Condensed Matter Physics, 1998, Vol. 1, No 1(13), p. 89–111

CONDENSED MATTER PHYSICS

Superconductivity in systems with local attractive interactions

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Received May 15, 1998

We discuss approaches based on the concepts of local electron pairing and the superconducting properties which they imply. The nature of the intermediate coupling regime is addressed and a recent progress in the BCS–Bose superconductivity crossover problem is outlined. We also survey the properties of systems with local attractive interactions consisting of a mixture of local electron pairs and itinerant fermions coupled via a charge exchange mechanism which mutually induces superconductivity in both subsystems. Finally, we briefly discuss the question of a pseudogap and a possible scenario of crossovers in high temperature superconductors.

Key words: *superconductivity, the Hubbard model, local attractive interaction, crossover*

PACS: *74.20.-z, 74.20.Mn, 71.10.-w, 71.10.Fd, 71.28.+d*

1. Introduction

Detailed muon spin rotation (μSR) studies by Uemura and his co-workers indicate that high temperature superconductors (HTS) (cuprates, doped barium bismuthates and fullerenes) and also other nonconventional superconductors, such as Chevrel phases and organic superconductors, form a unique group of superconductors characterized by high transition temperatures relative to the values of n_s/m^* (n_s is the superfluid carrier density and m^* the effective mass) [1,2]. These materials have their T_c proportional to $T_{\rm F}$ (the Fermi temperature) or $T_{\rm B}$ (the Bose-Einstein condensation temperature), with $T_{\rm B} \approx (3-30)T_c$ and $T_{\rm F} \approx (10-100)T_c$. They generally exhibit a low carrier density, a small value of the Fermi energy ($\propto 0.1 \div 0.3$ eV), a short coherence length ξ_0 and are *extreme type II superconductors* with the Ginzburg-Landau parameter $\kappa \gg 1$ [1–3]. The HTS and other nonconventional superconductors are placed in the intermediate crossover region between the BCS physics and the preformed electron pair scenario (table 1). For most HTS; $\xi_0 k_{\rm F} \sim 5-10$, whereas $\xi_0 k_{\rm F} \gg 1$ is needed for the BCS microscopic

theory to hold. The short coherence length is the average diameter of the pair in the condensate and limits the range of interaction, k_{F}^{-1} is the average distance between the carriers. Another quantity of interest is the average distance between the pairs d_{p}, which can be estimated as $1/d_{\mathrm{p}}^3 \sim N(0)\Delta$, where $N(0)$ is the density of states at E_{F} per spin and Δ is the superconducting energy gap. In HTS $\xi_0/d_{\mathrm{p}} \sim 1$, in contrast to conventional superconductors, where $\xi_0/d_{\mathrm{p}} \gg 1$[4].

Table 1. Schematic evolution from BCS to Pair Bose condensation. $d_{\mathrm{c}} \sim 1/k_{\mathrm{F}}$ – the average distance between carriers, d_{p} – the average distance between pairs.

$\frac{2\Delta}{k_{\mathrm{B}}T_c}$	$\xi_0 k_{\mathrm{F}}$	d_{p}/ξ_0			
~ 3.5	10^3	10^{-2}	BCS (Mean Field Theories)	(Al) ($\sim 10^3 \div 10^4$)	
$\geqslant 3.5$	10^2	10^{-1}	Eliashberg - Migdal	(Nb_3Ge)	
	10				$\Uparrow$ Theories of local short–range attraction
		1	(Charged bosons+fermions ?)	HIGH-T_c CUPRATES +others	
	1		$\Downarrow$		
$\gg 3.5$	10^{-1}	10	Bose Condensation (Blatt-Butler-Schafroth)		$\Downarrow$
	10^{-2}				

The above facts point out that the interactions responsible for pairing in HTS are *short-ranged*. The relation $\mathbf{T_c} \sim \frac{\mathbf{n_s}}{\mathbf{m^*}}$ (for small doping), discovered by Uemura et al. [1] is universal for cuprates and this suggests that pairing is *essentially nonretarded*.

Moreover, for many HTS, regardless of a specific microscopic mechanism leading to pairs, there are several universal trends in the scaled T_c versus the hole density, T_c versus the condensate density dependence, and T_c dependences of the pressure and the isotope effect coefficient [5]. These universal trends are consistent with *short-range, almost unretarded effective interactions responsible for pairing*, and, moreover, they suggest that there could be a *common condensation mechanism and thermodynamic description of short-coherence length superconductors* [7,4,6].

The above points also give some important hints for a phenomenological approach. Firstly, these systems have a small superfluid density in the underdoped regime as superconductivity comes through the doping of the insulating parent compound. Secondly, since $\xi_0/d_{\mathrm{p}} \sim 1$ for HTS, the phase fluctuations are important and determine T_c and can have a profound influence on the normal state properties [5,4,8]. Thirdly, the condition $\xi_0/d_{\mathrm{p}} \leqslant 1$ gives the region of applicability of the description in terms of real space XY type models as a lattice of Cooper pairs on effective sites $\mathbf{R}_i$ of extension ξ_0.

All the empirical constraints and the universal trends and observations dis-

cussed above put strong limitations on the microscopic theory of HTS and support the models with short-range, almost unretarded pairing interactions which will be discussed next.

2. Models of local electron pairing

The theoretical models of local pairing either start with a microscopic derivation of a local attractive interaction or simply postulate some effective Hamiltonian [6,7,4]. The simplest generic model, which can be thought of as a useful parametrization of the problem, is the extended Hubbard model (EHM) with an intra-site or inter-site attractive interaction:

$$H = \sum_{ij\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} + \frac{1}{2} \sum_{ij} W_{ij} n_i n_j - \sum_{i} (\mu - E_i) n_i, \qquad (1)$$

$$n = N_e/N = \frac{1}{N} \sum_{i} \langle n_i \rangle, \qquad (2)$$

$n_i = n_{i\uparrow} + n_{i\downarrow}, n_{i\sigma} = c^{\dagger}_{i\sigma} c_{i\sigma}$, t_{ij} denotes a transfer integral, U is an on-site and W_{ij} is an inter-site interaction between tight-binding electrons. μ is a chemical potential and E_i is a (random) site energy. Model (1) can be considered as rather general, resulting from a system of narrow band electrons strongly coupled to a bosonic field. The bosonic modes can be phonons, excitons, acoustic plasmons etc. The parameters of (1) t_{ij}, U, W_{ij} are effective ones (renormalized from their bare values). The typical microscopic mechanism leading to an effective short–range attraction is a strong electron–lattice coupling which can give rise to the formation of polarons. For this polaronic mechanism the local anharmonic modes can play an important role [9,10]. Also, the models introducing purely electronic ("chemical") mechanisms of a local attraction can be appropriately described by such an effective Hamiltonian.

Two cases have been extensively studied:

(i) $U_{\text{eff}} < 0, W_{\text{eff}} > 0$, when the induced local attraction outweights the on-site repulsion. This is the case of an *intra-site attraction* (or a negative U extended Hubbard model) and the problem of the formation of on-site electronic pairs in the strong $U < 0$ limit. The negative U Hubbard model is the simplest lattice model of a superconductor with a short coherence length displaying a crossover between the BCS-like superconductivity and the pair Bose condensation. It has been considered as an effective model of superconductivity and charge orderings in the family of barium bismuthates ($Ba_{1-x}K_xBiO_3$, $BaPb_xBi_{1-x}O_3$), fullerides, the Chevrel phases, as well as cuprates.

(ii) $W_{\text{eff}} < 0$ but $U_{\text{eff}} > 0$, i.e. the case of an *inter-site attraction* when the induced attraction is strong enough to dominate over the inter-site Coulomb repulsion. This is a model for systems with inter-site pairing of various pairing symmetries having the most direct relevance to the family of cuprate HTS, and to heavy fermion superconductors, where it can describe effective pairing of fermionic quasiparticles.

We should point out that, in certain cases, it is possible to introduce effective sites and the inter-site attraction can still be mapped onto a negative U extended Hubbard model. This is, for example, the case of Alexandrov and Ranninger model of bipolaronic superconductivity [11] which begins with an inter-site attraction but is mapped onto a $U < 0$ problem in the strong attraction limit.

