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On the critical behaviour of random anisotropy magnets: cubic anisotropy

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The critical behaviour of an m-vector model with a local anisotropy axis of random orientation is studied within the field-theoretical renormalization group approach for cubic distribution of anisotropy axis. Expressions for the renormalization group functions are calculated up to the two-loop order and investigated both by an $\varepsilon = 4 - d$ expansion and directly at space dimension $d = 3$ by means of the Padé-Borel resummation. One accessible stable fixed point indicating a 2nd order ferromagnetic phase transition with dilute Ising-like critical exponents is obtained.

Key words: *random anisotropy, renormalization group, critical exponents*

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1. Introduction

In statistical physics, low-temperature phases of many-particle spin models may possess qualitatively different features depending on the fact whether the corresponding spin Hamiltonian is of discrete or of continuous symmetry. An m-vector model described by the Hamiltonian [1] may serve as a textbook example:

$$\mathcal{H} = -\sum_{\mathbf{R},\mathbf{R}'} J_{\mathbf{R},\mathbf{R}'} \vec{S}_{\mathbf{R}} \vec{S}_{\mathbf{R}'}, \qquad (1)$$

where vectors $\mathbf{R}$,$\mathbf{R}'$ span sites of the d-dimensional (hyper)cubic lattice, $J_{\mathbf{R},\mathbf{R}'} > 0$ is a short-range ferromagnetic interaction and $\vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'}$ is a scalar product of classical m-component "spins" $\vec{S}_{\mathbf{R}} = (S^1_{\mathbf{R}'}, \ldots, S^m_{\mathbf{R}'})$. The Hamiltonian (1) possesses a global $O(m)$ symmetry: it remains invariant under rotations in the space of vectors $\{\vec{S}\}$.

However, continuous $O(m)$ symmetry turns to the discrete one for $m = 1$: this corresponds to the invariance of the Ising model Hamiltonian under discrete turn of the Ising spins to opposite direction. The consequences are well known: whereas in the Ising model a ferromagnetic phase exists for lattice dimensions greater than 1 [1] (i.e. the lower critical dimension $d_{\rm L} = 1$), $d_{\rm L} = 2$ for $m \geqslant 2$ [2]. For $d > 2$ a continuous symmetry can be spontaneously broken for any m, while $d_{\rm U} = 4$ is the upper critical dimension of the m-vector model: starting from $d = 4$ the magnetic phase transition is governed by the mean-field critical exponents.

Implementation of a weak structural (lattice) disorder into model (1) has crucial consequences for the existence of the ordered phase. For a particular reason of the present study of main interest to us will be the case when distribution of disorder can be characterized by a certain symmetry. The m-vector model with random anisotropy (a random anisotropy model, RAM) [3] may serve as an example:

$$\mathcal{H} = -\sum_{\mathbf{R},\mathbf{R}'} J_{\mathbf{R},\mathbf{R}'} \vec{S}_{\mathbf{R}} \vec{S}_{\mathbf{R}'} - D_0 \sum_{\mathbf{R}} (\hat{x}_{\mathbf{R}} \vec{S}_{\mathbf{R}})^2. \tag{2}$$

Here, the notations are as in (1), $D_0 > 0$ is an anisotropy strength, and $\hat{x}_{\mathbf{R}}$ is an unit vector pointing in the local (quenched) random direction of an uniaxial anisotropy.

The model Hamiltonian (2) possesses randomness only for $m > 1$: at $m = 1$ the second term is a constant and leads to a shift in a free energy of the resulting regular (Ising) model. It means that the randomness-induced behaviour in RAM may be observed only for spins of continuous symmetry. The low-temperature ordering in RAM is also influenced in addition to the global variables of a regular magnet (i.e. lattice dimension, type of interaction and spin symmetry) also by distribution of the random variables $\hat{x} \equiv \hat{x}_{\mathbf{R}}$ in (2). For non-correlated $\hat{x}_{\mathbf{R}}$ the low-temperature ordering depends on the probability distribution $p(\hat{x})$ of the direction of anisotropy on a single site. In particular, for the *isotropic* distribution $d_{\rm L} = 4$: a ferromagnetic order is absent for the lattice dimension less than 4. The absence of ferromagnetic ordering in isotropic RAM was first observed in the renormalization group study of [4] where no accessible fixed points of the renormalization group transformation were obtained for the model within $\varepsilon = 4 - d$ expansion. Recently, this result was corroborated by higher-order calculations refined by a resummation technique [5]. The proof of [6,7] used arguments similar to those applied by Imry and Ma [8] for a random-field Ising model and showed that the susceptibility of the ordered state diverges for $d < 4$. Explicit calculations for $m \to \infty$ were offered in [6]. Although the latter appeared to be erroneous [9], the value $d_{\rm L} = 4$ was further supported by an attempt of a Mermin-Wagner proof of the absence of ferromagnetism in RAM with the isotropic distribution of an anisotropy axis for $d < 4$ [10]. The proof of the work [10] uses the replica trick [11] and cannot be considered rigorous. However, the same paper studies RAM at low temperatures and thus the small anisotropy that avoids the application of replicas by means of the Migdal-Polyakov renormalization group technique and the fixed point structure obtained there in $d - 4$ dimensions confirms the absence of ferromagnetism below $d = 4$. The upper critical dimension for RAM with the isotropic distribution of an anisotropy axis was shown to be $d_U = 6$ [12].

