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Complex source point concept in the modelling of dynamic control for optical beam deflection

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Abstract. A rigorous analytical method for transient dynamics description in a cylindrical lens is developed and used to describe a possibility of beam controlled deflection via material tuning. The complex source point concept is used to simulate beam passing through the lens. Details of both the transient response and steady-state regime are described. The excited fields are described using a rigorous mathematical approach based on analytical solution in the Laplace transform domain and accurate evaluation of residues at singular points of the obtained functions.

Keywords: time-varying media, optical switching, optical beams, lens.

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1. Introduction

Electromagnetic wave propagation in time-varying media yields rise of new physical phenomena and possibilities for novel applications. Tuning the refractive index in time provides a fast frequency shift in the linear material dielectric resonator [1]. Half-restricted time-varying plasma causes focusing of a point source radiation at the plane boundary [2], which resembles action of a lens in the form of a plane layer with double-negative materials. Transient medium is used in the light-modulated photo-induced method for the development of a non-mechanical millimeter wave scanning technique [3]. Plasma based lenses with properties electronically adjusted can offer an alternative to the existing electronic beam steering systems by varying the density of plasma in time [4]. In practice, temporal switching of the material refractive index can be realized by varying the input signal in a nonlinear structure [5], by voltage control [6], by a focused laser beam as a local heat source [7], or by plasma injection of free carriers [8].

The main goal of this paper is to demonstrate a possibility of beam deflection control in a homogeneous lens of simple shape by adjusting its material parameters in time. The investigation based on a rigorous mathematical method that uses the Laplace transformation is aimed at deriving an analytical solution of the problem in a frequency domain. Time domain fields are recovered due to computation of the inverse Laplace transform via evaluation of residues at singular points. This approach provides accurate back transformation of the functions and allows to understand and look inside the fundamental processes. This method has already been successfully applied to solve the various time domain problems with different geometries [1, 9-11]. The accurate solution will reveal peculiarities of nonstationary electromagnetic processes in canonical objects, which will give a possibility to formulate recommendations for applications in new technologies to control electromagnetic radiation.

2. Formulation of the problem and its solution

Consider a 2D initial-boundary value problem of exciting a circular cylinder by an incident beam that is modeled by complex source point (CSP). To describe the fields, the cylindrical system of coordinates ρ, φ, z centered at the cylinder is introduced. The incident beam is generated by an external source $H_0^{(2)}\left(\omega\left|\vec{\rho}-\vec{\rho}_{cs}\right|/\nu\right)$ with time dependence $e^{i\omega_0 t}$, where ν is the phase velocity of background medium. Its position is described by a complex vector $\vec{\rho}_{cs}$ with the Cartesian coordinates [12]

$$\begin{cases} x_{cs} = x_0 + ib\cos\beta \\ y_{cs} = y_0 + ib\sin\beta, \end{cases} \tag{1}$$

where x_0, y_0, b, β are real numbers. In this case, the distance between the source and the point of observation is complex as well $|\vec{\rho} - \vec{\rho}_{cs}| = \sqrt{(x - x_{cs})^2 + (y - y_{cs})^2}$. The real point (x_0, y_0) corresponds to the center of the beam waist (Fig. 1). The beam width is controlled by the parameter $b\omega/c$ [13-15], and the beam direction is defined by the angle β. For the situation depicted in Fig. 1, the value of β is π. Using the addition theorem for Hankel functions, the incident field of CSP can be presented in the following form:

$$H_0^{(2)}\left(\omega|\rho - \rho_{cs}|/\nu\right) = \\ = \sum e^{ik(\varphi-\varphi_{cs})}\left[J_k\left(\omega\rho_{cs}/\nu\right)H_0^{(2)}(\omega\rho/\nu)\Theta\left(\rho - |\rho_{cs}|\right) + \right. \\ \left. + J_k\left(\omega\rho/\nu\right)H_0^{(2)}(\omega\rho_{cs}/\nu)\Theta\left(|\rho_{cs}| - \rho\right)\right] \tag{2}$$

where $\rho_{cs} = \sqrt{\rho_0^2 - b^2 + 2ib(x_0\cos\beta + y_0\sin\beta)}$, $\rho_0^2 = x_0^2 + y_0^2$, $\varphi_{cs} = \arccos((x_0 + ib\cos\beta)\rho_{cs}^{-1})$, $\Theta(...)$ is the Heaviside unit function.