Besides U and W, there are other inter-site interaction terms which are not included in (1) and which can be of importance in real narrow band systems. They are given by

$$H_1 = \sum_{ij\sigma} K_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} (n_{i,-\sigma} + n_{j,-\sigma}) + \frac{1}{2} \sum_{ij} J_{ij} \mathbf{s}_i \cdot \mathbf{s}_j + \sum_{ij} I_{ij} \rho^{+}_i \rho^{-}_j , \tag{3}$$

where $s^{+}_i = c^{\dagger}_{i\uparrow} c_{i\downarrow}$, $s^{-}_i = c^{\dagger}_{i\downarrow} c_{i\uparrow}$, $s^{z}_i = \frac{1}{2}((n_{i\uparrow} - n_{i\downarrow})$, $\rho^{+}_i = c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow}$, $\rho^{-}_i = c_{i\downarrow} c_{i\uparrow}$. They describe the correlated hopping (K_{ij})interaction, the electron spin (J_{ij}) and charge exchange (I_{ij} –pair hopping), respectively. Formally, J_{ij}, I_{ij} and K_{ij} are the off-diagonal terms of the Coulomb interaction $V(\mathbf{r} - \mathbf{r}')$: $J_{ij} = (ii|V(r)|jj)$, $\tilde{K}_{ij} = (ii|V(r)|ij)$. These terms, involving a bond charge density, result from the fact that due to the translational invariance the electron density operator is not diagonal in the Wannier representation. The typical range of electron- electron interaction parameters arising from the Coulomb potential is $U > W > K > |J|, |I|$ or, for a strongly screened potential, $U > K > W > |J|$.
A comprehensive study of the effects of these terms on the superconductivity of systems with inter-site pairing has been carried out by us [15,6,7] and independently by Hirsch and Marsiglio [13], and the correlated hopping term interaction has been even supposed to be a universal mechanism of s-wave superconductivity in the system with a low concentration of holes.

In what follows we will focus on three issues within the theory of superconductivity with local pairing:

- Superconductivity of preformed pairs (local pairs, bipolarons);
- Superconductivity with inter-site pairing (extended s-wave(s^*) vs d–wave symmetry);
- Crossover between the BCS and Bose condensation

In section 5 we will also comment on the realistic generalization of the theory for the case of coexisting local pairs and itinerant carriers: a model of charged bosons ($2e$) and fermions (1e).

3. Properties of superconductors with local electron pairing

3.1. Tightly bound local pairs

In the model of local electron pairs (LPs), superconductivity resembles the superfluid state of ^{4}HeII. The thermodynamic and electromagnetic properties of such

a system of charged (hard-core) bosons have been extensively studied (for a review see [6,8],[12]. In contrast to BCS superconductors, in the LP system the electron pairs (either on-site or inter-site) can exist above the transition temperature. T_c is determined by the center-of-mass motion of pairs; it increases with decreasing the local attraction and increases with increasing the bandwidth ($T_c \sim t^2/\mid U \mid$ in the case of on-site pairing). In such a way an enhancement of T_c with applying pressure is quite natural for the LP pair system. At some temperature $T_p > T_c$ the local pairs finally break up into electrons. Hence, there will be, in general, three temperature regions.

(i) A low temperature region where the pairs are in the superconducting state with the properties analogous to the superfluidity of charged bosons on a lattice. In the high density limit this phase can compete with the charge density wave (CDW) ordering (short- or long-range order).

(ii) An intermediate temperature regime with the state of dynamically disordered local pairs.

(iii) A high temperature regime above T_p around which a dissociation of pairs takes place.

The regions (i) and (ii) are separated either via a single λ-type transition ($SS \rightarrow NO$) or via a sequence of two (SS-CDW $\rightarrow$ CDW $\rightarrow$ NO) or three (SS $\rightarrow$ SS-CDW $\rightarrow$ CDW $\rightarrow$ NO) phase transitions in the case of on-site pairs.

It should be pointed out that since the gap in the single electron excitation spectrum persists across the SS transition, the single-electron conductivity of the normal phase (but below T_p) will be non-metallic and have an activated character. In such a case the transport properties can be dominated by charged LP.

In the regime well below T_p, the system with preformed local pairs can be described by the Hamiltonian of hard-core charged bosons on a lattice [6,8]

$$\bar{H} = -\sum_{ij} J_{ij} b_i^\dagger b_j + \sum_{ij} V_{ij} n_i n_j - \mu_B \sum_i n_i, \tag{4}$$

where $n_i = b_i^\dagger b_i, [b_i^\dagger, b_j] = (1 - 2n_i)\delta_{ij}$. The first term in equation (8) describes the transfer of electron pairs (hard-core bosons with charge 2e), whereas the second term stands for an effective Coulomb interaction between the pairs. The number of bosons is fixed by the condition $\tilde{n} = \frac{1}{N}\sum_i \langle n_i \rangle$. The operators $b_i^\dagger, b_j$ are commuting (Bose-like) for different sites, i.e. $[b_i^\dagger, b_j] = 0$ for $i \neq j$, but $(b_i)^2 = (b_i^\dagger)^2 = 0$, $b_i^\dagger b_i + b_i b_i^\dagger = 1$, for the same site, which reflects their fermionic nature.
In the case of on-site pairing the Hamiltonian (4) can be derived by an exact mapping of the EHM in the strong attraction limit[6] and $b_i^+ = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger$, $J_{ij} = 2t_{ij}^2/|U|$, $V_{ij} = J_{ij} + 2W_{ij}$, $\mu_B = 2\mu + |U| + J_0$, $J_0 = \sum_{j(\neq i)} J_{ij}$, $\tilde{n} = n/2$. It should be noted, however, that model (4) is common for the study of the superconductivity and CDW formation in systems with bound electron pairs, of either the on-site or inter-site nature. Also, model (4) is particularly useful for the extreme type II superconductors with a short coherence length, as far as the effects of phase fluctuations are concerned.

In the LP systems, which are equivalent to a hard core Bose gas on a lattice, T_c *strongly depends on n for any density.* In particular, for the low density limit $T_c \sim \tilde{n}^{2/3}(\tilde{n})$ for $d = 3(d = 2 + \epsilon)$, and such a dependence of T_c on n can be displayed over a wide range of electron densities.

For the case of the anisotropic layered lattice structure this subject has been studied within the self-consistent RPA [6,8]. It has been found that in the limit of small density ($n \ll 1$), the concentration dependence of T_c changes with the ratio $\alpha = J_\perp/J_\parallel$, where $J_\perp$and $J_\parallel$ are inter- and intra-layer values of J_{ij}, respectively. For $\alpha < 1$, T_c is well described by

$$k_{\rm B}T_c = 4.17 J_\parallel \alpha^{1/3}(2\tilde{n})^{2/3} = \frac{3.31(n^*)^{2/3}}{m^*}, \tag{5}$$

which is just the formula for a $3D$ (*anisotropic*) free Bose gas with the effective mass $m^* = (m_\perp m_\parallel^2)^{1/3}, m_\parallel = (2J_\parallel a^2)^{-1}, m_\perp = (2J_\perp d^2)^{-1}$, and with a density equal to the density of electron pairs in the system $n^* = \tilde{n}(a^2 d)^{-1}$. (The quantities a and d denote the intra- and inter-layer lattice spacing, respectively). However, for $\alpha \ll 1$, T_c is governed by the following expression:

$$\begin{aligned} k_{\rm B}T_c &= J_\parallel \frac{2\pi(1-|1-2\tilde{n}|)}{\ln(\frac{k_{\rm B}T_c}{J_\perp|n-1|})} \qquad (6) \\ &= \frac{2\pi n^* m^*}{m_\parallel \ln(k_{\rm B}T_c m_\perp d^2)}, \end{aligned}$$

which shows a linear in n behaviour and reduces to the formula for the non-interacting Bose gas with a quasi-two-dimensional spectrum $E_k = \frac{1}{2m_\parallel}(p_x^2 + p_y^2) + \frac{1}{m_\perp d^2}(1 - \cos k_z d)$ where the bandwidth in k_z direction $m_\perp d^{-2} \ll k_{\rm B}T_c$, and the density is n^*. The crossover of T_c versus $\tilde{n}$ behaviour from the Bose gas with anisotropic mass ($T_c \sim \tilde{n}^{2/3}$) to quasi $2d$(or $d = 2+\epsilon$) Bose gas ($T_c \sim \tilde{n}$) takes place for $\alpha \sim 10^{-2}$.