However, the above arguments do not concern *anisotropic* distributions $p(\hat{x})$. Here, the possibility of ferromagnetic ordering is to be studied for every particular case. We address the question of the existence of a ferromagnetic second order phase transition and its universal properties for a $d = 3$ RAM with an anisotropic distribution of a random axis, when the vector $\hat{x}_{\mathbf{R}}$ (1) points only along one of the $2m$ directions of axes $\hat{k}_i$ of a cubic lattice (the so-called cubic anisotropy):

$$p(\hat{x}) = \frac{1}{2m} \sum_{i=1}^{m} [\delta^{(m)}(\hat{x} - \hat{k}_i) + \delta^{(m)}(\hat{x} + \hat{k}_i)], \qquad (3)$$

where $\delta(\hat{y})$ are Kronecker's deltas. Besides a pure academic interest, such a choice has practical applications: typical examples of random-anisotropy magnets are amorphous rare-earth – transition metal alloys [13] and the cubic distribution (3) of a random axis mimics the situation when an amorphous magnet still "remembers" initial (cubic) lattice structure.

In the present paper, we study the RAM with the cubic distribution of a random-anisotropy axis (3) by means of a field theoretical renormalization group (RG) technique [14] and analyze the two-loop RG functions both by an $\varepsilon = 4 - d$ expansion and directly at space dimension $d = 3$. We show the existence of a second order phase transition and make our conclusions about its numerical characteristics based on the resummation technique applied to the resulting perturbation theory series. The paper is a direct continuation of our preceding work [5], where we applied similar tools to study RAM with an isotropic distribution $p(\hat{x})$ and we refer the reader there for a more extended review of the RAM general features.

The set-up of the paper is the following: in the next section 2 we describe the model and obtain the RG functions within the massive field theory scheme. In the RG analysis, the presence of a second order phase transition corresponds to the presence of a reachable stable fixed point of the RG transformation. The fixed points and their stability are analyzed in section 3 by means of an ε-expansion to order ε^2 and by resummation of a $d = 3$ series. We estimate the critical exponents values and display them in section 3 as well. Section 4 concludes our study and summarizes the results obtained.

2. The renormalization group functions

In order to apply the field theoretical RG approach to study the critical behaviour of the RAM (2) with quenched local anisotropy axis distributed according to (3) one should get an effective Hamiltonian of the model. Following the scheme of [4] for a given configuration of quenched random variables $\hat{x}_{\mathbf{R}}$ in (2) the partition function of RAM is written in the form of a functional integral of a Gibbs distribution with the effective Hamiltonian:

$$\mathcal{H}(\hat{x}_{\mathbf{R}}, \vec{\phi}) = -\int \mathrm{d}^d R \left\{ \frac{1}{2} \left[r_0 |\vec{\phi}|^2 + |\vec{\nabla}\vec{\phi}|^2 \right] - D_1 (\hat{x}_{\mathbf{R}} \vec{\phi})^2 + v_0 |\vec{\phi}|^4 + \ldots \right\}, \qquad (4)$$

where D_1 is proportional to D_0, r_0 and v_0 are defined by D_0 and the familiar bare couplings of an m-vector model, and $\vec{\phi} \equiv \vec{\phi}_{\mathbf{R}}$ is a m-dimensional vector. Implying quenched disorder and using therefore the replica trick [11] one arrives at the n-replicated configuration-dependent partition function. Performing then the average over random variables for the case of a cubic distribution (3) one ends up with the effective Hamiltonian [4]:

$$\mathcal{H}_{\text{eff}} = -\int \mathrm{d}^d R \left\{ \frac{1}{2}\left[{\mu_0}^2 |\vec{\varphi}|^2 + |\vec{\nabla}\vec{\varphi}|^2 \right] + u_0 |\vec{\varphi}|^4 + v_0 \sum_{\alpha=1}^{n} |\vec{\phi}^{\alpha}|^4 \right.$$
$$\left. + \, w_0 \sum_{i=1}^{m} \sum_{\alpha,\beta=1}^{n} (\phi_i^{\alpha})^2 (\phi_i^{\beta})^2 + y_0 \sum_{i=1}^{m} \sum_{\alpha=1}^{n} (\phi_i^{\alpha})^4 \right\}, \qquad (5)$$

which in the replica $n \to 0$ limit describes critical behaviour of the model (2) with cubic anisotropy distribution (3). Here, μ_0 is bare mass, bare couplings $u_0 > 0$, $v_0 > 0$, $w_0 < 0$ are defined by D_0 and familiar bare couplings of an m-vector model. The last term in (5) combines the symmetries of terms with coefficients v_0 and w_0. It does not result from the functional representation of the free energy but is generated by further application of the RG transformation. Therefore y_0 can be of either sign. ϕ_i^{α} are the components of an mn-dimensional order parameter field, $|\varphi_i|^2 = \sum_{\alpha} |\phi_i^{\alpha}|^2$. Values w_0 and u_0 are related to appropriate cumulants of the distribution function (3) in such a way that their ratio $w_0/u_0 = -m$ and thus determines a region of typical initial values in the $(u - v - w - y)$-space of couplings.