The scattered and transmitted fields are expanded, respectively, as follows:

$$U^{E,H} = \sum_{k=-\infty}^{\infty} A_k^{E,H} J_k\left(\omega_0\rho/v_1\right)e^{ik(\varphi-\varphi_{cs})}, \text{ if } \rho < a, \tag{3}$$

$$V^{E,H} = \sum_{k=-\infty}^{\infty} B_k^{E,H} H_0^{(2)}\left(\omega_0\rho/v\right)e^{ik(\varphi-\varphi_{cs})}, \text{ if } \rho > a. \tag{4}$$

Here, $U^{E,H}$ or $V^{E,H}$ represents z-coordinate of the electric or magnetic field for E- or H-polarized fields. Unknown expansion coefficients $A_k^{E,H}$ and $B_k^{E,H}$ are found from the boundary conditions, which requires continuity of tangential components of the total electric and magnetic fields. Assuming the external position of CSP ($\rho_0 > a$), the unknown coefficients can be obtained in the following form:

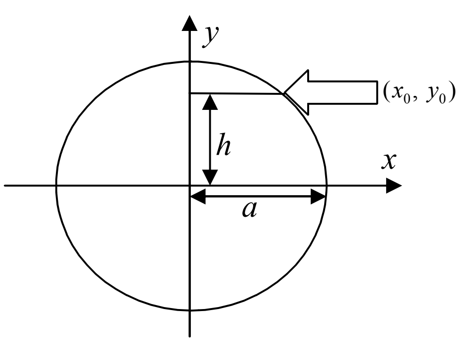

Fig. 1. Schematic diagram of the problem.

$$A_k^{E,H} = \frac{2}{\pi}\frac{vi}{\omega_0 a} \times \\ \times \frac{H_k^{(2)}(\omega_0\rho_{cs}/v)}{\alpha^{E,H} J_k'\left(\omega_0 a/v_1\right)H_0^{(2)}\left(\omega_0 a/v\right) - J_k\left(\omega_0 a/v_1\right)H_0^{(2)'}\left(\omega_0 a/v\right)}, \tag{5}$$

$$B_k^{E,H} = \\ \frac{J_k\left(\omega_0 a/v_1\right)J_k'\left(\omega_0 a/v\right) - \alpha^{E,H} J_k'\left(\omega_0 a/v_1\right)J_k\left(\omega_0 a/v\right)}{\alpha^{E,H} J_k'\left(\omega_0 a/v_1\right)H_0^{(2)}\left(\omega_0 a/v\right) - J_k\left(\omega_0 a/v_1\right)H_0^{(2)'}\left(\omega_0 a/v\right)} \times \\ \times H_k^{(2)}(\omega_0\rho_{cs}/v), \tag{6}$$

$$\alpha^{E,H} = \begin{cases} \sqrt{\varepsilon_1/\varepsilon}, & E\text{-pol}, \\ \sqrt{\varepsilon/\varepsilon_1}, & H\text{-pol}. \end{cases} \tag{7}$$

The prime here represents full differentiation with respect to the argument. Fig. 2 represents a near-field portrait of the CSP beam passing through a circular lens. The values of the parameters are: $b\omega/c = 1$, $x_0/a = 2$, $y_0/a = 0.5$, $\omega_0 a/c = 20\pi$, the refractive index of the material is $n = \sqrt{\varepsilon_1} = 1.5$.