At higher densities the LP superconductors display a non-monotonic behaviour of T_c versus n due to inter-pair repulsive interaction and the systems with on-site local pairs invariably

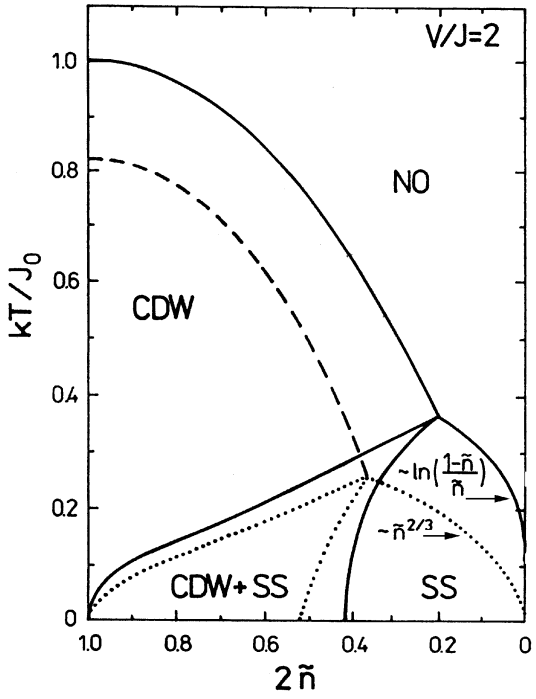

Figure 1. Phase diagram of the LP model (4) with n.n pair hopping (J) and n.n inter-pair repulsion (V), for sc lattice, and $V/J = 2$. Solid lines are from MFA, dotted ones are from RPA for the SS phase and a dashed line for CDW is from the BPW approximation. In the CDW+SS state two types of a long-range-order (LRO) may coexist forming a phase separated state or a supersolid depending on the range of inter-pair interactions. NO is a phase without LRO.

show a maximum of T_c near the phase boundary between the SS and CDW phases. In figure 1 we show an example of the finite temperature phase diagram of model (4) with near-neighbour (n.n) interactions for a simple cubic lattice calculated in MFA, RPA and BPW (Bethe-Peierls-Weiss) approximations. Indeed, MFA, being an exact theory for $d \to \infty$, substantially overestimates $T_c's$ for $d \leqslant 3$. The collective excitation and quantum fluctuation effects extend the range of existence of a homogeneous SS phase and provide a power law $\tilde{n}$ dependence of T_c. Notice, that this phase diagram displays the main phases found in the doped barium bismuthates [6]. For these materials the source of local attractive interaction is a chemical tendency toward disproportionation ($Bi^{4+} \to Bi^{3+}$+Bi^{5+}) (valence skipping) and the electron-phonon coupling.

We remark that there are other factors not considered above which can reduce T_c of charged bosons and also modify the concentration dependence of T_c[6,8]. It concerns: (i) the effects of the diagonal and off-diagonal disorder (ii) reduction of T_c by inter-subsystem density-density interaction if the LP coexist with itinerant carriers (section 5) and (iii) suppression of T_c due to the Wigner crystallization in the low density regime (effects of a long-range Coulomb interaction).

The LP superconductors are expected to have a *short coherence length* due to the short-range coupling between the pairs and the small radius of a pair. This leads to a relatively weak sensitivity of the SS phase in the low concentration limit to the presence of non-magnetic impurities (except for SS coupled to CDW), by analogy with the behaviour of the superfluid phase of ^{4}HeII in the presence of a disorder. It also provides *an enlarged width of the critical regime* which should be experimentally accessible and a true critical behaviour of the quantum X-Y, s=1/2 model should be observable. For d=2 a Kosterlitz-Thouless type phase transition can occur [6].

The electromagnetic properties of LP superconductors are essentially different from those of BCS [6,8,12]. The major effect of a magnetic field in a LP system occurs via its coupling to the orbital motion of the charged local pair. This leads to very enhanced values of H_{c_2}, proportionally reduced values of H_{c_1}, no Clogston limit for H_{c_2} as $T \to 0$ ($H_{c_2}(0) \sim E_{\text{binding}} \gg k_{\text{B}}T_c$) and a strongly enhanced penetration depth. Moreover, one obtains an upward curvature of H_{c_2} near T_c , with $H_{c_2} \sim (1-(T/T_c)^{3/2})^{3/2}$ for the 3D system in the dilute limit.
The temperature dependences of λ and H_{c_1} can also be non-standard. In particular, for the screened Coulomb interaction in the dilute limit, there are three different regimes ($d > 2$) [8]:
(i) a low-T "phonon" region for $\tau = T/T_c < \tau_0 = T_0/T_c$ with
$1-(\lambda_0/\lambda(T))^2 \sim \tau^{d+1}$, (where $k_{\text{B}}T_0 \approx \tilde{n}\bar{V}_0$, $\bar{V}_0 = 2(J_0+V_0)$),
(ii) an intermediate "free particle" region for $\tau_0 < \tau < \tau_x \sim 1-\tau_0$ with $1-(\lambda(0)/\lambda(T))^2 \sim \tau^{d/2}$ and
(iii) the critical region for $\tau_x < \tau < 2-\tau_x$ with the XY model type critical behaviour and $(\lambda_0/\lambda(T))^2 = (1-\tau)^\nu$, $\nu \approx 2/3$ for $d = 3$ (λ_0 being the penetration depth amplitude). With increasing $\tilde{n}$ and/or $\bar{V}_0$ regions (i) and (iii) will expand at the cost of suppression of region (ii). Thus, the universal features of $[\lambda(0)/\lambda(T)]^2$ in

the considered model for d=3 are the T^4 behaviour in the $T \to 0$ limit and the $3D\ XY$ critical point behaviour close to T_c. The temperature in the intermediate region of T depends on $\tilde{n}$ and $\bar{V}_0$ and reflects the location of the crossover from the dilute to the dense limit in terms of the exponent x in the formal expression: $[\lambda(0)/\lambda(T)]^2 = 1 - (T/T_c)^x$, with $3/2 < x < 4$ ($x \to 3/2$ for $\tilde{n}\bar{V}_0 \to 0$ and $x \to 4$ in the dense case). It is important to note that in the $d = 2 + \epsilon$ case there will be a regime of a linear in T dependence of the superfluid density far from the critical region.

The superfluid properties of the interacting hard–core charged bosons on a lattice for short–range and long–range inter-site interactions are summarized in table 2 [8].

Table 2. Superfluid characteristics of the hard–core boson model.

Quantity	Short–range repulsion				Long–range repulsion
	$0 < T < T_0$		$T_0 < T < T_c$ ⋆		$0 < T \ll T_c$
	$d = 3$	$d = 2 + \varepsilon$	$d = 3$	$d = 2 + \varepsilon$	$d = 3, n \ll 1$
$\langle\rho^x(0)\rangle - \langle\rho^x(T)\rangle$	T^2	T	$T^{3/2}$	T	$T^{1/2}\exp(-\bar{\Delta}/k_{\mathrm{B}}T)$
$E(T) - E(0)$	T^4	T^3	$T^{5/2}$	T^2	$T^{3/2}\exp(-\bar{\Delta}/k_{\mathrm{B}}T)$
$C(T)$	T^3	T^2	$T^{3/2}$	T	$T^{-1/2}\exp(-\bar{\Delta}/k_{\mathrm{B}}T)$
$\rho_s - \rho_s(T) \sim$ $1 - (\lambda_L(0)/\lambda_L(T))^2$	T^4	T^3	$T^{3/2}$	T	$T^{3/2}\exp(-\bar{\Delta}/k_{\mathrm{B}}T)$

⋆ only for low concentration and beyond the critical regime

$\Delta = \hbar\Omega_0^*$ for $\hbar\Omega_0^* \ll J_0$;

$\Delta = \min E_k < J_0$ for $\hbar\Omega_0^* > J_0$

$(\hbar\Omega_0^*)^2 = \frac{4\pi\bar{e}^2\langle\rho^x(0)\rangle^2}{\epsilon_0 m^*}$, $\langle\rho^x(0)\rangle^2 \approx \tilde{n}$. Ω_0^* is the plasma frequency.

