Numerical modeling of the magnetosphere with data based internal magnetic field and arbitrary magnetopause

P. S. Dobreva, M. D. Kartalev

Institute of Mechanics, Bulgarian Academy of Sciences, Acad G. Bonchev Street block 4, Sofia 1113, Bulgaria
polya2006@yahoo.com

We present a new model of the magnetospheric magnetic field. Using the finite element method, Chapman-Ferraro problem is solved numerically in the considered approach. The whole magnetic field is a sum of: the dipole field, the field, produced by the internal current systems (cross-tail, Birkeland, ring currents) and the field induced by the magnetopause currents. In contrast to similar earlier models, the internal magnetospheric magnetic fields are taken from Tsyganenko data-based model. The magnetosphere boundary could be arbitrary (generally non-axisymmetric). Input model parameters are the solar wind parameters, the Dst index and the dipole tilt angle. We discuss some results, obtained in three dimensional solution of the Neumann-Dirichlet problem corresponding to a closed magnetosphere.

Introduction

When the solar wind interacts with and is deflected by the Earth's intrinsic magnetic field, the magnetosphere is formed. Since the discovery of the Earth's magnetosphere in 1958 by Explorer 1, considerable efforts of many researchers were devoted to its description. Many models, describing both the magnetospheric magnetic field and the magnetopause boundary itself, have been created during the last decades. The early models have two principal limitations: the shape of the magnetopause is given by a surface (a paraboloid, an ellipsoid, a semi-sphere and a cylinder), which is usually axisymmetric and does not depend on the parameters of the solar wind. The second main shortcoming in the magnetosphere modeling is the inability to include the effect of all main magnetospheric sources.

In one of the first magnetospheric models a vacuum magnetospheric magnetic field configuration is considered, containing only the dipole source at the origin and its corresponding shielding field [8]. The model is concentrated on a solution of the problem of calculating the shape of the magnetospheric boundary. An iterative algorithm is used, and the criteria for reaching convergency is achievement of the pressure balance. It is supposed that at any boundary point a local balance between the ram pressure of the solar wind and the magnetic pressure inside the cavity is maintained.

The contributions of all main current systems inside the magnetosphere, taken from Tsyganenko 96 [19] model, are included in the model of Sotirelis [13], where the pressure outside is estimated within the Newtonian approximation. Before that, in the Kartalev model [4] outside pressure was calculated solving gas dynamics equations in the magnetosheath. Magnetic field inside is presented by the model of Stern [14].

Many models are based on an alternative approach, where the magnetopause is not a result of pressure balance equation, but has a given predetermined shape. Numerical technics, solving the Chapman-Ferraro problem, were applied in the models of Voigt [21, 22], Stern [14], Toffoletto [6, 16]. The problem of finding a solution of the Laplace equation for the magnetic field potential is solved analytically in [14, 21, 22], and numerically, using the finite element method in [6]. The magnetic field potential is calculated mainly for the dipole field [21], the ring current is included in [14]. Numerical solution, shielding the internal magnetic field from Hilmer & Voigt [2] model is presented in [6]. A paraboloid magnetopause shape is used in [14], a hemisphere elongated by a cylinder in the tail in [21], and a fixed arbitrary form in [6].

Satellite missions provided an opportunity to gather a great amount of magnetic field measurements. During the last two decades, magnetospheric magnetic field models based on empirical measurements, such as those of Tsyganenko & Usmanov 1982 [17], Tsyganenko 1995 [19], Tsyganenko 2001 [20], are widely used. Latest versions of Tsyganenko models — Tsyganenko 1996 and Tsyganenko 2001, referred here as T96 and T01, include separate contributions from all magnetospheric sources — the ring, cross-tail and Birkeland currents, and a magnetopause, based on real measurements — the form of Sibeck 1991 [11] is used in T96, and that of Shue 1998 [10] — in T01.

We describe some numerical aspects of the presented magnetospheric magnetic field model. Next section presents the technique for solving the Chapman-Ferraro problem using an application of the finite element method. The capabilities of the numerical method in simulation of some typical features of magnetic field topology are presented in the last section.

Computational method

The section is devoted to a short description of a method of calculation the Chapman-Ferraro currents. This magnetospheric model is based on the development and improvement of earlier created — 2D, Kartalev 1995 [3] and simplified 3D, Koitchev 1999 [7] — models.