Suppose that at zero moment of time, the dielectric permittivity value inside the cylinder changes from ε_1 to ε_2 in response to some external source. The transformation of the initial CSP field caused by time change of the medium is going to be studied with a particular emphasis on the transient processes and steady-state regime occurring in such a simple dynamic lens. It means that we have an initial-boundary value problem, where transformed fields have to satisfy the wave equations

$$\Delta W(\vec{\rho}, t) - 1/v_2^2 \cdot \partial_{tt}^2 W(\vec{\rho}, t) = 0, \quad \rho < a, \tag{8}$$

$$\Delta W(\vec{\rho}, t) - 1/v^2 \cdot \partial_{tt}^2 W(\vec{\rho}, t) = 0, \quad \rho > a. \tag{9}$$

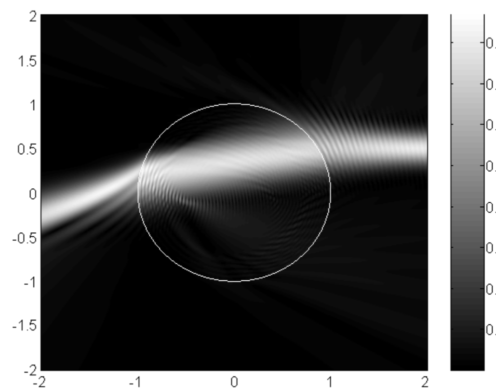

Fig. 2. CSP beam passage through a circular dielectric lens (near-field distribution).

Here, W represents the electric E_z or magnetic H_z field components after zero moment of time, and

$$\Delta = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} .$$

The corresponding initial conditions are provided by continuity of the electric flux density $\vec{D}$ and the magnetic flux density $\vec{B}$ at zero moment of time. In the transient area inside the cylinder, they have the following form [1]

$$E(t=0^+) = \varepsilon_1/\varepsilon_2\, E(t=0^-) ,$$
$$\partial_t E(t=0^+) = \varepsilon_1/\varepsilon_2\, \partial_t E(t=0^-) ; \qquad (10)$$
$$H(t=0^+) = H(t=0^-) ,$$
$$\partial_t H(t=0^+) = \varepsilon_1/\varepsilon_2\, \partial_t H(t=0^-) . \qquad (11)$$

In the steady-state outer region, initial conditions are:

$$E(t=0^+) = E(t=0^-) ,\ \partial_t E(t=0^+) = \partial_t E(t=0^-) ; \qquad (12)$$

$$H(t=0^+) = H(t=0^-) ,$$
$$\partial_t H(t=0^+) = \partial_t H(t=0^-) . \qquad (13)$$

The Laplace transform $L(p) = \int\limits_0^\infty W(t) e^{-pt} dt$ is employed directly to the wave Eqs (8), (9). It follows from the previous works [9-11] that the solution has to be written in the form:

$$L^{E,H} = \frac{v_2^2 (p + i\omega_0)}{p^2 v_1^2 + \omega_0^2 v_2^2} U^{E,H} + \sum_{k=-\infty}^{\infty} C_k^{E,H} I_k\left(n_2 \frac{p}{c}\rho\right) e^{ik(\varphi - \varphi_{cs})} , \qquad (14)$$

$$L^{L,H} = \frac{1}{p - i\omega_0} V^{E,H} + \sum_{k=-\infty}^{\infty} D_k^{E,H} K_k(p\rho/c) e^{ik(\varphi-\varphi_{cs})} . \qquad (15)$$

The unknown coefficients can be found using the boundary conditions:

$$C_k^{E,H}(p) = A_k \frac{p(v_1^2 - v_2^2)}{(p - i\omega_0)(v_1^2 p^2 + \omega_0^2 v_2^2)} \frac{P_k^{E,H}}{R_k^{E,H}} , \qquad (16)$$

$$D_k^{E,H}(p) = A_k \frac{p(v_1^2 - v_2^2)}{(p - i\omega_0)(v_1^2 p^2 + \omega_0^2 v_2^2)} \frac{Q_k^{E,H}}{R_k^{E,H}} , \qquad (17)$$

$$P_k^E = \sqrt{\varepsilon_1/\varepsilon}\,\omega_0 J_k'(\omega_0 a/v_1) K_k(\omega_0 a/v) - p J_k(\omega_0 a/v_1) K_k'(\omega_0 a/v) , \qquad (18)$$

$$P_k^H = -i\omega_0 J_k'(\omega_0 a/v_1) K_k(\omega_0 a/v) - ip\sqrt{\varepsilon/\varepsilon_1} J_k(\omega_0 a/v_1) K_k'(\omega_0 a/v) , \qquad (19)$$