We note that the exponential T dependences of the thermodynamic characteristics for the case of unscreened Coulomb interactions (cf. table 2) will occur in a restricted temperature range. With increasing T, for $d > 2$ a crossover to power–low characteristics, $c_v(T) \sim T^{d/2}, \langle\rho^x - \rho^x(T)\rangle \sim \rho_s - \rho_s(T) \sim T^{d/2}$, can take place at higher T, if $k_{\mathrm{B}}T_c > k_{\mathrm{B}}T > \bar{\Delta} = \min E_{\mathbf{k}} \leqslant \hbar\Omega_0^*$, where $E_{\mathbf{k}}$ is the collective excitation spectrum with a long-range Coulomb interaction.

As far as other universal trends are concerned, we should point out the plots of T_c versus $\lambda^{-2}(0)$ first reported by Uemura et al. [1,2]. In [4,14], the plots of $\bar{T}_c = T_c/T_c^m$ versus $\bar{\lambda}^{-2}(0) = \left(\lambda_{||}^m(0)/\lambda_{||}(0)\right)^2$ for the hard core Bose gas on a lattice have been obtained with the use of a selfconsistent RPA theory for T_c [6,8] and $\lambda^{-2}(0)$, for several values of V/J. (T_c^m corresponds to the maximum critical temperature in the $0 < \bar{n} < 1$ interval and $\lambda_{||}^m(0)$ is in plane penetration depth at $T = 0K$ attained for T_c^m). These scaled curves compare well with the corresponding experimental plots for various families of superconducting cuprates and Chevrel phases, for both the underdoped and overdoped regimes. The theory predicts an almost universal $\bar{T}_c$ versus $\bar{\lambda}^{-2}(0)$ behaviour (being only very weakly dependent

on V/J) in the underdoped regime (low concentration)[4,14], and possible deviations from the universality for the systems in the overdoped regime. The general trends including the fact that the overdoped systems have lower T_c and suppressed superconductivity are qualitatively reproduced [14].

Other points which can distinguish the LP superconductivity from the BCS [6,8,12] are:
(i) The collective excitation spectrum with a sound wave-like excitation branch in the case of a screened Coulomb interaction. A reduced plasma frequency in the case of an unscreened Coulomb interaction. The existence of two energy gaps in the latter case: the gap in the single electron excitation spectrum which remains almost a T independent well in the normal state region ($2\Delta/k_{\rm B}T_c \gg 1$) and the gap in the two electron spectrum $\Delta_1(2\Delta_1/k_{\rm B}T_c \sim 1)$.
(ii) The heat capacity jump at T_c : $\Delta c/nk_{\rm B} \leqslant 1$ ($\ll 1$ for BCS), and a possibility of λ-like anomaly in the heat capacity.
(iii) The relaxation rate of nuclear spins $T_1^{-1} \sim \exp(-\Delta/k_{\rm B}T)$ (only thermally activated electrons can interact with the nuclear spin) (iv) Temperature dependent sound velocity ($s(T)$) with a negative temperature gradient, a jump of the 1st derivative of s(T) at T_c and the sound wave damping $\Gamma \sim T(\Gamma \sim \exp(-\Delta/k_{\rm B}T)$ in BCS). (v) Essentially the same effect of nonmagnetic and magnetic impurities for s-type LP on T_c.
(vi) A possibility of the disorder induced LP superconductivity and superconducting glass behaviour of an LP superconductor.

The normal state properties for the on-site and inter-site local pairs can be different, due to the existence of triplet states for the latter. The main features common for both types of pair carriers are: (i) diamagnetic (or Van Vleck type) susceptibility (ii) field independent resistivity ρ and thermoelectric power S up to a very high magnetic field($\sim \Delta$) (iii) the possibility of a linear in T dependence of ρ and a small value of thermopower in a wide range of temperature, in the case of a nondegenerate gas of charged bosons [12].

3.2. The extended Hubbard model with inter-site pairing

The superconductivity and magnetism in the extended Hubbard model (1) with on-site repulsion and inter-site attractive interaction, i.e. $U_{\rm eff} > 0, W_{\rm eff} < 0$ (and E_i=const) have been studied in both the weak ($U < 2zt$) and the strong ($U \gg t$) correlation limits, within various approximation schemes [15,6,4]. This is a generic model incorporating magnetic correlations and inter-site pairing, having relevance for high$-T_c$ materials, as well as heavy fermion superconductors and organic superconductors. The model can be considered as the simplest model of oxygen holes pairing in high$-T_c$ cuprates due to the polaronic mechanism [15,12] or due to purely electronic mechanisms ([15],[4]) or as an effective model of quasiparticle (AF spin polarons) pairing ([16,17]). These studies, including the analysis of various types of anisotropic superconducting pairings, the spin-density waves (SDW) and the phase separated state (electronic droplets formation) have been essentially carried out for a $2D$ square lattice with near neighbour (n.n) hopping

and for arbitrary electron density. The effects of next-nearest neighbour (n.n.n) hopping, antiferromagnetic exchange, the correlated hopping and longer range Coulomb repulsion have also been analyzed. In many of these studies the intersite interaction terms have been decoupled in the broken symmetry HFA, which, due to the extended nature of the pairing potential, is here better justified than in the case of on-site interaction. In contrast to the original BCS treatment for the phonon-mediated attraction, we did not impose any cutoff in either momentum or frequency. The effective short-range attraction in the considered model is essentially instantaneous on the time scale of the inverse bandwidth.

Let us briefly summarize the essential properties of the system with an intersite attraction having in mind that due to the complexity of the problem only partial and mainly qualitative conclusions can be drawn.

The nature of the ordered state depends on the band filling and the values of the parameters involved. In figure 2 we show the ground state phase diagram at half-filling. For $n \neq 1$ and for the nearest neighbour hopping only, the sequence of transitions $d \Leftrightarrow p \Leftrightarrow s^*$ is possible with lowering the electron density (from $n = 1$) for $U > 0$(or small negative U). Close to half-filling the d-wave pairing dominates, which competes with the SDW state. The extended s-wave pairing is stable for lower electron densities and T_c^s versus n, with increasing n, and increases first, then goes through the maximum and drops to zero above some critical density (cf. figures in [15]). Increasing $|W|$ expands the range of stability of a d-wave pairing towards higher values of $|n-1|$. The stable superconducting solutions in the present model can be obtained upon an appropriate tuning of the parameters. Above some critical value of $|W|$, dependent on n, a condensed state of electronic droplets-phase separation becomes stable. The phase separation line rapidly shifts towards higher values of $|W|/D$ upon increasing the long-range repulsive Coulomb interaction.

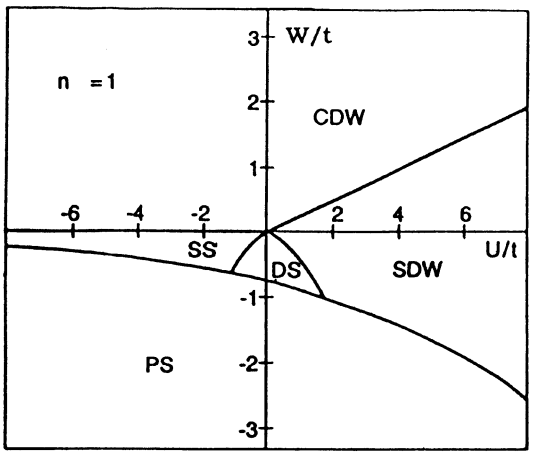

Figure 2. Ground state phase diagram of the $t-U-W$ model at a half-filled band for the two dimensional square lattice for near neighbour hopping, from the mean-field analysis. SDW – a spin density wave state, CDW – a charge density wave state, PS – phase separation, SS and DS are superconducting states with s and $d_{x^2-y^2}$ symmetry, respectively [15,16].

The mutual stability phase diagrams of anisotropic superconducting solutions and the condensed state for a $2D$ square lattice were given in [15,16]. The increasing on-site repulsion suppresses the s-wave state and restricts it to low densities only.

The effects of next-near neighbour hopping(t_2) are appreciable and can radically modify the mutual stability of superconducting and SDW states, as well as

the variation of superconducting T_c with the electron density [15]. In the presence of t_2 the electron dispersion is of the form:
$\varepsilon_{\mathbf{k}} = -2t_1\left(\cos(k_x a) + \cos(k_y a)\right) - 4t_2 \cos(k_x a)\cos(k_y a)$, where t_1 and t_2 are the near neighbour and next-near neighbour hopping parameters, respectively. For $t_2 < 0$ (which is the case suggested to reproduce the Fermi surface for hole doped cuprates), with increasing the ratio $|t_2/t_1|$ the sequence of transitions with n can be changed from $s^* \to p \to d \to s^*$ to $s^* \to d \to p \to s^*$ and then to $d \to p \to s^*$ (see figure 14 in [4]). The p-wave pairing is strongly suppressed by an antiferromagnetic exchange interaction which, in turn, enhances the extended s and d- wave singlet pairings. An example of the mutual stability diagram of s-d pairings (with excluded triplet p-wave pairing) for the square lattice with n.n and n.n.n hoppings in a $t_2/t_1 - n$ plane is shown in figure 3.