At each point inside the cavity of the magnetosphere, the net magnetic field $\mathbf{B}$ is presented as a sum:

$$\mathbf{B} = \mathbf{B}_d + \mathbf{B}_t + \mathbf{B}_r + \mathbf{B}_b + \mathbf{B}_s, \tag{1}$$

where $\mathbf{B}_d$ is the dipole field, and $\mathbf{B}_t$, $\mathbf{B}_r$, $\mathbf{B}_b$ are the fields of the cross-tail, ring and Birkeland currents correspondingly, $\mathbf{B}_s$ is the magnetopause currents field, that has to be determined in the process of solution. The sources of magnetic field, such as the ring, tail and Birkeland currents, are incorporated in our model through the empirical model of Tsyganenko. We use two of the latest variants of Tsyganenko models — T96 and T01. The Tsyganenko model is modified in order to satisfy the current physical problem formulation. A prescribed magnetopause shape, based on the Sibeck 1991 [11] ellipsoid is used in T96, and the Shue 98 [10] form in T01. Instead of using the above mentioned empirical forms, we fit the Tsyganenko model boundary to an arbitrary surface. That surface can be obtained numerically, from the numerical magnetosheath-magnetosphere model, developed in Geospace hydrodynamics laboratory at the Institute of Mechanics, part of which is the magnetospheric model, described here. In that case the magnetopause surface is determined by requiring the pressure balance relation to be fulfiled. The magnetopause can also be initialized as having a data-based shape (see examples in [9, 10, 11]).

As the magnetopause field is a function of its shape, it has to be calculated in correspondence to the given magnetopause surface. Hence the field induced by the magnetopause currents in Tsyganenko model is replaced by numerically calculated one.

It is supposed, that the unknown field $\mathbf{B}_s$ is divergence and curl free:

$$\operatorname{div} \mathbf{B}_s = 0, \qquad \operatorname{rot} \mathbf{B}_s = 0, \tag{2}$$

thus the field potential U, a harmonic scalar function, exists. The potential is found out from the Laplace equation:

$$\triangle U = 0, \tag{3}$$

with Neumann boundary condition:

$$(\mathbf{B}_s, \mathbf{n}) = \partial U / \partial \mathbf{n} = -\left[(\mathbf{B}_d, \mathbf{n}) + (\mathbf{B}_t, \mathbf{n}) + (\mathbf{B}_r, \mathbf{n}) + (\mathbf{B}_b, \mathbf{n}) \right], \tag{4}$$

applied at the boundary points, where $\mathbf{n}$ is the vector normal to the surface. The restriction of zero normal magnetic field component at the boundary is implied in the present case, corresponding to a closed magnetosphere configuration. When the field potential is already calculated from (3) and (4), the magnetic field $\mathbf{B}_s$ is determined as its gradient:

$$\mathbf{B}_s = \nabla U. \tag{5}$$

Beside physics, another aspect of prime importance is the computational one. Development of adequate magnetosphere models requires not only understanding the physical processes in the magnetosphere, but also finding an appropriate method for solving such problems. The reliability of the method is indicated by the accuracy of solution, the flexibility in the description of three-dimensional regions with a complicated geometry, and last but not least, by the required calculation time.

Numerical procedure, based on the finite element method (see [1] and [15]) lies in solving equations (3) and (4). The modeled region of the magnetosphere is divided into 3D elements, which are accepted to be twenty-node isoparametric serendipity hexahedrons. The elements satisfy the consistency conditions [15]. This type of elements is accepted for at least two reasons: the type is desirable for achieving the necessary smoothness of the approximated functions and their derivatives. It also allows a precise approximation of 3D curvilinear boundaries, such as the magnetosphere shape.

We replace the continuum formulation by a discrete representation, thus converting the equation (3) and the boundary condition (4) into a linear algebraic system with a semi-definite matrix. Alternating subspace iteration method [5] is used for the numerical solution of the system. More details about the method and the discretization algorithm can be found in the earlier papers (see [3, 7]). The method of solving the algebraic system [5] allows the computer code to be run on a personal computer with mean capabilities in a reasonable time frame.

Some results

Some features of the model described in the previous section are considered. Input parameters for the model are: dynamic pressure of the solar wind (Dp), two components, $\mathbf{B}_y$ and $\mathbf{B}_z$, of the interplanetary magnetic field, Dst index and the angle of the dipole inclination (tilt angle).

The distribution of the total magnetic field in the main meridional (noon-midnight) plane is presented in Fig. 1. The magnetic field inside is presented by all main sources: the Earth' internal field, the field of cross-tail, ring, Birkeland currents, and the field of the magnetopause currents. The magnetopause field is calculated numerically in our model, and the field from the other magnetospheric sources (the cross-tail, ring and Birkeland currents) is given by the model of Tsyganenko — T01. The magnetospheric boundary is described by the empirical shape of Shue 98 [10] (for Dp = 2 nPa and $\mathbf{B}_z = 0$ nT). The input parameters are mentioned in the figure.

The magnetic field distribution calculated in our model can be compared with the results, obtained in similar way and published earlier in the literature. The distribution of total magnetic field, calculated in the main meridional plane is presented in Fig. 5 in [12]. Figure 5 in [12] illustrates similar behavior of the magnetic field, although it is obtained under different conditions — the magnetic field inside is given by a version of Tsyganenko 87 [18] model.

