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Pc1-pulsations: the parallel structure in the magnetosphere plasma with the admixture of the heavy ions

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The paper deals with the parallel structure of ultra-low frequency waves. To obtain the function describing the wave field we use the quasi-perpendicular approximation ($k_\perp \gg k_{||}$). Different regions of the wave propagation are studied.

Key words: MHD waves and instabilities; planetary magnetospheres; waves in plasma

INTRODUCTION

The paper is devoted to the parallel structure of the geomagnetic pulsations. The geomagnetic pulsations are hydro-magnetic waves in the terrestrial magnetosphere. They are divided into nine types. The type depends on the character and the period of a pulsation. In the present paper we consider the Pc1 range of geomagnetic pulsations. These oscillations are called "the pearls" because their oscillogram looks like a pearl necklace. The wave frequency is supposed to be of the same order as the gyrofrequency of heavy ions. For a long time the common model describing the pearl forming was the bouncing wave packet model [1, 3]. According this model the Pc1-pulsation is an Alfvén wave which propagates along the magnetic field line and reflects from the conjugate ionospheres [1, 3]. But, some recent studies have thrown doubt on this interpretation [6].

In the present paper the parallel structure of Pc1 pearls at the different parts of the magnetic field line is considered and the possibility of excitation of the eigenmodes in the near-ionospheric transparent regions is studied.

COORDINATE SYSTEM

We use the axial-symmetric model of magnetosphere. We introduce the orthogonal field-aligned coordinate system $\{x^1, x^2, x^3\}$ in which x^1 and x^2 coordinates are the radial and the azimuthal coordinates, respectively, and x^3 is directed along the ambient magnetic field lines. The length element in this system is given by the expression:

$$dl^2 = g_1(dx^1)^2 + g_2(dx^2)^2 + g_3(dx^3)^2, \qquad (1)$$

where $g_i = g_{ii}(x^1, x^3)$ are diagonal elements of the metric tensor, $g = g_1 g_2 g_3$ is the determinant of the metric tensor. The summation over the repeated index is not implied here.

ULF-WAVES AT THE EQUATOR

We consider the ULF-waves (ultra-low frequency waves) in the space plasma. Plasma consists of electrons, protons and heavy ions. We should take into account the admixture of heavy ions, because the density of oxygen ions in the Earth's magnetosphere may be as much as the proton density [8]. The plasma is described by the permittivity tensor

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & -i\eta & 0 \\ i\eta & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_{||} \end{pmatrix},$$

whose elements are given by:

$$\varepsilon_{||} = -\infty, \quad \varepsilon_\perp = \frac{\Omega_{pp}^2}{\Omega_{cp}^2 - \omega^2} + \frac{\Omega_{ph}^2}{\Omega_{ch}^2 - \omega^2},$$

$$\eta = \frac{\Omega_{pe}^2}{\omega \Omega_{ce}} - \frac{\Omega_{cp}}{\omega}\frac{\Omega_{pp}^2}{\Omega_{cp}^2 - \omega^2} - \frac{\Omega_{ch}}{\omega}\frac{\Omega_{ph}^2}{\Omega_{ch}^2 - \omega^2},$$

where Ω_p and Ω_c denote the local plasma and the local cyclotron frequencies, respectively, and the second index designates corresponding particle: proton (p), heavy ion (h) and electron (e).

Taking into account the finitness of the ratio ω/ω_c one can get the dispersion relation of the cold plasma in a form [7]:

$$\left(\frac{\omega^2}{c^2}\varepsilon_\perp - k_{||}^2\right)\left(\frac{\omega^2}{c^2}\varepsilon_\perp - k_{||}^2 - k_\perp^2\right) = \frac{\omega^4}{c^4}\eta^2, \qquad (2)$$

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where ω is the frequency, c is the speed of light, $k_{||}$ and $k_{\perp}$ are the wave vector components parallel and transverse to the ambient magnetic field.