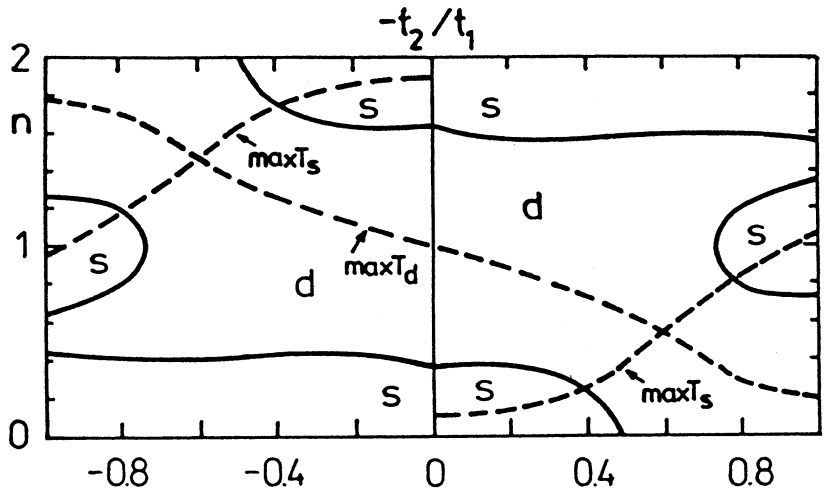

Figure 3. Mutual stability diagram of extended s and d-wave pairings for the case of the n.n inter-site attractive interaction ($(|W| + 3J/2)/4t_1 = 0.5$, J is the antiferromagnetic exchange)and n.n and n.n.n hoppings on the square lattice.

As we see, for $t_2 < 0$, with increasing $|t_2/t_1|$ the range of stability of the d-wave pairing can extend up to $n = 0$, while the $d - s^*$ phase boundary for small $(2 - n)$ is only weakly dependent on $|t_2/t_1|$.

One should bear in mind recent calculations of van der Marel [18] which show that for singlet pairing potential g_1, larger than the critical value (dependent on n), the ground state can be of a mixed s^* and d wave symmetry and the region of $s^* - d$ mixing almost coincides with the region of the p-wave symmetry if we use a spin independent interaction ($|W|$ instead of g_1) (cf. figure 14 in [4]).

In the strong correlation limit ($U \gg t$) model (1) can be mapped onto the generalized $t - J$ model with the additional inter-site interaction term ($t - J - W$ model) and as such can generally result from the mapping of the multiple band extended Hubbard model to the single band problem [6].

Determination of possible superconducting, magnetic and CDW solutions of the EHM on a $2D$ lattice is still a challenging problem. For a square lattice an

electron pairing in the antiferromagnetic background via the spin-bag mechanism should also be considered.

In the dilute limit, formation of real bound inter-site pairs is possible. The properties of the extended bound states and resonances (formed by two lattice fermions interacting via a nonretarded potential) were studied in[6,19]. This two-body problem is exactly soluble on any periodic lattice. The pair binding energies for different lattices and the pairing symmetry were determined [6,19]. The symmetry of the stable bound state depends on the form of the pairing potential and the band structure. For model (1) with the interactions and hopping restricted to n.n., the two-body ground state is an extended s-wave for $U \geqslant 0$. The inclusion of n.n.n. hopping of the sign reversed to t_1 and the n.n.n. repulsion ($W_2 > 0$) favour the pairing of the $d_{x^2-y^2}$ symmetry (compare figure 3). Moreover, both these factors reduce the minimum value of $|W_1|$ necessary for the bound state formation.

Figure 4 shows the ranges of parameters for which the bound state of a given symmetry first appears below the continuum band in the case of n.n attraction ($W_1 < 0$) and n.n.n. hopping(t_2) and n.n.n. repulsion ($W_2 > 0$). [19]. It is interesting to note that taking $t_2 = -0.45t_1$ (which is close to the tight binding parameters inferred from the band structure calculations for 123 and 214 cuprates), one finds that the strength of the next neighbour repulsion must be at least $0.35t_1$ in order to have for the ground state a d-wave symmetry.

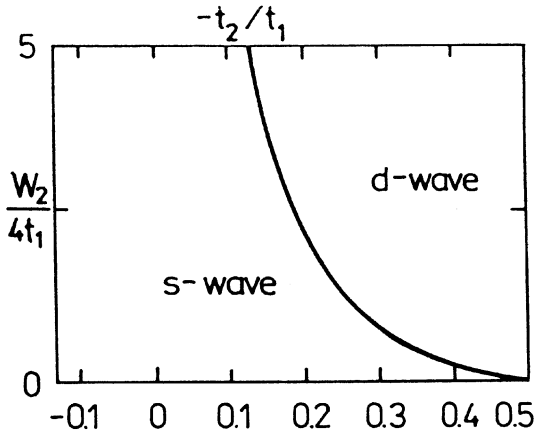

Figure 4. Mutual stability diagram of the extended s and d-wave bound states in the plane of $W_2/4t_1$ and t_2/t_1 obtained for n.n. attraction $W_1 < 0$ and $U = \infty$. (2D square lattice. After [19]).

We stress that the effective mass of a strongly bound inter-site pair can be small and even comparable with the mass of its constituent fermions, in contrast to the case of an on-site pairing. This is due to the fact that unlike the case of a strong on–site attraction, where the pair moves via virtual ionization only, the inter-site pair can easily move without breaking its bond, if the n.n.n. hopping is included. For example, in a square lattice $m_f = \hbar^2/2(t_1+2t_2)a^2$, whereas the mass of a strongly bound inter-site pair $m_b = \hbar^2/|t_2|a^2 = 2(t_1+2t_2)m_f/|t_2|$ [19].

The superconducting state requires a Bose condensation of such real bound pairs. For a low electron density the system behaves as a dilute gas of two-electron molecules and the problem can be mapped onto that of a hard-core charged Bose gas on a dual lattice, whose properties were discussed in section 3.1.

In the weak local attraction case, i.e. beyond the limit of real space pair formation, the physics becomes much more similar to BCS than in the strong coupling regime. However, even then, there remain some essential differences in comparison

with BCS superconductors. The fact that the attraction is static (without a cut-off in the frequency dependence of the interaction), implies that many electrons inside the Fermi surface can contribute to the pairing and that the effective half-bandwidth D (instead of ω_D) will determine the energy scale. One also expects enhanced pair fluctuation effects which are beyond the standard mean-field BCS approach. The consequences of this are the following. Firstly, T_c and the energy gap Δ in the BCS expressions are enhanced, since ω_D is replaced by D. Secondly, both these quantities are explicitly dependent on the electron density (see below). Thirdly, the ratio $2\Delta/k_{\rm B}T_c$ can deviate from the BCS value, being a function of the lattice structure (DOS), electron concentration, and the strength of attractive interaction.

At very small densities, analytic expressions for T_c^s can be obtained if the pairing potential is restricted either to on-site or nearest neighbour attraction: $T_c^s \sim n^{1/3}(n^{1/2})$ for $d = 3(d = 2)$ lattice. However, if the attractive couplings fall off gradually with a distance: $T_c^s \sim n^{2/3}(n)$ for $d = 3(d = 2)$ lattice, i.e. one gets the same n dependence of T_c as for LP superconductors.

In the systems with a local pairing interaction several types of phase separated states can develop besides homogeneous phases, particularly, if the long–range Coulomb interactions are strongly screened. In these states, which can be favourable close to the half–filling of the band, the system breaks into coexisting domains of two different charge densities and different types of electron ordering. For the on–site pairing these are the CDW–SS or CDW–NO states, whereas for the inter-site pairing the SDW–SC (s,d or p type), the CDW-SC or the state of electron droplets [15,16,21]. In real materials the size of the domains will be finite and determined by the long–range Coulomb repulsion and disorder effects (structural imperfections, disorder of doped ions, etc).

