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Instability of 2DEG interacting with drifting 3DEG

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Abstract. Dispersion law is studied for high-frequency longitudinal plasma waves in 2DEG (the plane z = 0), separated by thin dielectric layer from half-limited 3DEG. It is shown that drift of 3DEG provokes for special conditions instability of considered plasma waves.

Keywords: instability, electron gas, plasma.

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1. Introduction

The problem of generating the high-frequency electromagnetic waves attracts attention of a great number of investigators. Now various low-dimensional electronic systems are presented as candidates for designing the suitable generators. There is a lot of distinguished variants; all of them have their own advantages and shortages. We propose here to investigate one variant of well-known beam instability in plasma (see for instance [1]), where an activating beam is not actually a beam but plays the significant role of source of energy transferred to plasma waves. Controlling this source gives a possibility to manage the frequency increment of unstable waves.

2. Dielectric susceptibility

Investigated in this work is the dielectric susceptibility for the system of charged carriers, which includes two-dimensional electron gas located in the plane $z = 0$ and separated by an insulating layer from conducting substance occupying the half-space $z \geq l$ (see Figure). The system like that was considered in [2] and [3], similar systems – in papers [4-6].

We assume here that the dispersion laws for two-dimensional and three-dimensional band carriers are isotropic and parabolic:

$$\varepsilon(\vec{k}_\perp) = \hbar^2 k_\perp^2 / 2m_2 \ ; \ \ \varepsilon(\vec{k}) = \hbar^2 k^2 / 2m_3 . \quad (1)$$

The dispersion equation for longitudinal high-frequency (collisionless) plasmons has the form

$$\varepsilon_S(\omega, \vec{q}_\perp) + \Delta\varepsilon(\omega, \vec{q}_\perp) = 0 . \quad (2)$$

Here (see [2])

$$\varepsilon_S(\omega, \vec{q}_\perp) = \frac{1}{2}\left[\varepsilon_1 + \varepsilon_2 \frac{\beta(\omega, \vec{q}_\perp)\tanh(q_\perp l) + 1}{\beta(\omega, \vec{q}_\perp) + \tanh(q_\perp l)}\right]; \quad (3)$$

$$\beta(\omega, \vec{q}_\perp) = \frac{\varepsilon_2 q_\perp}{\pi} \int_{-\infty}^{\infty} \frac{dq_z}{q^2[\varepsilon_3 + \Delta\varepsilon_3(\omega, \vec{q})]} ; \quad (4)$$

$$\Delta\varepsilon(\omega, \vec{q}_\perp) = = \frac{e^2}{\pi \hbar q_\perp} \int d^2\vec{k}_\perp \frac{f_2(\vec{k}_\perp) - f_2(\vec{k}_\perp - \vec{q}_\perp)}{\omega + \hbar(q_\perp^2 - 2\vec{k}_\perp \vec{q}_\perp)/2m_2 + i0} ; \quad (5)$$

$$\Delta\varepsilon_3(\omega, \vec{q}) = = \frac{e^2}{\pi^2 \hbar q^2} \int d^3\vec{k} \ \frac{f_3(\vec{k}) - f_3(\vec{k} - \vec{q})}{\omega + \hbar(q^2 - 2\vec{k}\vec{q})/2m_3 + i0} ; \quad (6)$$

f_2 and f_3 are distribution functions for 2DEG and 3DEG.

At $\varepsilon_1 = \varepsilon_2$ and $\beta(\omega, \vec{q}_\perp) \to 0$ (the latter corresponds to application of a metallic field electrode) $\varepsilon_S(\omega, \vec{q}_\perp) = \varepsilon_1 / [1 - \exp(-2q_\perp l)]$.

At $q_\perp l \to \infty$, that is in the case of the infinitely separated three-dimensional electrode, it follows from (3):
$\varepsilon_S(\omega, \vec{q}_\perp) = (\varepsilon_1 + \varepsilon_2)/2$.

Limit our consideration here, for shortness, by non-degenerated 2DEG and 3DEG. In this case, the equilibrium distribution function for 2DEG has the form

$$f_2^0(\vec{k}_\perp) = (2\pi n_2 / k_{3T}^2) \exp(-k^2 / k_{2T}^2) ;$$
$$k_{2T} = (2m_2 k_B T)^{1/2} / \hbar = m_2 v_{2T} / \hbar . \quad (7)$$

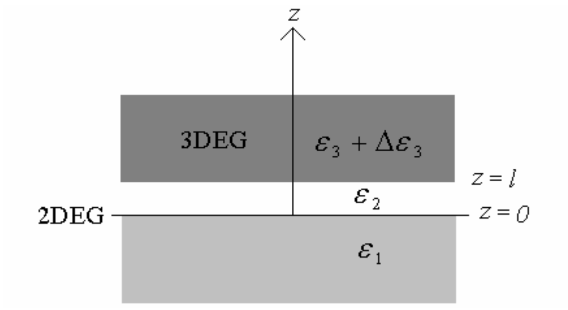

Fig. 1.

