

Letters to Editor

Probing of quantum turbulence with the emitting vortex loops

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The statistics of vortex loops emitted from the domain with quantum turbulence is studied. The investigation is performed on the supposition that the vortex loops have the Brownian or random walking structure with the generalized Wiener distribution. The main goal is to relate the properties of the emitted vortex loops with the parameters of quantum turbulence. The motivation of this work is connected with recent studies, both numerical and experimental, on study of emitted vortex loops. This technique opens up new opportunities to probe superfluid turbulence. We demonstrated how the statistics of emitted loops is expressed in terms of the vortex tangle parameters and performed the comparison with numerical simulations.

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Scientific background and motivations

Quantum turbulence (QT) in superfluids is one of the most fascinating phenomena in the theory of quantum fluids [1]. In general, the vortex tangle composing QT, consists of a set of vortex loops of different lengths and having a random structure. Question of an arrangement of the vortex tangle is a key problem of the theory of QT. Recently, a number of works devoted to the study of the vortex tangle structure with the use of loops emitted from the turbulent domain have been performed [2–5]. This technique opens up new opportunities for research QT.

One of the main problem in this activity is to relate the properties of the emitted vortex loops with the parameters of QT. In the present work we propose an analytical approach that allows to relate the statistics of vortex loops with the parameters of the real vortex tangle. This approach is based on the Gaussian model of the vortex tangle, which describes the latter as a set of vortex loops having a random walking structure with the generalized Wiener distribution [6]. Then we apply our results for data processing of numerical work [2] who studied QT driven by an oscillating sphere. Generation of QT by oscillating objects is very important topic in this field (see, e.g., [7–9]).

Flux of vortex loops emitted from quantum turbulence

In this paragraph we very briefly describe main ideas leading to the theory of emission of vortex loops, details can be found in paper by the author [10]. Vortex loops composing the vortex tangle can move as a whole with a drift velocity V_l depending on their structure and their length l . The flux of the line length, energy, momentum etc., executed by the moving vortex loops takes place. In the case of inhomogeneous vortex tangle the net flux \mathbf{J} of the vortex length due to the gradient of concentration of the vortex line density $\mathcal{L}(x, t)$ appears. The situation here is exactly the same as in classical kinetic theory with the difference being that the “carriers” are not the point particles but the extended objects (vortex loops), which possess an infinite number of degrees of freedom with very involved dynamics.

To develop the theory of the transport processes fulfilled by vortex loops (in spirit of classical kinetic theory) we need to know the drift velocity V_l and the free path $\lambda(l)$ for the loop of size l . Referring to the paper [10] we write down here the following result. The drift velocity V_l and the $\lambda(l)$ for the loop of size l are

$$V_l = C_v \beta / \sqrt{l \xi_0}, \quad \lambda(l) = 1/2 l b_m \mathcal{L}. \quad (1)$$

Quantity β is $(\kappa/4\pi)\ln(\mathcal{L}^{-1/2}/a_0)$, where κ is the quantum of circulation, a_0 is the core radius, C_v is numerical factor of the order of unity, and b_m is the numerical factor, approximately equal to $b_m \approx 0.2$. The ξ_0 is the parameter of the generalized Wiener distribution, it is of order of the interline space $\mathcal{L}^{-1/2}$. The probability $P(x)$ for the loop of length l to fly the distance x without collision is $P(x) = (1/\lambda(l))\exp(-x/\lambda(l))$. Knowing the averaged velocity V_l of loops, and the $P(x)$ (both quantities are l -dependent), we can evaluate the spatial flux \mathbf{J} of the vortex loops.

Let us consider the small area element placed at some point of the boundary of domain containing QT and oriented perpendicularly to axis x (see for details Fig. 2 of paper [10]). The x -component of flux \mathbf{J} of the number of loops executed by loops of sizes l , placed in θ, φ direction, and remote from the area element at distance R , can be written as

$$J_x(l) = \frac{1}{4\pi} \int n(l, R, \theta, \varphi) (V_l \cos \theta) P(R) \sin \theta d\theta d\varphi dR. \quad (2)$$

Here the quantity $V_l \cos \theta$ is just the x component of the drift velocity, the factor $P(R)dR$ is introduced to control an attenuation of flux, due to collisions. In the spirit of classical kinetic theory, we assume the local equilibrium is established.