4. Crossover from BCS to the local electron pair limit

In the previous section, we presented properties of systems with a local electron pairing derived in the LP and weak coupling regimes.

Let us now briefly summarize the main physical ideas regarding the crossover from the cooperative Cooper pairing(BCS) to Bose condensation (BC) (see figure 5, cf. also figures 3 and 13 in [6]) [6,7,22,23].

1. In both regimes there is only one phase transition at T_c, as long as no other broken symmetry phases intervene or the system does not undergo a phase separation. The transition is from the normal state to the superfluid one with an off-diagonal long range order (ODLRO)(or algebraic order in 2D).

2. The nature of the phase transition in both limits is quite different. In the BCS limit a formation of Cooper pairs and condensation at T_c takes place simultaneously and $T_c \sim \exp(-1/N(0)V)$ ($N(0)$-the density of states per spin, V-the parameter of attractive coupling). The first deviation from this scenario can be described in terms of superconducting fluctuations. In the preformed pair regime, however, the pair formation (T_p) and their condensation (T_c) are independent pro-

cesses. T_p and T_c are widely separated and T_p is a characteristic energy scale, not a phase transition temperature (at least for non-frustrated lattices). T_c will decrease with the increase of coupling constant V. For $T > T_p$ local pairs are thermally dissociated.

3. In the weak coupling limit, below T_c, we have a BCS condensate of a large number of overlapping Cooper pairs ($\xi \gg a$). Thermodynamics and T_c are determined by single particle excitations (broken Cooper pairs) with an exponentially small gap. In the opposite, strong coupling regime one has the Bose condensate of tightly bound local pairs ($\xi \sim a$), and the thermodynamics and T_c are governed by the collective modes. With increasing coupling there is a smooth reduction of $k_{\mathrm{B}}T_c/E_g(0)$(T_c over the energy gap) ratio from the BCS value. In the intermediate and strong coupling regime T_c does not scale with the energy gap and $k_{\mathrm{B}}T_c/E_g(0)$ decreases with the reduced concentration.

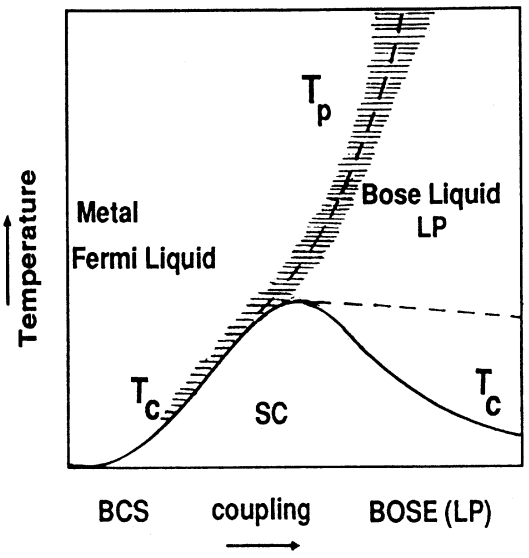

Figure 5. Schematic diagram of the evolution from weak coupling BCS to preformed local pair regimes. Solid lines are for a lattice fermion model like the attractive Hubbard model (where $T_c \sim \exp(-t/|U|)$ in a weak coupling, whereas $T_c \sim t^2/|U|$ and $T_p \sim |U|$ in strong coupling). Dashed line is continuation of T_c in the strong coupling regime for continuum fermion models with attraction. Hatched region is a crossover scale around which pairing correlations show up in the normal state and T_p marks a characteristic temperature of the pair formation.

The collective modes evolve smoothly between the two regimes. In the BCS regime we have Anderson–Bogolubov modes for a neutral case and plasmons for a charged case, respectively. In the BC limit there are either sound wave Bogolubov modes for the screened Coulomb repulsion or plasma modes for the charged boson superfluid. As far as electromagnetic properties are concerned, there is a smooth evolution of the Meissner kernel from a Pippard type to a London type behaviour at $T = 0$K [22]. Also, λ_L, H_c and the coherence length evolve smoothly from the BCS to BC regime [24]. In the BC case: $\kappa = \lambda_L/\xi \gg 1$, superconductivity is clearly of extreme type II, H_{c2} is very large, $H_{c2}(T)$ can exhibit a positive curvature vs T and $H_{c1}/H_{c2} \ll 1$.

4. For the BCS superconductor the normal state is a Fermi liquid. In the BC regime one has for $T_c < T < T_p$ a normal Bose liquid (of bound pairs). The evolution of the normal state from the Fermi to Bose liquid is quite unusual, especially in 2D systems. From the recent studies it appears that even for a moderate attraction the (degenerate) Fermi liquid regime shows anomalous features due to

pairing correlations such as a pseudogap, anomalous behaviour of spin susceptibility and a spin gap in the normal state. [6,22,23,25–30] (cf. section 6). It is of interest that these anomalies (superfluid precursor effects) are similar to those experimentally observed in NMR and optical conductivity on underdoped high T_c cuprates. From these studies it follows that deviations from the conventional Fermi liquid behaviour are generic to the normal state of short–coherence length superconductors.

The above results were mostly derived from the studies of one-component models. Both, for the case of on-site pairing and that of inter-site pairing *the crossover between the "BCS" and the local pair limits can occur either by increasing the coupling strength, or by decreasing the carrier concentration (in the intermediate coupling regime)*. We should stress the importance of a mixed model of coexisting bosons and fermions [6,31,33]. This model can be naturally considered as an extension of the extreme bosonic limit and a candidate to describe the intermediate coupling crossover regime.

Regarding open problems we point out the question of a crossover for anisotropic pairings of s or d–wave symmetry in systems of reduced dimensionality. Enhanced thermodynamic fluctuations and short coherence length effects are clearly challenging issues for the successful theory of cuprates and organic superconductors.

5. Coexisting local pairs and itinerant carriers

The coexistence of bound pairs, itinerant electrons and the effects resulting from interactions between these two species constitute an important problem for understanding the intermediate crossover regime and application to real materials. Such a model of the mixture of local pairs and electrons (the mixture of charged Bosons and Fermions) interacting via a charge exchange was introduced by us a few years ago [31] and its extended version has been extensively analyzed in a number of more recent papers [32–46]. It has been shown that in this type of systems a new mechanism of superconductivity can develop. It results from the intersubsystem charge exchange coupling, both hybridization induced and direct, and leads to the superconducting state involving both types of particles. The physical properties of such a mixture of interacting charged bosons (bound electron pairs) and electrons can show features which are intermediate between the features of pure local pair superconductors and those of classical BCS systems. The model may also have relevance to the problem of a single band system with a short-range attraction in the intermediate coupling regime where the bound and ionized pairs coexist [6,7,22]. An effective Hamiltonian of coexisting local d-electron pairs and itinerant c-electrons can be written as [31,32,6,7]:

$$H = H_0 + H_1 + H_C, \tag{7}$$

$$H_0 = \sum_{\mathbf{k}\sigma}(\epsilon_{\mathbf{k}} - \mu)c^{\dagger}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma} + 2\sum_i(\Delta_0 - \mu)b^{\dagger}_i b_i - \sum_{ij} J_{ij}b^{\dagger}_i b_j, \tag{8}$$

$$H_1 = \frac{1}{2N}\sum_{i\mathbf{k}\mathbf{k}'}(I_{\mathbf{k}',-\mathbf{k}}\exp[i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{R}_i]c^\dagger_{\mathbf{k}'\uparrow}c^\dagger_{-\mathbf{k}\downarrow}b_i + H.c.) \quad (9)$$
$$+ \frac{1}{2N}\sum_{i\mathbf{k}\mathbf{k}'\sigma} V_{\mathbf{k}'\mathbf{k}}\exp[-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_i]c^\dagger_{\mathbf{k}'\sigma}c_{\mathbf{k}\sigma}b^\dagger_i b_i,$$

$\epsilon_{\mathbf{k}}$ refers to the energy band of c-electrons, Δ_0 measures the relative position of the local pair level with respect to the bottom of the c-electron band. μ is the chemical potential which ensures that the total number of particles in the system is constant, i.e.,

$$n = \frac{1}{N}\left(\sum_{\mathbf{k}\sigma}\langle c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}\rangle + 2\sum_i\langle b^\dagger_i b_i\rangle\right) = n_c + 2n_d.$$

J_{ij} is the pair hopping integral, n_d is the number of pairs per (effective) site. $I_{\mathbf{k},\mathbf{k}'}$, represents the transverse component of the charge-charge coupling between the two subsystems. (For the sake of simplicity we take intersubsystem pair hopping of s-wave type). $V_{\mathbf{k},\mathbf{k}'}$ is the density-density intersubsystem interaction. H_C includes the remaining c-c and d-d Coulomb interactions between charge carriers. The charge operators for local pairs $b^\dagger_i, b_i$ obey the Pauli spin 1/2 commutation rules which exclude multiple occupancy of a given pairing center. For on-site pairs $b^\dagger_i = d^\dagger_{i\uparrow}d^\dagger_{i\downarrow}, b_i = d_{i\downarrow}d_{i\uparrow}, 2b^\dagger_i b_i = \sum_\sigma d^\dagger_{i\sigma}d_{i\sigma}$ (where $d^\dagger_{i\sigma}, d_{i\sigma}$ operate in the subspace with single occupancies excluded).