In the case of drifting 3DEG, we use the hydrodynamic approximation for the non-equilibrium distribution function:

$$f_3(\vec{k}) = f_3{}^0(\vec{k} - m_3\vec{u}_\perp/\hbar). \quad (8)$$

Here $\vec{u}_\perp$ is drift velocity in $\{x, y\}$ -plane. Then

$$f_3(\vec{k}) = (2\pi^{3/2} n_3 / k_{3T}) \exp[-(\vec{k} - m_3\vec{u}_\perp/\hbar)^2 / k_{3T}^2];$$

$$k_{3T} = (2 m_3 k_B T)^{1/2}/\hbar = m_3 v_{3T}/\hbar. \quad (9)$$

Substituting the expressions (7) and (9) in Eqs. (5) and (6) and integrating the obtained expression over the wave vectors $\vec{k}_\perp$ and $\vec{k}$, we obtain dielectric functions in the form:

$$\Delta\varepsilon_2^0(\omega, \vec{q}_\perp) = \frac{2\pi e^2 n_2}{k_B T q_\perp} \times \left[\frac{1}{\lambda - \gamma_2{}^2} F\left(\frac{\lambda - \gamma_2{}^2}{2\gamma_2}\right) - \frac{1}{\lambda + \gamma^2} F\left(\frac{\lambda + \gamma_2{}^2}{2\gamma_2}\right)\right]; \quad (10)$$

$$\Delta\varepsilon_3(\omega, \vec{q}) = \frac{4\pi e^2 n_3}{k_B T q^2}\left[\frac{1}{\lambda_u - \gamma_3{}^2} \times F\left(\frac{\lambda_u - \gamma_3{}^2}{2\gamma_3}\right) - \frac{1}{\lambda_u + \gamma_3{}^2} F\left(\frac{\lambda_u + \gamma_3{}^2}{2\gamma_3}\right)\right]. \quad (11)$$

Here we have used the following designations (see [1] and [2]):

$\lambda = \hbar\omega / k_B T$; $\lambda_u = \hbar(\omega - \vec{q}\vec{u}_\perp)/k_B T$;

$\gamma_3 = q / k_{3T}$; $\gamma_2 = q_\perp / k_{2T}$;

$$F(s) = \frac{s}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-t^2)}{t - s - i0} dt. \quad (12)$$

Now turn to the classical limit. Adopting the inequality $1 >> \lambda >> \gamma_{2,3}^2$, that is

$$k_B T >> \hbar\omega >> \hbar^2 q^2 / 2m_3 \text{ and}$$
$$k_B T >> \hbar\omega >> \hbar^2 q_\perp^2 / 2m_2, \quad (13)$$

and considering the case of high frequencies, where

$$\omega / q_\perp v_{2T} >> 1; \quad \omega / q v_{3T} >> 1, \quad (14)$$

one obtains from (10) and (11):

$$\Delta\varepsilon(\omega, \vec{q}_\perp) = -\frac{2\pi e^2 n_2 q_\perp}{m_2 \omega^2} = -\frac{\Omega_2^2(q_\perp)}{\omega^2}; \quad (15)$$

$$\Delta\varepsilon_3(\omega, \vec{q}) = -\frac{4\pi e^2 n_3}{m_3(\omega - \vec{q}\vec{u}_\perp)^2} = -\varepsilon_3 \frac{\Omega_3^2}{(\omega - \vec{q}\vec{u}_\perp)^2}. \quad (16)$$

Here

$$\Omega_2^2(q_\perp) = \frac{2\pi e^2 n_2 q_\perp}{m_2}; \quad \Omega_3^2 = \frac{4\pi e^2 n_3}{\varepsilon_3 m_3}. \quad (17)$$

Substituting (16) into (4), we have

$$\beta(\omega, \vec{q}_\perp) = \varepsilon_2(\omega - \vec{q}\vec{u}_\perp)^2 / \varepsilon_3[(\omega - \vec{q}\vec{u}_\perp)^2 - \Omega_3^2]. \quad (18)$$

3. Instability

Possible instability of 2DEG for the system shown in the figure was investigated in [3]. In this paper, the role of a field electrode was limited by creation of a node for EM field on the boundary of the electrode. In this case, the dispersion law for two-dimensional plasma waves does not depend on the state of 3DEG. Now, we consider quite another instability. It appears for a limited conductivity of drifting 3DEG.