To study the parallel structure of the wave we use the quasi-perpendicular approximation ($k_{\perp} >> k_{||}$). Hence the relation (2) takes a form [4]:

$$\frac{\omega^2}{c^2}\varepsilon_{\perp} - k_{||}^2 = -\frac{\omega^4}{c^4}\frac{\eta^2}{k_{\perp}^2}. \qquad (3)$$

Thus, we obtain the parallel component of the wave vector [4, 5]:

$$k_{||}^2 = \frac{\omega^2}{A_p^2\left(1-\dfrac{\omega^2}{\Omega_{cp}^2}\right)} + \frac{\omega^2}{A_h^2\left(1-\dfrac{\omega^2}{\Omega_{ch}^2}\right)}, \qquad (4)$$

where A_p and A_h are the Alfvén velocities determined for protons and the ions, respectively. We suppose that $\Omega_{cp} \gg \Omega_{ch}$. There is a point on the magnetic field line where the wave frequency equals the heavy ion gyrofrequency and the parallel wave vector tends to infinity. At the equator the magnetic field is minimum, thus at the equator $k_{||}^2 > 0$, and approaching the parallel singular point along the field line from the equator, one gets the point where $k_{||}^2 \to \infty$. So, we conclude, that somewhere between the equator and the singular point there must be a point where $k_{||}^2 = 0$. This point is called a turn point. Thus, in the case of the quasi-perpendicular approximation we have a resonator at the equatorial part of the magnetic field line (Fig. 1). The resonator serves as a wave energy reservoir. Its eigenfrequencies determine the wave frequency. Due to (4) we can use a parabolic approximation for the $k_{||}^2(l)$ dependence. Using the parabolic approximation and the Bohr-Sommerfeld quantization condition we obtain the wave frequency [4, 5]:

$$\omega_n^2 = \left(1+\frac{\rho_h}{\rho_p}\right)\Omega_{ch}^2 + (2n+1)\frac{\rho_h}{\rho_p}\frac{A_h\Omega_{ch}}{r_{eq}}, \qquad (5)$$

here ρ_h and ρ_p are the densities of heavy ions and protons, respectively, and r_{eq} is the equatorial radius of the magnetic field line curvature.

There are two opaque regions ($k_{||}^2 < 0$) and two near-ionospheric transparent regions ($k_{||}^2 > 0$) (Fig. 1).

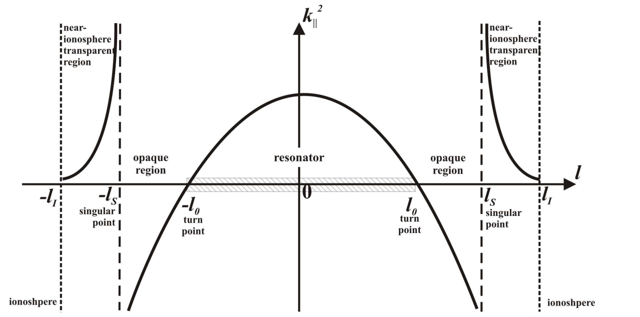

Fig. 1: The wave vector parallel component squared as a function of the longitudinal coordinate.

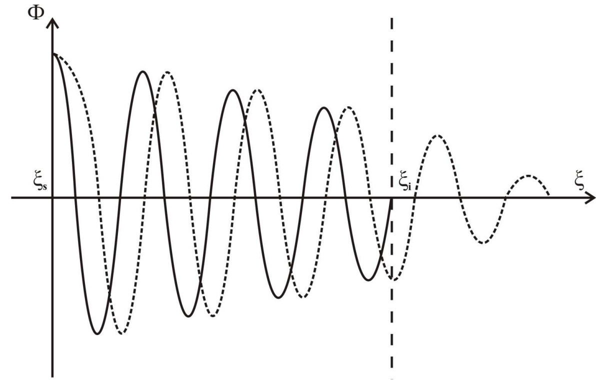

Fig. 2: The parallel structure of the ULF-wave when different boundary conditions take place. Solid line presents the case of the ideal conductive ionosphere, dotted line presents the case of the nonideal conductive ionosphere.

PARALLEL STRUCTURE OF ULF-WAVE IN OPAQUE REGION

Some part of the wave energy can tunnel through the opaque region to the near-ionospheric transparent region where a standing wave is formed. At the opaque region near the equatorial resonator there is one dumped oscillation mode, so we can conclude, that the penetration of the wave to the near-ionospheric region is possible. That is why the question about the existence of such mode near the near-ionospheric transparent region arises. In the opaque region the wave structure is given by

$$\Phi = \sqrt{\xi-\xi_s}\left[\left(a_2 e^{i\frac{3\pi}{2}} - a_1\right) I_1 \times \right.$$
$$\left. \times\left(2\sqrt{\alpha(\xi-\xi_s)}\right)\frac{2}{\pi}a_2 K_1\left(2\sqrt{\alpha(\xi-\xi_s)}\right)\right], \qquad (6)$$