In this study we demonstrated, that the method, applied in calculation the Chapman-Ferraro currents for a given magnetopause form and a data-based related internal field, gives reliable magnetic field distribution in the magnetosphere.

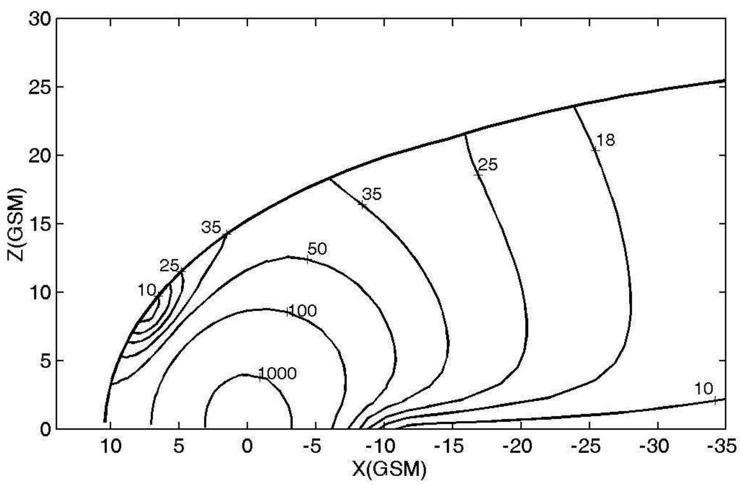

Figure 1: Magnitude of total magnetic filed $|\mathbf{B}|$ (in nT) in the main meridional XZ (GSM) plane. The field of the magnetopause currents is numerically calculated by our model. The empirical form of Shue 98 is used as a boundary of the magnetosphere. Input parameters: Dp = 2 nPa, $\mathbf{B}_y = \mathbf{B}_z = 2$ nT, Dst = -10, tilt= $0°$.

Acknowledgement

We would like to thank the Bulgarian "Evrika" Foundation for financial support. This work is also supported by the European Social Fund and Bulgarian Ministry of Education, Youth and Science under Operative Program "Human Resources Development", Grant BG051PO001-3.3.04/40.

References

[1] Bathe K. J., Wilson E. L. 'Numerical Methods in Finite Element Analysis', Prentice-Hall, Englewood Cliffs, NJ, (1977)

[2] Hilmer R. V., Voigt G.-H. J. Geophys. Res., V. 100(A4), pp. 5613-5626 (1995)

[3] Kartalev M. D., Kaschiev M. S., Koitchev D. K. J. Comp. Phys., V. 119, pp. 220-230 (1995)

[4] Kartalev M. D., Nikolova V. I., Kamenetsky V. E., Mastikov I. P. Planet. Space Sci., V. 44, pp. 1195-1208 (1996)

[5] Kaschiev M.S. 'Computational Processes', Moskow, Science, V. B, pp. 161-170 (1988)

[6] Kloucek P., Toffoletto F. R. J. Comp. Phys., V. 150, pp. 549-560 (1999)

[7] Koitchev D. K., Kaschiev M. S., Kartalev M. D. 'Finite Difference Methods: Theory and Applications', Nova Science Publishers, Commack, NY, U.S.A., pp. 127-138, (1999)

[8] Mead G. D., Beard D. B. J. Geophys. Res., V. 69, pp. 1169-1179 (1964)

[9] Petrinec S. M., Russell C. T. J. Geophys. Res., V. 101(A1), pp. 137-152 (1996)

[10] Shue J.-H., Song P., Russell C. T. et al. J. Geophys. Res., V. 103(A8), pp. 17691-17700 (1998)

[11] Sibeck D. G., Lopez R. E., Roelof E. C. J. Geophys. Res., V. 96(A4), pp. 5489-5495 (1991)

[12] Sotirelis T. J. Geophys. Res., V. 101, pp. 15255-15264 (1996)

[13] Sotirelis T., Meng C. I. J. Geophys. Res., V. 104, pp. 6889-6898 (1999)

[14] Stern D. P. J. Geophys. Res., V. 90, pp. 10851-10863 (1985)

[15] Strang G., Fix G. 'An Analysis of the Finite Element Method', Prentice-Hall, Englewood Cliffs, NJ, (1977)

[16] Toffoletto F. R. Geophys. Res. Lett., V. 90, pp. 621-624 (1994)

[17] Tsyganenko N. A., Usmanov A. V. Planet. Space Sci., V. 30, pp. 985-998 (1982)

[18] Tsyganenko N. A. Planet. Space Sci., V. 35, pp. 1347-1358 (1987)

[19] Tsyganenko N. A. J. Geophys. Res., V. 100(A4), pp. 5599-5612 (1995)

[20] Tsyganenko N. A. J. Geophys. Res., V. 107(A8), pp. 1179-1195 (2002)

[21] Voigt G.-H. Z. Geophys., V. 38, pp. 319-346 (1972)

[22] Voigt G.-H. Planet. Space Sci., V. 29, pp. 1-20 (1981)