In paper [10] Eq. (2) was the starting point to develop the theory of diffusion of vortex loops in QT, therefore the density of loops $n(l)$ was supposed to depend on spacial position, in spherical coordinates $n = n(l, R, \theta, \varphi)$. Here we put another goal to study radiation of loops from the domain with uniform vortex tangle. Supposing that inside domain the density of loops does not depend on spacial coordinates ($n(l, R, \theta, \varphi) = n(l)$), and integrating out over solid angle $d\theta d\varphi$ and over position of loops dR we obtain the x -component of the loop flux through the domain boundary

$$J_x = \frac{1}{4} \int n(l) \left(\frac{\beta}{\sqrt{l\xi_0}} \right) dl. \quad (3)$$

The flux J_x , described by formula (3) carries the loops of different sizes in the normal to boundary direction and provides information on the distribution of loops inside of the turbulent domain. However, since the speed of loops depends on their sizes the initial distribution changes as the vortex “cloud” propagates. For instance, in some time the small vortex loops (practically rings) get ahead, and the measurements in a short time will detect only small loops. That means that the experimental data essentially depends on the position of detector and time of registration. In a stationary situation when the turbulence is maintained by some means (counterflow or oscillating structures), the intensity of detected loops should be steady in time.

Processing of experimental and numerical works

As an illustration let's treat the numerical work [2] on the statistics of emitted loops. Unfortunately, the pure experimental works are not too reliable for a proper quantitative treatment. In work [2] the authors have studied the statistics of vortex loops emitted from QT driven by an oscillating sphere of radius $1 \mu\text{m}$ with the frequency 3 kHz , and the amplitude $5.31 \mu\text{m}$. Counting of loops was made on the spherical boundary of radius $30 \mu\text{m}$ (centered at the center of the oscillating sphere). The result of this counting is illustrated in Fig. 1(a), where the a probability density function (PDF) $Pr_{\text{num}}(l)$ of the length of the emitted vortex loops is presented.

Let us consider this result from the position of the theory stated above. Formula (3) describes the flux of loops per unit area. The integrand in (3) is distribution of loops over their lengths in the propagating loop “jet”. The total number of emitted loops (per unit time) is $N(l) = \int \mathbf{J} d\mathbf{S}$. The Gaussian model, used in our consideration predicts that the density of loops $n(l)$ inside the domain with vortex tangle is the power-like function $n(l) = Al^{-5/2}$. This behavior, however, has a low cutoff near the length $l \sim \xi_0$. Below the cutoff there are a few loops of smaller sizes which do not essentially affect the whole theory. From the graph for PDF $Pr_{\text{num}}(l)$