To get a qualitative picture of a critical behaviour it is standard now to rely on the field-theoretical RG approach [14]. In this approach, finiteness of the (renormalized) vertex functions $\Gamma_R^{(n)}$ is ensured by imposing certain normalizing conditions. In turn, this leads to different renormalization schemes. Here, we will make use of the renormalization at fixed mass and zero external momenta $\{k\}$ [15]. Normalization conditions are written then for a fixed space dimension d and read:

$$\begin{aligned}
\Gamma_R^{(2)}(0; \mu^2, u, v, w, y) &= \mu^2, \\
\frac{\partial}{\partial k^2} \Gamma_R^{(2)}(k; \mu^2, u, v, w, y)\Big|_{k^2=0} &= 1, \\
\Gamma^{(4)}_{u\ R}(\{0\}; \mu^2, u, v, w, y) &= \mu^{4-d} u, \\
\Gamma^{(4)}_{v\ R}(\{0\}; \mu^2, u, v, w, y) &= \mu^{4-d} v, \\
\Gamma^{(4)}_{w\ R}(\{0\}; \mu^2, u, v, w, y) &= \mu^{4-d} w, \\
\Gamma^{(4)}_{y\ R}(\{0\}; \mu^2, u, v, w, y) &= \mu^{4-d} y; \\
\Gamma_R^{(2,1)}(p; k; \mu^2, u, v, w, y)\Big|_{p^2=k^2=0} &= 1.
\end{aligned} \qquad (6)$$

Here, μ, u, v, w, y are renormalized mass and dimensionless couplings, $\Gamma_R^{(2,1)}$ is renormalized vertex function with ϕ^2 insertion, and the vertices $\Gamma_u^{(4)}$, $\Gamma_v^{(4)}$,$\Gamma_w^{(4)}$, $\Gamma_y^{(4)}$ are parts of a full vertex function

$$\Gamma^{(4)}{}^{i_1 i_2 i_3 i_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} = \Gamma_u^{(4)} S^{i_1 i_2 i_3 i_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} + \Gamma_v^{(4)} S_{i_1 i_2 i_3 i_4} F_{\alpha_1\alpha_2\alpha_3\alpha_4} + \Gamma_w^{(4)} F_{i_1 i_2 i_3 i_4} S_{\alpha_1\alpha_2\alpha_3\alpha_4}$$

$$+\,\Gamma_y^{(4)} F_{i_1 i_2 i_3 i_4} F_{\alpha_1\alpha_2\alpha_3\alpha_4}, \tag{7}$$

where

$$\begin{aligned} F_{ijkl} &= \delta_{ij}\delta_{ik}\delta_{il}, \\ S_{ijkl} &= \frac{1}{3}\left(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\right), \\ S_{ijkl}^{\alpha\beta\gamma\tau} &= \frac{1}{3}\left(\delta_{ij}\delta_{kl}\delta_{\alpha\beta}\delta_{\gamma\tau}+\delta_{ik}\delta_{jl}\delta_{\alpha\gamma}\delta_{\beta\tau}+\delta_{il}\delta_{jk}\delta_{\alpha\tau}\delta_{\beta\gamma}\right), \end{aligned} \tag{8}$$

δ_{ab} is Kronecker's delta. Tensors in (8) correspond to terms of different symmetry in the effective Hamiltonian (5), the Latin symbols are the spin indices and the Greek symbols are the replica indices.

The mass is renormalized by: $\mu = Z_\phi \Gamma^{(2)}(0;\mu_0;\{u_{i,0}\})$, with $u_i = u, v, w, y$ and Z_ϕ being the field renormalizing factor. Finiteness of $\Gamma^{(2,1)}$ is secured by the factor $\bar{Z}_{\phi^2}$. The renormalizing factors of couplings Z_{u_i} relate the bare couplings to the renormalized ones

$$u_{i,0} = \mu^{4-d}\frac{Z_{u_i}}{Z_\phi^2}u_i. \tag{9}$$

All renormalizing factors are defined by conditions (6). A change of the couplings u_i and Z-factors under the RG transformation is described by the β- and γ-functions:

$$\beta_{u_i} = \frac{\partial u_i}{\partial \ln\mu}, \qquad \gamma_\phi = \frac{\partial \ln Z_\phi}{\partial \ln\mu}, \qquad \bar{\gamma}_{\phi^2} = -\frac{\partial \ln \bar{Z}_{\phi^2}}{\partial \ln\mu}, \tag{10}$$

determining the approach of the system to criticality. Namely, the fixed point (FP) $\{u_i^*\}$ of the RG transformation defined as a solution of equations

$$\beta_{u_i}(\{u_j^*\}) = 0 \tag{11}$$

if stable, may correspond to the critical point. The condition of the FP stability reads:

$$\left|\frac{\partial \beta_{u_i}}{\partial u_j}(\{u_j^*\}) - \omega_i\delta_{ij}\right| = 0, \qquad \mathrm{Re}\,\omega_i > 0. \tag{12}$$

However, the correspondence of a stable FP to the critical point of a system implies that this FP is reachable from the initial conditions (initial values of the couplings). In the stable FP, the correlation length critical exponent and the pair correlation function critical exponent are defined by:

$$\nu^{-1} = 2 - \bar{\gamma}_{\phi^2}(\{u_i^*\}) - \gamma_\phi(\{u_i^*\}), \tag{13}$$

$$\eta = \gamma_\phi(\{u_i^*\}). \tag{14}$$

The rest of critical exponents may be derived from the familiar scaling relations.

Applying the renormalization scheme (6) we get the RG functions of the model (5) in a two-loop approximation. In the replica limit $n = 0$ they read:

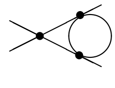
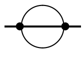
$$\beta_u = -\varepsilon\Big\{u - \frac{1}{6}\left[8u^2+2\,(m+2)\,uv+2vw+4uw+6uy\right] + \frac{1}{9}\Big[44u^3+48u^2w$$

$$+12w^2u{+}24\,(m+2)\,vu^2{+}2\,(3m{+}6)\,uv^2{+}4w^2v{+}4v^2w{+}72u^2y{+}18y^2u$$
$$+60uvw{+}36uvy{+}36uwy\Big]i_1+\frac{2}{9}\Big[2u^3{+}2\,(m{+}2)\,vu^2{+}\,(m{+}2)\,uv^2$$
$$+2w^2u{+}6uvw{+}4u^2w{+}6u^2y{+}6uvy{+}6uwy{+}3y^2u\Big]i_2\Big\}, \qquad (15)$$

$$\beta_v = -\varepsilon\, v\Big\{1-\frac{1}{6}\left[(m{+}8)\,v{+}12u{+}4w{+}6y\right]+\frac{1}{9}\Big[2\,(5m{+}22)\,v^2{+}8\,(3m{+}15)\,vu$$
$$+84u^2{+}12w^2{+}68vw{+}72vy{+}18y^2{+}72uw{+}108uy{+}36wy\Big]i_1{+}\frac{2}{9}\Big[(m{+}2)v^2$$
$$+2\,(m+2)\,uv{+}2u^2{+}2w^2{+}6vw{+}4uw{+}6uy{+}6vy{+}6wy{+}3y^2\Big]i_2\Big\}, \qquad (16)$$

$$\beta_w = -\varepsilon\, w\Big\{1-\frac{1}{6}\left[8w{+}12u{+}4v{+}6y\right]+\frac{1}{9}\Big[44w^2{+}84u^2{+}120wu{+}2\,(m{+}6)\,v^2$$
$$+68vw{+}72wy{+}18y^2{+}2\,(6m+36)\,uv{+}108uy{+}36vy\Big]i_1{+}\frac{2}{9}\Big[(m{+}2)v^2$$
$$+2\,(m+2)\,uv{+}2u^2{+}2w^2{+}6vw{+}4uw{+}6uy{+}6vy{+}6wy{+}3y^2\Big]i_2\Big\}, \qquad (17)$$

$$\beta_y = -\varepsilon\Big\{y-\frac{1}{6}\left[9y^2{+}8vw{+}12uy{+}12vy{+}12wy\right]+\frac{1}{9}\Big[(4m+72)\,v^2w{+}72w^2v$$
$$+54y^3{+}84u^2y{+}\,(6m+84)\,v^2y{+}84w^2y{+}144y^2u{+}144y^2v{+}144y^2w$$
$$+96uvw{+}2\,(6m+84)\,uvy{+}252vwy{+}168uwy\Big]i_1{+}\frac{2}{9}\Big[2u^2y{+}2\,(m+2)\,uvy$$
$$+\,(m{+}2)\,v^2y{+}2w^2y{+}6vwy{+}4uwy{+}6y^2u{+}6y^2v{+}6y^2w{+}3y^3\Big]i_2\Big\}, \qquad (18)$$
$$\gamma_\phi = -\frac{\varepsilon}{9}\left(2u^2+2\,(m+2)uv+(m+2)v^2+6vw+2w^2+4uw+6uy\right.$$
$$\left.+6vy+6wy+3y^2\right)i_2, \qquad (19)$$
$$\bar\gamma_{\phi^2} = \frac{\varepsilon}{3}\Big\{\frac{1}{2}\Big(2u+(m+2)\,v+2w+y\Big)-\Big(2u^2+2\,(m+2)\,uv+(m+2)\,v^2$$
$$+6vw+2w^2+4uw+6uy+6vy+6wy+3y^2\Big)\,i_1\Big\}, \qquad (20)$$

where $\varepsilon = 4-d$ and i_1, i_2 are loop integrals [16] of the diagrams  and $\frac{\partial}{\partial k^2}$  $|_{k^2=0}$ correspondingly. Analysing the RG functions (15)–(20) in the fixed $d=3$ scheme [15] one substitutes the loop integrals by their numerical values i_1 $(d=3)=1/6$, $i_2(d=3)=-2/27$ [17] and then deals with the expansions (15)–(20) in renormalized couplings. However, the ε-expansion technique [18] is also applicable to the massive scheme. To this end, the loop integrals are to be substituted by their ε-expansion: $i_1 \simeq 1/2 + \varepsilon/4 + \ldots$, $i_2 \simeq -\varepsilon/8 + \ldots$ [19] and the perturbation theory is constructed both in ε and in renormalized couplings. Both schemes will be used in our analysis of the expressions (15)–(20) in the next section.

To conclude this section, let us note that the one-loop parts of the RG functions (15)–(20) reproduce ε-expansion results of works [4,20]. Moreover, the RAM with the cubic distribution of random axis may be regained from the more general model of the article [21] describing phase transition in crystals with low-symmetry point defects. There, the corresponding RG functions were written to order ε^2. The ε–expansion of RG functions (15)–(20) does not coincide with the appropriate functions of the work [21] as far as the renormalization schemes differ. However, as it will be shown in the next section, the observables obtained in ε-expansion on their basis do coincide, as expected.

3. The fixed points and the critical exponents

3.1. An ε-expansion

The first RG study of the RAM [4] reported an evidence of 14 FPs for the cubic distribution of an anisotropy axis. This result was obtained in the first order in ε and may be easily reproduced based on the functions (15)–(18) putting two-loop contributions equal to zero and applying the ε-expansion scheme as described at the end of section 2. We list coordinates of the FPs I–XIV in the table 1 (in order to recover the results of [4] we extract the value of the one-loop integral $\sim 1/\varepsilon$ from conventionally normalized couplings: see note [16]).

The results of linear in ε analysis [4] state that among the FPs with $u > 0, v > 0, w < 0$ only a "polymer" $O(n = 0)$ FP III is stable at all m for $\varepsilon > 0$, but it is not reachable from the initial values of couplings (see figure 1). The reason is a separatrix joining the unstable FPs I and VII and separating initial values of couplings (shown by a cross in figure 1) and FP III. Possible runaway behaviour of RG flows lead Aharony [4] to the conclusion about smearing of the phase transition as T_c approaches.