Introducing the expressions (3), (15) and (18) into (2), we find the dispersion law for two-dimensional plasma waves:

$$(\omega - \vec{q}_\perp\vec{u}_\perp)^2(\omega^2 - Q_2^2) - \omega^2 Q_3^2 + Q_1^2 Q_2^2 = 0 \quad (19)$$

Here

$$\alpha = \tanh(q_\perp l); \quad Q_1^2 = \alpha\varepsilon_3 \Omega_3^2 / (\alpha\varepsilon_3 + \varepsilon_2);$$
$$Q_2^2 = 2\Omega_2^2(\alpha\varepsilon_3 + \varepsilon_2) / [\varepsilon_1(\alpha\varepsilon_3 + \varepsilon_2) + \varepsilon_2(\alpha\varepsilon_2 + \varepsilon_3)];$$
$$Q_3^2 = \Omega_3^2 \varepsilon_3(\alpha\varepsilon_1 + \varepsilon_2) / [\varepsilon_1(\alpha\varepsilon_3 + \varepsilon_2) + \varepsilon_2(\alpha\varepsilon_2 + \varepsilon_3)]. \quad (20)$$

Consider the following limit: $q_\perp l \to \infty$. Then one obtains a usual law for two-dimensional plasma waves:

$$\omega^2 = Q_2^2(\alpha = 1) = 2\Omega_2^2 / (\varepsilon_1 + \varepsilon_2). \quad (21)$$

More interesting is the opposite limit: $q_\perp l \to 0$. Then, as a consequence of the expressions (19) and (20) we have the dispersion law of this form:

$$(\omega - \vec{q}_\perp\vec{u}_\perp)^2[\omega^2 - 2\Omega_2^2/(\varepsilon_1 + \varepsilon_3)] - \omega^2\Omega_3^2\varepsilon_3/(\varepsilon_1 + \varepsilon_3) = 0. \quad (22)$$

One can see that for frequencies

$$\omega^2 < 2\Omega_2^2 / (\varepsilon_1 + \varepsilon_3) \quad (23)$$

a real solution of the Eq. (22) does not exist.

In the case

$$\omega^2 << 2\Omega_2^2/(\varepsilon_1+\varepsilon_3), \quad (24)$$

the equation (22) takes the simple form:

$$2(\omega-\vec{q}_\perp\vec{u}_\perp)^2\,\Omega_2^2+\omega^2\,\Omega_3^2\,\varepsilon_3=0. \quad (25)$$

Solving this equation, we find

$$\omega=\vec{q}_\perp\vec{u}_\perp\frac{1\pm i\Omega_3\sqrt{\varepsilon_3}/\Omega_2\sqrt{2}}{1+\varepsilon_3\Omega_3^2/2\Omega_2^2}. \quad (26)$$

Here we see an evident instable branch (Im $\omega > 0$). The form (26) is typical for the so-called beam instability. Note that for the considered case metallic three-dimensional electrode ($\Omega_3 \rightarrow \infty$) used in [3] is not suitable totally.

4. Discussion and conclusion

The condition of compatibility of the solution (26) and accepted earlier conditions (13), (14) and (24) has the form of the following set of inequalities:

$$q_\perp v_{2T} << q_\perp u[1+\Omega_3\sqrt{\varepsilon_3}/\Omega_2(q_\perp)\sqrt{2}]^{-1} <<$$
$$<< \Omega_2(q_\perp)\sqrt{2/(\varepsilon_1+\varepsilon_3)};$$
$$v_{3T} << u[1+\Omega_2(q_\perp)\sqrt{2}/\Omega_3\sqrt{\varepsilon_3}]^{-1}. \quad (27)$$

It is seen from here that the drift velocity of 3DEG has to exceed significantly the thermal velocities of 2DEG and 3DEG.

It follows from the expression (26) that for the given wave vector $q_\perp$ increment of instability attends a maximal value at the condition $\Omega_2(q_\perp)\sqrt{2}=\Omega_3\sqrt{\varepsilon_3}$, or

$$q_\perp = n_3 m_2/n_2 m_3 \quad (28)$$

Frequency of waves for the given value of $\vec{q}_\perp\vec{u}_\perp$ increases, if the ratio $\varepsilon_3\Omega_3^2/2\Omega_2^2$ goes down.

Consider now another area of frequencies:

$$\omega << \vec{q}_\perp\vec{u}_\perp. \quad (29)$$

In this case, it follows from (22):

$$\omega^2=\frac{2\Omega_2^2(\vec{q}_\perp\vec{u}_\perp)^2}{(\vec{q}_\perp\vec{u}_\perp)^2(\varepsilon_1+\varepsilon_3)-\varepsilon_3\Omega_3^2}. \quad (30)$$

Instability of two-dimensional plasma waves appears at the condition

$$(\vec{q}_\perp\vec{u}_\perp)^2(\varepsilon_1+\varepsilon_3)-\varepsilon_3\Omega_3^2<0. \quad (31)$$

The inequality (29) can be represented in the form

$$(\vec{q}_\perp\vec{u}_\perp)^2(\varepsilon_1+\varepsilon_3) << \varepsilon_3\Omega_3^2-2\Omega_2^2<0. \quad (32)$$

The latter inequality limits from the top both the drift velocity and the wave vector $q_\perp$. Simultaneously, the condition $\omega/q_\perp v_{3T} >> 1$ (see (14)) has to be satisfied.

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