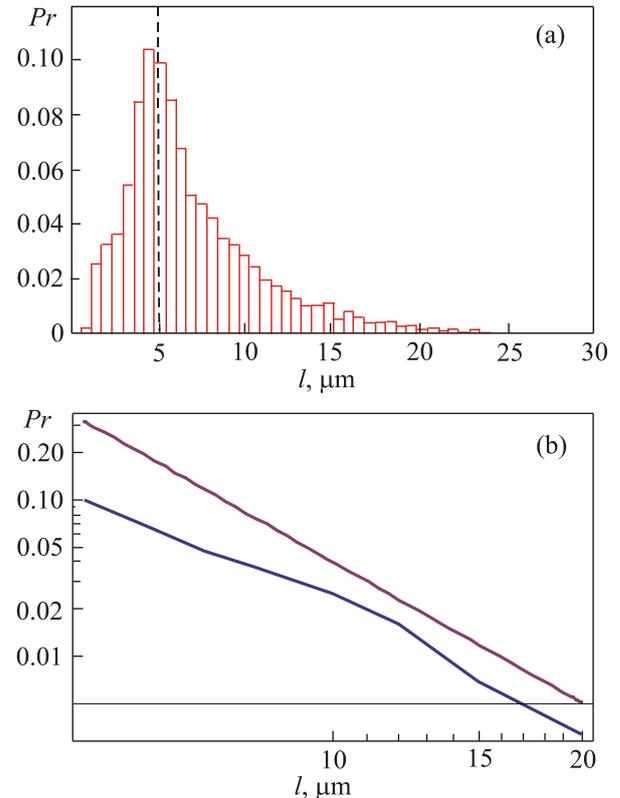


Fig. 1. (Color online) The PDF of the length of emitted vortex loops obtained in [2] (a). Lower line is the PDF from picture (a) depicted in logarithmic coordinates. The upper curve is the analytical PDF $Pr_{\text{an}}(l) \approx 50l^{-3}$, obtained in this work (b).

in Fig. 1 it is seen that the cutoff is $l \approx 5 \mu\text{m}$. The size $l \approx 5 \mu\text{m}$ is the point of the maximum of the PDF $Pr_{\text{num}}(l)$, where the power-like behavior ceases. Using (3) we get that the analytical PDF is $Pr_{\text{an}}(l) = N(l) / \int N(l) dl \propto l^{-3}$. Furthermore, taking into account that in the normalization factor $\int N(l) dl$ the value of integral is accumulated near the lower limit $l \approx 5 \mu\text{m}$, we get that the analytical PDF approximately is $Pr_{\text{an}}(l) \approx 50l^{-3}$.

In Fig 1(b) we depicted PDF $Pr_{\text{num}}(l)$ obtained in work [2] and our $Pr_{\text{an}}(l) \approx 50l^{-3}$ in the logarithmic coordinates. It can be seen that the power -3 is indeed close to the numerical data, deviations appear near the low cutoff where the Gaussian model does not work properly. As for absolute values of the PDFs (numerical and analytical) they differ by a factor about 2–3 (for large loops). This can be explained by the fact that the vortex turbulence, produced in numerical simulation is not the dense and uniform structure, which was required in the analytical consideration. In fact, the interline space obtained from the data on total length (see Fig. 2 of paper [2]) is about $20 \mu\text{m}$ which is comparable with size of largest loops. This implies that the QT is rather dilute.

Conclusion

Summarizing we can conclude that the “clouds” of vortex loops emitted from the turbulent superfluid helium provides an important information on the structure of QT. However, due to very complicated dynamics of the network of vortex loops, this information is hidden and should

be extracted with the use of appropriate formalism. It should be understood that the described above procedure is greatly simplified, many real features such as a possible anisotropy, the mutual friction, the specific conditions of the turbulence generation have not been considered. This is supposed to be performed in future.

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1. S.K. Nemirovskii, *Phys. Rep.* **85**, 524 (2013).
2. A. Nakatsuji, M. Tsubota, and H. Yano, *Phys. Rev. B* **89**, 174520 (2014).
3. Y. Nago, A. Nishijima, H. Kubo, T. Ogawa, K. Obara, H. Yano, O. Ishikawa, and T. Hata, *Phys. Rev. B* **87**, 024511 (2013).
4. L. Kondaurova and S.K. Nemirovskii, *Phys. Rev. B* **86**, 134506 (2012).
5. P.M. Walmsley, P.A. Tompsett, D.E. Zmeev, and A.I. Golov, *ArXiv e-prints* (2013).
6. S.K. Nemirovskii, *Phys. Rev. B* **77**, 214509 (2008).
7. V. Chagovets, I. Gritsenko, E. Rudavskii, G. Sheshin, and A. Zadorozhko, *J. Phys.: Conf. Series* **150**, 032014 (2009).
8. G.A. Sheshin, A.A. Zadorozhko, E.Ya. Rudavskii, V.K. Chagovets, L. Skrbek, and M. Blazhkova, *Fiz. Nizk. Temp.* **34**, 1111 (2008) [*Low Temp. Phys.* **34**, 875 (2008)].
9. I. Gritsenko and G. Sheshin, *J. Low Temp. Phys.* **175**, 91 (2014).
10. S.K. Nemirovskii, *Phys. Rev. B* **81**, 064512 (2010).