However, the subsequent study of Mukamel and Grinstein [20] brought about a possibility of a second order phase transition with the scenario of a weakly diluted quenched Ising model [22]. Indeed, performing perturbation theory expansion to the order ε^2 we get not only the corrections to the coordinates of the FPs I–XIV (listed in the tables 1, 2) but the new FPs XV, XVI, XVII (see the bottom of the table 1). The appearance of the pairs of the FPs XV and XVI is caused by the well known fact that the β-functions β_w, β_y at $u = v = 0$ (β_u, β_v at $w = y = 0$, correspondingly) are degenerated at the one loop level. Expressions of FPs coordinates XV, XVI in the table 1 are a familiar $\sqrt{\varepsilon}$ expansion of the FP of weakly diluted quenched Ising model [22]. The $\sqrt{\varepsilon}$ expansion of the FP XVII holds for $m = 2$ and is caused by the one-loop degeneracy of the β_u, β_v, β_y functions for $w = 0$ (c.f. singularity at $m = 2$ in the ε-expansion of the FP IX).

Table 1. FPs of the RAM with the random cubic anisotropy distribution (ε-expansion). Here, $\alpha_\pm = (m-4\pm\sqrt{m^2+48})/8$, $\beta_\pm = -(m+12\pm\sqrt{m^2+48})/6$, $A_{\pm\pm} = 6\alpha_\pm + 3\beta_\pm + m + 6$. Note, that the fixed points XV–XVII appear only in the two-loop approximation due to the degeneracy of the corresponding one-loop functions. Expressions for some two-loop contributions (indexed by Roman numbers) are too cumbersome, their numerical values are listed in table 2 for some m.

	u^*	v^*	w^*	y^*
I.	0	0	0	0
II.	0	$\frac{6}{m+8}\varepsilon+18\frac{(3m+14)}{(m+8)^3}\varepsilon^2$	0	0
III.	$\frac{6}{8}\varepsilon+\frac{63}{128}\varepsilon^2$	0	0	0
IV.	0	0	$\frac{6}{8}\varepsilon+\frac{63}{128}\varepsilon^2$	0
V.	0	0	0	$\frac{6}{9}\varepsilon+\frac{34}{81}\varepsilon^2$
VI.	$\frac{6(m-4)}{16(m-1)}\varepsilon+u_{\rm VI}\varepsilon^2$	$\frac{6}{4(m-1)}\varepsilon+v_{\rm VI}\varepsilon^2$	0	0
VII.	$\frac{3}{2}\varepsilon+\frac{3}{4}\varepsilon^2$	0	$-\frac{3}{2}\varepsilon-\frac{3}{4}\varepsilon^2$	0
VIII.	0	$\frac{2}{m}\varepsilon+v_{\rm VIII}\varepsilon^2$	0	$\frac{2(m-4)}{3m}\varepsilon+y_{\rm VIII}\varepsilon^2$
IX.	$\frac{m-4}{4(m-2)}\varepsilon+u_{\rm IX}\varepsilon^2$	$\frac{1}{m-2}\varepsilon+v_{\rm IX}\varepsilon^2$	0	$\frac{m-4}{3(m-2)}\varepsilon+y_{\rm IX}\varepsilon^2$
X.	$\frac{1}{2}\varepsilon+\frac{25}{108}\varepsilon^2$	0	$-\frac{1}{2}\varepsilon-\frac{25}{108}\varepsilon^2$	$\frac{2}{3}\varepsilon+\frac{34}{81}\varepsilon^2$
XI.$\alpha_+\beta_+$	$\frac{3\alpha_+}{A_{++}}\varepsilon+u_{\rm XI}\varepsilon^2$	$\frac{3}{A_{++}}\varepsilon+v_{\rm XI}\varepsilon^2$	$\frac{3(m+4)}{4A_{++}}\varepsilon+w_{\rm XI}\varepsilon^2$	$\frac{3\beta_+}{A_{++}}\varepsilon+y_{\rm XI}\varepsilon^2$
XII.$\alpha_+\beta_-$	$\frac{3\alpha_+}{A_{+-}}\varepsilon+u_{\rm XII}\varepsilon^2$	$\frac{3}{A_{+-}}\varepsilon+v_{\rm XII}\varepsilon^2$	$\frac{3(m+4)}{4A_{+-}}\varepsilon+w_{\rm XII}\varepsilon^2$	$\frac{3\beta_-}{A_{+-}}\varepsilon+y_{\rm XII}\varepsilon^2$
XIII.$\alpha_-\beta_+$	$\frac{3\alpha_-}{A_{-+}}\varepsilon+u_{\rm XIII}\varepsilon^2$	$\frac{3}{A_{-+}}\varepsilon+v_{\rm XIII}\varepsilon^2$	$\frac{3(m+4)}{4A_{-+}}\varepsilon+w_{\rm XIII}\varepsilon^2$	$\frac{3\beta_+}{A_{-+}}\varepsilon+y_{\rm XIII}\varepsilon^2$
XIV.$\alpha_-\beta_-$	$\frac{3\alpha_-}{A_{--}}\varepsilon+u_{\rm IV}\varepsilon^2$	$\frac{3}{A_{--}}\varepsilon+v_{\rm IV}\varepsilon^2$	$\frac{3(m+4)}{4A_{--}}\varepsilon+w_{\rm IV}\varepsilon^2$	$\frac{3\beta_-}{A_{--}}\varepsilon+y_{\rm IV}\varepsilon^2$
XV.	0	0	$\mp\sqrt{\frac{54}{53}\varepsilon}$	$\pm\frac{4}{3}\sqrt{\frac{54}{53}\varepsilon}$
XVI.	$\mp\sqrt{\frac{54}{53}\varepsilon}$	0	0	$\pm\frac{4}{3}\sqrt{\frac{54}{53}\varepsilon}$
XVII.$_{m=2}$	$\pm\sqrt{\frac{54}{53}\varepsilon}$	$\mp2\sqrt{\frac{54}{53}\varepsilon}$	0	$\pm\frac{4}{3}\sqrt{\frac{54}{53}\varepsilon}$

Checking the stability of new FPs XV–XVIII we find that all of them are unstable except for the FP with $w < 0$, $y > 0$ from the pair XV. Moreover, this point is reachable from the initial values of the couplings. As far as it is the FP of the diluted Ising model one concludes, that in the critical region, RAM with cubic distribution of random anisotropy axis (3) decouples into m independent dilute Ising models and the phase transition is governed by the familiar random Ising model critical exponents [23].

