 1. To resolve this problem, we remember the speculations above (see the discussion around (14)). The idea is to normalize the limiting process (40), using instead of (13), (41) some modification $M_n(\omega_x)$, where ω_x follows from the condition (14). In our model, such a requirement evidently corresponds to the situation when both the real and imaginary parts of the Stokes force (12) are of the same order. Equating them one can evaluate the corresponding frequency as

$$\omega_x(T) = \frac{x^2\eta(T)}{2\rho_n(T)R_n^2} \quad x^2 = 9/2 . \quad (42)$$

If so the value of associated mass (13) is reduced to

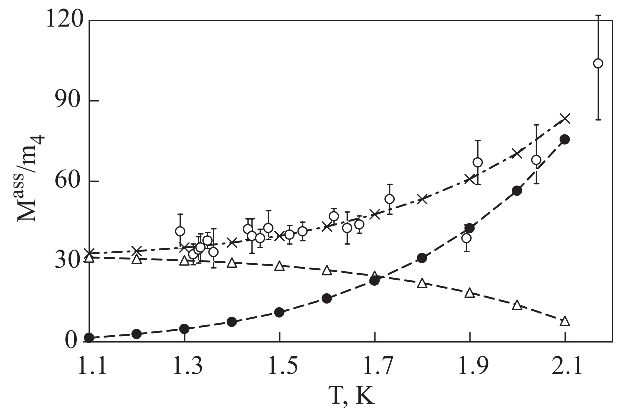

Fig. 1. The temperature dependencies of $M_s^{(\text{ass})}(T)$ according to Eqs. (7), (8) (triangles), $M_n^{(\text{ass})}(T)$ (see Eq. (43), black dots) and their sum $M_s^{(\text{ass})}(T) + M_n^{(\text{ass})}(T)$ (crosses) are presented separately. The dependence $\rho_n(T)$ is taken from Ref. 4. The experimental data (open circles with error bars) are taken from Ref. 4.

$$M_n^{(\text{ass})}(\omega_x) = 3\pi R_n^3 \rho_n \frac{1}{x}\left(1 + \frac{2}{9}x\right) . \quad (43)$$

The fitting of the total effective mass Eq. (6) with the definitions (7), (8) for $M_s^{(\text{ass})}$ and (43) for $M_n^{(\text{ass})}$ plus the simplifications $R_{\text{id}} \simeq R_n$, $M^{(\text{sol})} = 0$, where R_n is adjustable parameter, demonstrates a good agreement with the experimentally observed temperature dependence in the wide range of temperatures (see Fig. 1). This fact qualitatively supports the idea to use two-liquid scenario and to account for the viscous velocity distribution in the frameworks of the Navier–Stokes flow picture.

It is necessary to mention that understanding of possible viscous origin of the temperature dependence $M_n^{(\text{ass})}(T)$ was already contained in the early paper [4]. We support ideas from Ref. 4, demonstrate the basic reasons for unusual features in $M_n(T,\omega)$ behavior, liberate the theory from snowball Atkins [2] assumptions, introduce two different radiuses: R_{id} and R_n, repair the fitting program [4] to explain the data Fig. 1 and discuss briefly the modern (after [4]) experimental situation.

In conclusion, it should be emphasized that the observed temperature (actually, frequency) dependence of the snowball effective mass is interesting not only in itself. It also turns out sensitive indicator revealing qualitative difference in the velocity distribution field around the moving sphere in either viscous or ideal regime. This difference has been known for a long time. However, it did not attract much attention since in the applications (calculation of the Stokes drag force) slow decrease (of the $1/r$ type) of the velocity field does not result in any divergence in its real part. The situation is quite different in the calculation of the imaginary part, and the above outlined insight in this field is one of important conclusions of this paper.

Certainly, the hydrodynamic treatment of the $M(T,\omega)$ dependence imposes some restrictions on explanation of the available experimental data obtained at low temperatures (in the vicinity of several mK). However, the alternative kinetic language suitable for description of the ballistic regime confronts in substantial difficulties when the sphere effective mass is calculated even in the laminar limit. Details of this formalism will be reported elsewhere.

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