where $\xi = \sqrt{\dfrac{g_1}{g_2}}l_{||}$, a_1 and a_2 are some coefficients describing the wave amplitude, ξ_s is the coordinate of the singular point in the new notation, α is a positive constant, I_1 and K_1 are the modified Bessel functions. By finding the relation between the coefficients a_1 and a_2, we obtain in the case of the ideal conductive ionosphere:

$$a_1 = -a_2 \left[\frac{1 - e^{i\frac{3\pi}{2}} e^{-i4\sqrt{\alpha}\sqrt{\xi_i - \xi_s}}}{1 + e^{i\frac{3\pi}{2}} e^{-i4\sqrt{\alpha}\sqrt{\xi_i - \xi_s}}} \right], \qquad (7)$$

where ξ_i is the coordinate of the ionosphere, and in the case of the nonideal conductive ionosphere we have:

$$a_1 = -a_2 \left[\frac{1 - e^{i\frac{3\pi}{2}} e^{-i4\sqrt{\alpha}\sqrt{\xi_i - \xi_s}}\Theta}{1 + e^{i\frac{3\pi}{2}} e^{-i4\sqrt{\alpha}\sqrt{\xi_i - \xi_s}}\Theta} \right], \qquad (8)$$

where

$$\Theta = \frac{1 - 4i\sqrt{\alpha}(\xi_i - \xi_s)^{1/4}}{1 + 4i\sqrt{\alpha}(\xi_i - \xi_s)^{1/4}}.$$

It is seen that one coefficient is expressed via another. So, neither the ideal conduction of the ionosphere (7) nor the nonideal conduction (8) allow the appearance of the only one wave mode, which can become the cause of the near-ionospheric eigenmodes. There are two modes always. One of them is dumped, another one increases.

PARALLEL STRUCTURE IN THE NEAR-IONOSPHERIC REGION

In the near-ionospheric transparent region the structure of the ULF-wave is obtained by using the WKB-approximation:

$$\Phi = \frac{A_1(\xi - \xi_s)}{\alpha^{1/4}} e^{i2\sqrt{\alpha(\xi-\xi_s)}} + \frac{A_2(\xi - \xi_s)}{\alpha^{1/4}} e^{-i2\sqrt{\alpha(\xi-\xi_s)}}, \qquad (9)$$

where A_1 and A_2 are the coefficients which may be obtained from boundary conditions.

But near the singular point this WKB-approximation is not available, so we have to obtain the wave structure as a sum of linear-independent Bessel functions. Then in the vicinity of singular point the ULF-wave structure takes the form:

$$\Phi = (\xi - \xi_s)^{1/2} \left[a_1 J_1(2\sqrt{\alpha(\xi - \xi_s)}) + a_2 Y_1(2\sqrt{\alpha(\xi - \xi_s)}) \right], \qquad (10)$$

here J_1 and Y_1 are Bessel functions.

In the very vicinity of the singularity the solution takes the form

$$\Phi = \sqrt{\alpha} \left[a_1(\xi - \xi_s) + \frac{a_2}{\pi\sqrt{\alpha}} \sqrt{\xi - \xi_s} \ln \alpha(\xi - \xi_s) \right]. \qquad (11)$$

It is seen that the wave amplitude near the singular point is finite. The relation between a_1 and a_2 also may be obtained from the boundary conditions. In the Fig. 2 the obtained parallel structure is shown.

SUMMARY AND DISCUSSION

If we use the quasi-parallel approximation ($k_{||} \gg k_\perp$) to consider the dispersion relation then the equation (2) has the form [2]

$$k_{||\pm}^2 = \frac{\omega^2}{c^2}(\varepsilon_\perp \pm \eta). \qquad (12)$$

In this case a transverse resonator occurs between the magnetic shells. The resonator also has two turn points which separate the resonator from the opaque regions. The conditions of the resonator occurrence differ from the conditions of the parallel resonator occurrence. So, we can see that the equatorial resonator occurs in both the quasi-parallel and the quasi-perpendicular approximations. It was obtained that in the near-ionospheric region the excitation of the eigenmodes is impossible, because there are two modes in the opaque region. So, the ULF-waves occur only in the resonator and propagate inside between the turn points. Since the resonator eigenfrequencies are very close to each other, simultaneous excitation of many harmonics can result in formation of beating, which resembles the characteristic structure of the Pc1 pearls. Some part of the wave energy leaks out from the resonator and penetrates to the atmosphere, where many ground-based satellites observe it.

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