However, let us keep in mind that the above picture is obtained in the frames of the "naive" analysis of ε (and $\sqrt{\varepsilon}$) expansion and it is highly desirable to confirm it by a more reliable analysis of FPs and their stability. This will be done below.

3.2. A $d = 3$ series

Table 2. Numerical values of some contributions to the fixed points coordinates of the table 1 for $m = 2, 3, 4$.

m	u_{IV}	u_{IX}	u_{XI}	u_{XII}	u_{XIII}	u_{XIV}
$2^{\pm}$	–3.8906	$\pm\infty$	–29.0018	0.2158	0.0598	0.3838
3	–0.6665	1.2133	–5.3683	0.2319	–0.0161	–9.5594
$4^{\pm}$	–0.2292	0.5	–2.1797	0.2433	–0.0120	$\pm\infty$
	v_{IV}	v_{VIII}	v_{IX}	v_{XI}	v_{XII}	v_{XIII}
$2^{\pm}$	3.2578	0.6296	$\mp\infty$	–29.4605	0.1785	0.0372
3	0.8346	0.2689	–1.6030	–2.9857	0.1277	0.1395
4	0.5	0.1042	–0.3958	–0.7266	0.0923	0.2959
	v_{XIV}	w_{XI}	w_{XII}	w_{XIII}	w_{XIV}	y_{VIII}
2	–0.0627	–54.2734	0.2556	–0.1332	–0.2614	–0.4198
3	10.3646	–8.8086	0.2288	–0.0863	18.5654	0.1079
$4^{\pm}$	$\mp\infty$	–3.5156	0.2054	0.0762	$\mp\infty$	0.3333
	y_{IX}	y_{XI}	y_{XII}	y_{XIII}	y_{XIV}	
$2^{\pm}$	$\pm\infty$	135.4989	–0.1911	0.4562	0.2200	
3	0.9788	20.9753	–0.1543	0.3749	–12.5612	
$4^{\pm}$	0.3333	8.1563	0.1231	0.1289	$\pm\infty$	

The next step in our analysis will be to consider the series (15)–(20) for the RG functions directly at fixed space dimension $d = 3$. In the field theory, expansions in renormalized couplings are known to be asymptotic at best and certain resummation procedure is needed in order to obtain reliable data on their basis. Here, we will make use of the Padé-Borel resummation techniques [24]. It consists in the following steps. For the given initial polynomial in several (in our case in four) variables for any series of $\beta = \beta_{u_i}$ from the expressions (15)–(18)

$$\beta(u, v, w, y) = \sum_{1 \leqslant i+j+k+l \leqslant 3} a_{i,j,k,l}\, u^i v^j w^k y^l \tag{21}$$

one introduces a "resolvent" polynomial [25] in one auxiliary variable λ by:

$$F(u,v,w,y;\lambda)=\sum_{1\leqslant i+j+k+l\leqslant 3} a_{i,j,k,l}\, u^i v^j w^k y^l \lambda^{i+j+k+l-1}. \tag{22}$$

with obvious relation $F(u,v,w,y;\lambda=1)=\beta(u,v,w,y)$. Then, the Borel image of (22) is defined as:

$$F^{\mathrm{B}}(u,v,w,y;\lambda)=\sum_{1\leqslant i+j+k+l\leqslant 3} \frac{a_{i,j,k,l} u^i v^j w^k y^l \lambda^{i+j+k+l-1}}{(i+j+k+l-1)!}. \tag{23}$$

Truncated series (23) is approximated by Padé-approximant $[1/1](\lambda)$. Then the resummed β-function is obtained from the formula:

$$\beta^{\mathrm{res}}(u,v,w,y)=\int_0^\infty \mathrm{d}t \exp(-t)[1/1](t). \tag{24}$$

Similar technique is used for resummation of the expression $\nu^{-1}=2-\gamma_\phi(\{u_i^*\})-\bar{\gamma}_{\phi^2}(\{u_i^*\})$. The pair correlation function critical exponent η is obtained by a direct substitution of FPs values into (14).