Depending on the relative concentration of "c" and "d" electrons we distinguish three essentially different physical situations.

(i) $\Delta_0 < 0$, so that all the available electrons form local pairs of "d" electrons (the d-regime or the "local pair" $n_d \gg n_c$)
(ii) $\Delta_0 > 0$, so that the "c" electron band is filled up to the Fermi level $\mu = \Delta_0$ and the remaining electrons are in the form of local pairs of "d" electrons (the c+d -regime or "intermediate", $0 < n_d, n_c < 2$).
(iii) $\Delta_0 > 0$, so that the Fermi level $\mu < \Delta_0$ and, consequently, at $T = 0$K all the available electrons occupy "c" electron states (the c-regime or "BCS", $n_c \gg n_d$).

For $J_{ij} = 0$, in case (ii), superconductivity is caused by a perpetual interchange between local pairs of "d" electrons and pairs of "c" electrons. In this process "c" electrons become "polarized" into Cooper pairs and "d" electron pairs increase their mobility by decaying into "c" electron pairs. In this intermediate case neither the standard BCS picture nor the picture of local pairs fits, and superconductivity has a "mixed" character with a correlation length of the order of several interatomic spacings. The system shows features which are intermediate between the BCS and preformed local pair regimes. This concerns the energy gap in the single electron excitation spectrum, $k_{\mathrm{B}}T_c/E_g(0)$ ratio, the thermodynamic critical field, the Ginzburg ratio κ, the width of the critical regime and the normal state properties.

In case (i) the local pairs of "d" electrons move via virtual excitations into empty c-electrons states. Such a mechanism gives rise to the long-range hopping of pairs of "d" electrons (analogy to the Rudermann-Kittel-Kasuya-Yosida (RKKY)

interaction for s-d mechanism in the magnetic equivalent). The superconducting properties are analogous to those of a pure local pair superconductor, eventually with a reduced critical region due to the extended range of pair hopping.

In case (iii), on the contrary, we find a situation which is similar to the BCS case: pairs of "c" electrons with opposite momenta and spins are exchanged via virtual transitions into local pair states.

The indirect long-range character of the charge exchange between the local pairs is an essential feature of the mixed model. This should be contrasted with the previously considered models of a local pair superconductivity in which the pair hopping term resulting from the kinetic exchange mechanism ($J_{ij} = 2t_{ij}^2/|U|$) is obviously short-ranged. Thus, an indirect charge exchange can be effective even if the local pair centers are well separated in space. The case of a small number of local pairs coupled by a long-range interaction resembles an RKKY "spin glass" and it might equally well exhibit a "superconducting glass" state or a "charge density wave glass" state.

The main features of the mixture of wide band electrons and local pairs are summarized below [31–33,6,7].

(1). The model avoids problems with small pair mobility (of on-site pairs) and can provide a screening mechanism of a long-range Coulomb interaction between charged bosons.

(2).The origin of the energy gap can be distinct from BCS. An energy gap in a wide band can open due to the pair Bose condensate ($\langle b \rangle \neq 0$).

(3). As we proceed from the case of predominantly local pairs to that of predominantly wide band-electrons, we observe a non-monotonic behaviour of T_c , which passes through a maximum of order I_0^2/D when the two constituents have roughly equal concentrations and drops to zero when we approach regions (i) and (iii).

(4). The ratio $E_g(0)/k_{\mathrm{B}}T_c$ (the energy gap over T_c) - is not universal. It varies around the BCS value 3.52 as the relative proportion of local pairs to wide band electrons is changed. Where T_c is maximum, $E_g(0)/k_{\mathrm{B}}T_c$ has a shallow minimum; it approaches the BCS value for predominantly wide band electrons and surpasses it as the concentration of local pairs increases above that of wide band electrons.

(5). $\xi, H_c, \lambda, \kappa_{GL}$ evolve with a change of position of the LP level Δ_0 from "LP" to "Crossover" and finally to a "BCS"-like regime.

As for the evolution of superconducting properties with increasing the total number of carriers, there are two possible types of change-overs [33,34]: (i) for $\Delta_0 > 0$, "BCS" $\rightarrow$ "Crossover"$\rightarrow$ "BCS", and (ii) for $\Delta_0 \leqslant 0$, "LP" $\rightarrow$ "BCS". The latter case is relevant to the doping dependence of superconducting characteristics observed in high T_c cuprates. Only, in the case when the local pair level is deeply located below the bottom of the fermionic band, the system remains in the d-regime for any $n \leqslant 2$.

(6) The local pairs exist above T_c together with itinerant fermions. As far as the state above T_c is concerned, it has been shown that a system of local pairs and wide band electrons can exhibit a linear in T resistivity in the normal state where the Fermi level decreases linearly with T [35,36]. The normal state properties deviate

from the Fermi-liquid. In particular, numerical studies of the boson -fermion model ($T > T_c$) show: the existence of a pseudogap in single particle DOS, anomalies in one–electron self–energy $\Sigma(\mathbf{k}, \omega)$, anomalies in charge and magnetic responses, which are similar to those observed in cuprates [37]. Assuming a uniform distribution of LP states near E_F it was demonstrated [38] that: $Im\Sigma(\mathbf{k}, \omega) \sim -|\omega|$ (V–shaped form) (as in the marginal Fermi-liquid scenario). It can explain many normal state anomalies in HTS linear-like in T resistivity, tunneling conductance ($g(V) = g_0 + g_1 V$), photoemission data and optical conductivity.
(7). The model also involves a Kondo lattice problem, but for charged pairs (double valence fluctuations) ($U_{\text{eff}} < 0$) rather than for spins ($U_{\text{eff}} > 0$) [31,39]. The Kondo type coupling for charge operators

$$\sum_i \left[I(\rho_{di}^+ \rho_{ci}^- + H.c.) + V \rho_{di}^z \rho_{ci}^z \right]$$

instead of $\sum_i J\sigma_i \cdot \mathbf{S}_i$ ($\rho_{di}^+ = b_i^\dagger, 2\rho_{di}^z + 1 = b_i^\dagger b_i, \rho_{ci}^+ = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger, 2\rho_{ci}^z + 1 = \sum_\sigma c_{i\sigma}^\dagger c_{i\sigma}$). Increasing I reduces the charge moment of local pairs. For I comparable or greater than the c-electron bandwidth, *the charge Kondo lattice state* with a local charge moment compensation (isospin singlet) can develop, suppressing superconductivity and CDW. At low T the narrow quasiparticle band appearing near E_F is split by the coherence gap E_c. E_c disappears when T increases and at higher temperatures the system enters the incoherent charge Kondo regime, and then into the regime with properties similar to those of a single charge Kondo impurity. This new charge Kondo fluid may have potential applications for the normal state of systems with enhanced double-valence charge fluctuations, like doped $BaBiO_3$ oxides.

Various aspects of superconductivity in the boson-fermion model have been recently studied by many authors [40–46]. A generalization to anisotropic pairing of extended s–wave or d-wave is also possible [41,44,46].

6. Scenario of crossovers in HTS

There is a growing consensus that the normal state of cuprate HTS is characterized by a pseudogap or a quasiparticle gap. Several experiments pointed out that above T_c there is an anomalous reduction of spin response at temperatures much higher than the critical one (spin gap). This feature was initially associated with the bilayer nature of some cuprate families, however, it is now documented to occur in most of cuprate HTS.