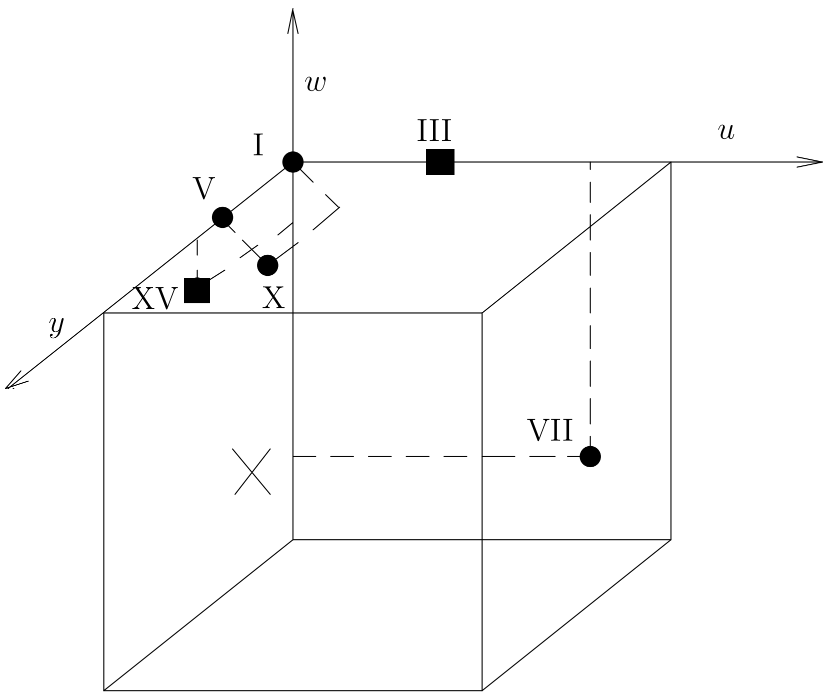


Figure 1. Fixed points of the RAM with cubic distribution of a local anisotropy axis for $v=0$. The only FPs located in the region $u>0, w<0$ are shown. Filled boxes show the stable FPs, a cross denotes the region of the typical initial values of couplings.

Applying the resummation procedure (22)–(24) to the β-functions (15)–(18) we get 16 FPs. In table 3 we present numerical values of FPs coordinates with $u^*>0$,

$v^* > 0$, $w^* < 0$. We visualize the FP picture in figure 1 for $v = 0$. The last FP XV in table 3 corresponds to the stable FP of $\sqrt{\varepsilon}$-expansion of pair XV in table 1. It has coordinates with $u^* = v^* = 0$, $w^* < 0$ and $y^* > 0$ and is accessible from the typical initial values of couplings (marked by a cross in figure 1).

Table 3. Resummed values of the FPs and critical exponents for the cubic distribution in the two-loop approximation at $d = 3$. Only FPs with $u^* > 0$, $v^* > 0$, $w^* < 0$ are shown. The only FPs III and XV are stable.

FP	m	u^*	v^*	w^*	y^*	ν	η
I	$\forall m$	0	0	0	0	1/2	0
II	2	0	0.9107	0	0	0.663	0.027
	3	0	0.8102	0	0	0.693	0.027
	4	0	0.7275	0	0	0.720	0.026
III	$\forall m$	1.1857	0	0	0	0.590	0.023
V	$\forall m$	0	0	0	1.0339	0.628	0.026
VI	3	0.1733	0.6460	0	0	0.659	0.027
	4	0.2867	0.4851	0	0	0.653	0.027
VII	$\forall m$	2.1112	0	–2.1112	0	1/2	0
VIII	2	0	1.5508	0	–1.0339	0.628	0.026
	3	0	0.8393	0	–0.0485	0.693	0.027
	4	0	0.5259	0	0.3624	0.709	0.026
IX	3	0.1695	0.7096	0	–0.1022	0.659	0.027
	4	0.2751	0.4190	0	0.1432	0.653	0.027
X	$\forall m$	0.6678	0	–0.6678	1.0339	0.628	0.026
XV	$\forall m$	0	0	–0.4401	1.5933	0.676	0.031

Applying the resummation procedure (22)–(24) we have not found any other stable FPs in the region of interest.Thus we are drawn to the conclusion that the effective Hamiltonian (5) in critical regime reduces to a product of m effective Hamiltonians of a weakly diluted quenched random site Ising model. This means that for any value of m the system is characterized by the same set of critical exponents which are those of a weakly diluted random site quenched Ising model.

In the other FPs, we recover the familiar two-loop numerical results for the Gaussian (FPs I, VII), m-vector (FP II), polymer $O(n = 0)$ (FP III), Ising (FPs V, X), diluted m-vector (FP VI), and cubic (FP VIII) models. FP IX belongs to the new universality class. In table 3 we give the numerical values of the critical exponents in these FPs as well: if the flow from the initial values of couplings pass near these FPs, one may observe an effective critical behaviour governed by these critical exponents [23].

4. Conclusions

In this paper, we presented an analysis of an m-vector model with quenched disorder of a random anisotropy type as described by the Hamiltonian (2). It possesses randomness only for $m > 1$ and the randomness-induced behaviour in RAM may be observed only for spins of continuous symmetry. We were interested in a possibility of a ferromagnetic ordering of RAM for certain anisotropic distribution of a random anisotropy axis. In particular, we studied the case when the local anisotropy axis points along the edges of an m-dimensional hypercube.

We used the field theoretical RG approach, obtaining RG functions in the two-loop approximation and analysing them both by an ε-expansion as well as by resummation of the expansion for fixed space dimension $d = 3$. In the RG language, the critical point of a system corresponds to the accessible stable FP of the RG transformation. In our analysis, we get two stable FPs. One of them (FP III in figure 1) is not accessible for the flows from the region of initial values of couplings, but the other one FP XV may be reached from these values. Taken that the FP XV is of the random site Ising type, we conclude that RAM with cubic distribution of random anisotropy axis is governed by a set of critical exponents of a weakly diluted quenched Ising model [23]. There is a simple physical interpretation of the phenomena observed: since the m easy axes of RAM with cubic distribution are mutually orthogonal, a spin oriented along a given axis feels only the presence of near-neighbour spins constrained to lie upon the same axis. The system, therefore, decomposes into m independent diluted Ising models [20,21,26]. Note once more, that this behaviour is characteristic only of RAM with cubic distribution of random anisotropy axis, described by the effective Hamiltonian (5). A distribution of random anisotropy axis is relevant, i.e., for isotropic distribution, all investigations bring about an absence of a second order phase transition for $d \leqslant 4$ [4–10,12].

To conclude we want to attract attention to a certain similarity in the critical behaviour of both random-site [22] and random-anisotropy [3] quenched magnets: if at all there appears any *new critical behaviour* it always *is governed by critical exponents of site-diluted Ising type*. Thus, in a random-anisotropy system the situation may occur that the critical behaviour of a system of spins of *continuous symmetry* is the same as that of a random-site system with *discrete* (Ising) spins. The above calculations of a critical behaviour of RAM were based on a two-loop expansion improved by a resummation technique. Once the qualitative picture became clear there is no need to go into higher orders of a perturbation theory as far as the critical exponents of the site-diluted Ising model are known by now with high accuracy [23].

As a possible generalization of the RAM one may consider a case when quenched randomness is present in both random-site and random-anisotropy forms. Then, one arrives [4] to the effective Hamiltonian (5) where the coupling u_0 may be of either sign. We have checked the region $u < 0$ for the presence of new FPs and verified that they are absent. Therefore, again FP XV is the only one reachable stable FP and the observed critical behaviour is unique.

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References

1. see e.g. Stanley H. Introduction to Phase Transitions and Critical Phenomena. Oxford, Claredon Press, 1971.
2. Absence of the phase transition with spontaneous order parameter for the systems with continuous symmetry and a short range interaction at $d = 2$ follows from the Mermin-Wagner theorem: Mermin N.D., Wagner N. // Phys. Rev. Lett., 1966, vol. 17, p. 1133; exact solutions for 2d classical $O(m)$ model for $m = 4$ and $m = 3$ also demonstrate the absence of magnetic ordering: Polyakov A.M., Wiegman P.B. // Phys. Lett., 1983, vol. B131, p. 121; Wiegman P.W. // Pis'ma v JETP, 1985, vol. 41, p. 79.
3. Harris R., Plischke M., Zuckermann M.J. // Phys. Rev. Lett., 1973, vol. 31, p. 160.
4. Aharony A. // Phys. Rev. B, 1975, vol. 12, p. 1038.
5. Dudka M., Folk R., Holovatch Yu. // Condens. Matter Phys., 2001, vol. 4, p. 77.
6. Pelcovits R.A., Pytte E., Rudnick J. // Phys. Rev. Lett., 1978, vol. 40, p. 476.
7. Ma S.-k., Rudnick J. // Phys. Rev. Lett., 1978, vol. 40, p. 589.
8. Imry Y., Ma S.-k. // Phys. Rev. Lett., 1975, vol. 35, p. 1399.
9. Pelcovits R.A., Pytte E., Rudnick J. // Phys. Rev. Lett., 1982, vol. 48, p. 1297.
10. Pelcovits R.A. // Phys. Rev. B, 1979, vol. 19, p. 465;
11. Emery V.J. // Phys. Rev. B, 1975, vol. 11, p. 239.
12. Pytte E. // Phys. Rev. B, 1978, vol. 18, p. 5046.
13. Cochrane R.W., Harris R., Zuckermann M.J. // Phys. Rep., 1978, vol. 48, p. 1.
14. Amit D.J. Field Theory, the Renormalization Group, and Critical Phenomena. Singapore, World Scientific, 1984; Zinn-Justin J. Quantum Field Theory and Critical Phenomena. Oxford, Oxford University Press, 1989; Kleinert H., Schulte-Frohlinde V. Critical Properties of ϕ^4-Theories. Singapore, World Scientific, 2001.
15. Parisi G. 1973 (unpublished); J. Stat. Phys., 1980, vol. 23, p. 49.
16. We absorb the value of a one-loop integral into normalization of couplings and two-loop integrals.
17. Nickel B.G., Meiron D.I., Baker G.A. Jr. // Univ. of Guelph Report, 1977 (unpublished).
18. Wilson K., Fisher M.E. // Phys. Rev. Lett., 1972, vol. 28, p. 240.
19. Brezin E., Le Guillou J.C., Zinn-Justin J. // Phys. Rev. D, 1973, vol. 8, p. 434.
20. Mukamel D., Grinstein G. // Phys. Rev. B, 1982, vol. 25, p. 381.
21. Korzhenevskii A.L., Luzhkov A.A. // Sov. Phys. JETP, 1988, vol. 67, p. 1229.
22. Grinstein G., Luther A. // Phys. Rev. B, 1976, vol. 13, p. 1329.
23. See e.g. Folk R., Holovatch Yu., Yavors'kii T. // Phys. Rev. B, 2000, vol. 61, p. 15114 for a recent review on random $d = 3$ Ising model.
24. Baker G.A. Jr., Nickel B.G., Meiron D.I. // Phys. Rev. B, 1978, vol. 17, p. 1365.
25. Watson P.J.S. // J. Phys. A., 1974, vol. 7, p. L167.
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Критична поведінка магнетиків із випадковою анізотропією: кубічна анізотропія

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Критична поведінка m-векторної моделі з локальними осями анізотропії випадкової орієнтації досліджується для кубічного розподілу осей анізотропії за допомогою методу теоретико-польової ренормалізаційної групи. Вирази для ренормгрупових функцій обчислюються у двопетлевому наближенні і досліджуються як $\varepsilon = 4 - d$ розкладом, так і безпосередньо при вимірності простору $d = 3$ пересумовуванням Паде-Бореля. Отримується одна досяжна стійка фіксована точка, яка вказує на фазовий перехід другого роду з критичними показниками розведеної моделі Ізинґа.

Ключові слова: *випадкова анізотропія, ренормалізаційна група, критичні показники*

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