The existence of a pseudogap (as probed by the spin and charge responses) in the excitation spectrum of the cuprates, opening at the characteristic temperature $T^\star$, which can be much above T_c in the underdoped regime, is well confirmed by a variety of measurements, including resistivity and Hall effect[47], specific heat[48], infrared studies[49,50], NMR [51], as well as by ARPES spectroscopy [52–54]. In particular, the latter, for Bi2212, shows a gap in the normal state, which is almost nearly doping independent (~ 33 meV), with a k dependence consistent with the $d_{x^2-y^2}$ symmetry. $T^\star$ and T_c are widely separated in the underdoped regime

and eventually merge near the optimum doping [52,53] The recent tunneling spectroscopy data give evidence that a superconducting gap is temperature independent up to T_c where it merges into a pseudogap. In the tunneling spectroscopy data a pseudogap is found to be present both in underdoped and overdoped samples and it scales with a superconducting gap [55].

These results and the evidence of a pseudogap behaviour in the normal state of doped barium bismuthates: $BaPb_{1-x}Bi_xO_3$ [56] and $Ba_{1-x}K_xBiO_3$ [57], together with universal features and trends of HTS discussed in the previous sections, give a strong support for the theories of superconductivity with a local pairing. In our opinion, these results are central for the microscopic theory of HTS and show that the conventional BCS theory, regardless of the symmetry of the order parameter, cannot describe superconductivity in HTS materials in the whole range of doping concentration. They suggest that the physics of these materials should be considered in terms of the crossover between "Bose(BE)(LP)" and "BCS" limits (as it was pointed out in [6] and [2]). Schematic plot in figure 6 illustrates possible crossovers in cuprate HTS. In this scenario we identify the pseudogap phenomena observed below T^* as a result of singlet pairing in the normal state. In that region a low temperature gap weakly depends on doping, while T_c is proportional to the carrier density. With increasing the doping the state with preformed pairs without a long-range phase coherence crosses over to a "metallic" state and eventually to a Fermi-liquid. In the underdoped regime the phase fluctuations drive transition at T_c. Beyond the optimum doping the amplitude fluctuations control T_c and a "BCS"-like behaviour is expected. As suggested by the Ginzburg-Landau phenomenology and the scaling theory of critical phenomena, with increasing the doping there is also possible a crossover from the essentially 2D behaviour of weakly coupled (Josephson type) CuO_2 planes to the (anisotropic) three-dimensional behaviour with the 3DXY critical point [58]. At point x_1 in figure 6 there can be a quantum phase transition from insulator to superconductor. It has been recently observed in Zn-substituted high-T_c cuprates in the underdoped region [59], that this transition is characterized by the universal 2D resistance $\rho_0 \to h/4e^2 = 6.45k\Omega$, as predicted by the scaling theory for insulator–superconductor transition in a bosonic system.

It is of interest to note that the character of several normal state anomalies related to tunneling, Raman scattering, optical conductivity and pseudogap, in cuprate HTS and doped bismuthates is quite similar. As we have discussed, a model of coexisting local pairs and itinerant fermions (which is essentially a two-band model) can provide a basis for the explanation of normal state anomalies and superconductivity in HTS. Moreover, there is a possibility that the mixed model describes generic features of the intermediate crossover regime of one-band models with a local attraction in the dilute limit.

The question is a proper description of the intermediate crossover regime (cf. table 1). For 2D systems (cuprates and organic superconductors), bound states can be formed due to a variety of mechanisms in the underdoped regime, while for 3D systems (bismuthates and fullerides), bound states occur if attraction is strong enough. A crossover from the Bose limit to the BCS regime can be reached

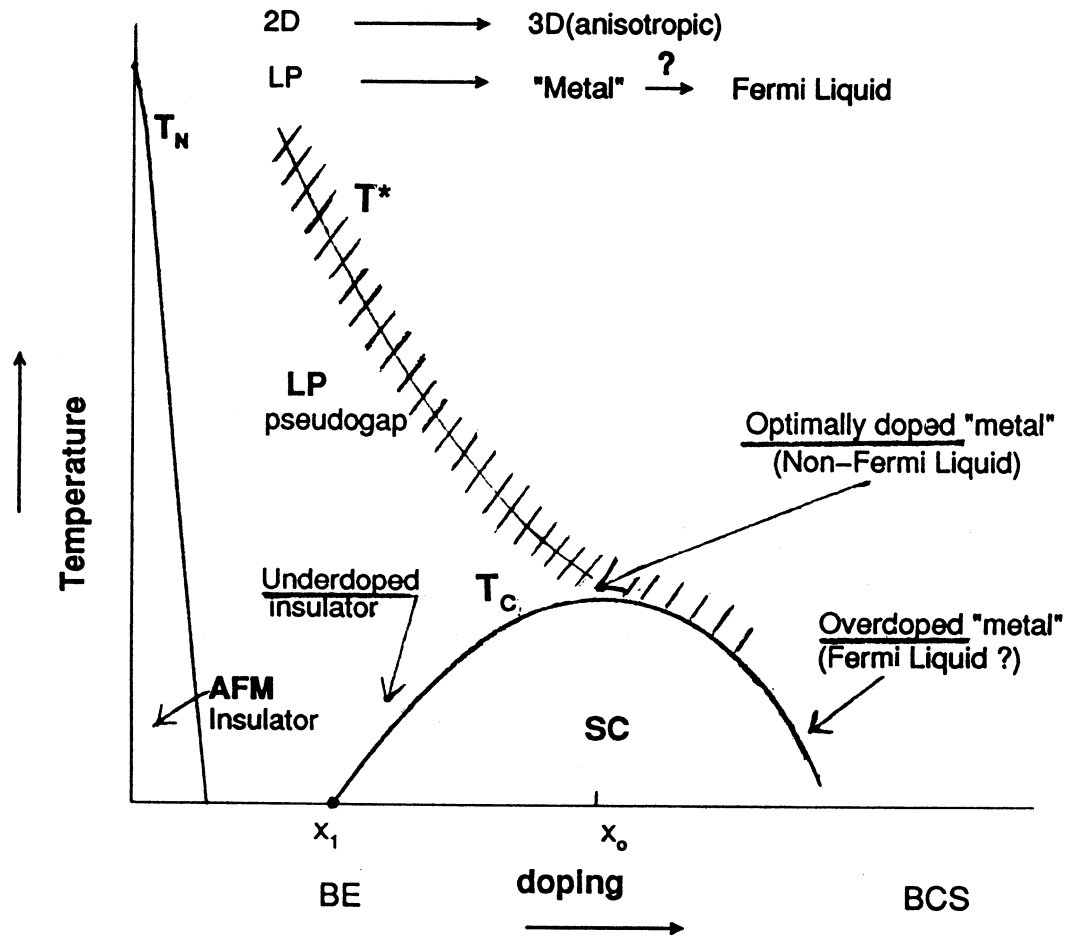

Figure 6. Scenario for crossovers in cuprate HTS with possible BE-BCS and 2D-3D dimensional crossover. The dashed line marks a region of the onset of local pairing with a characteristic T^*, the heavy solid line is for the superconducting transition (T_c). Regime with local pairs is characterized by a pseudogap in one-electron spectrum. AFM - antiferromagnet, SC - superconductor, x_0 corresponds to optimum doping, x_1 denotes a quantum critical point.

either upon varying the carrier concentration or decreasing the coupling strength, or varying the position of the bosonic level in a two-component model.

Acknowledgements

This work has been supported by the Committee of Scientific Research (K.B.N.) Poland, project numbers: 2 PO3B 056 14 and 2 PO3 B 104 11.

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Надпровідність в системах з локальним притяганням

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Отримано 15 травня 1998 р.

Обговорюються підходи, які базуються на концепціях локального електронного спарювання, і надпровідні властивості, до яких вони приводять. Звертається увага на природу проміжного зв'язку і окреслюються останні досягнення в проблемі кросоверу БКШ - Бозе надпровідність. Зроблено також огляд властивостей систем з локальними притягальними взаємодіями, які складаються з суміші локальних електронних пар і блукаючих ферміонів, що взаємодіють через механізм обміну зарядом, який одночасно індукує надпровідність в обох підсистемах. Наприкінці коротко обговорюється проблема псевдощілини і можливий сценарій кросоверів у високотемпературних надпровідниках.

Ключові слова: *надпровідність, модель Хаббарда, локальна притягальна взаємодія, кросовер*

PACS: *74.20.-z, 74.20.Mn, 71.10.-w, 71.10.Fd, 71.28.+d*