$$Q_k^E = \sqrt{\varepsilon_1/\varepsilon}\,\omega_0 J_k'(\omega_0 a/v_1) I_k(pa/v_2) - \sqrt{\varepsilon_2/\varepsilon}\, p J_k(\omega_0 a/v_1) I_k'(pa/v_2) , \qquad (20)$$

$$Q_k^H = -i\sqrt{\varepsilon/\varepsilon_1}\, p J_k'(\omega_0 a/v_1) I_k(pa/v_2) - i\sqrt{\varepsilon/\varepsilon_2}\,\omega_0 I_k'(pa/v_2) J_k(\omega_0 a/v_1) , \qquad (21)$$

$$R_k^{E,H} = \beta^{E,H} I_k'(pa/v_2) K_k(pa/v) - I_k(pa/v_2) K_k'(pa/v) , \qquad (22)$$

$$\beta^{E,H} = \begin{cases} \sqrt{\varepsilon_2/\varepsilon}, & E\text{-pol}, \\ \sqrt{\varepsilon/\varepsilon_2}, & H\text{-pol}. \end{cases} \qquad (23)$$

Behaviour of the back transformed function is defined by its singular points, hence the inverse Laplace transform can be evaluated by calculation of the residues at its poles and the integral along the branch cut. The functions (14) and (15) involve an infinite number of simple poles associated with the eigenfrequencies of the cylinder and given by zeros of the functions $R_k^{E,H}$ (22). Besides, apart from these singularities there is another one associated with the carrier frequency ω_0 of the incident beam. The functions (14) and (15) also have the branch point at $p = 0$, therefore a branch-cut should be introduced, for instance, along the negative $\mathrm{Re}(p) = 0$ axis. All residues can be calculated analytically, while the integral along the branch cut should be calculated numerically. After permittivity change, the eigenmodes of the cylinder are excited, which causes transients. It should be mentioned that all eigenfrequencies are complex valued, so this 'ringing' is observable during limited time interval. Here, the electromagnetic field is defined by the residue at the the singular point $p = i\omega_0$ and coincides with the harmonic source field in the cylinder with dielectric permittivity ε_2. So, transformation of the near-field distribution during the transient period is observable. However, in the steady-state regime the field behaviour is determined only by the residue at $p = i\omega_0$.

3. Numerical results

Being based on the above approach, controlling the deflection angle of the beam passing through the isolated dielectric cylinder by adjusting the material parameters in time should be demonstrated. In what follows, with no loss of generality, the *E*-polarized fields are estimated numerically. Fig. 3 shows the radiation pattern of the CSP beam passing through the cylinder. It is evident that the angle of deflection crucially depends on the position of the incident beam (the value of h in Fig. 1). It is seen

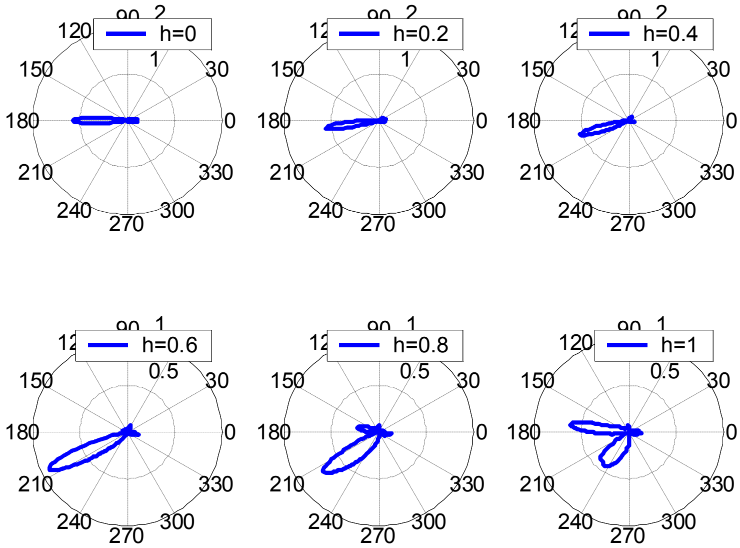

Fig. 3. CSP beam passage through a circular dielectric lens (far-field distribution).

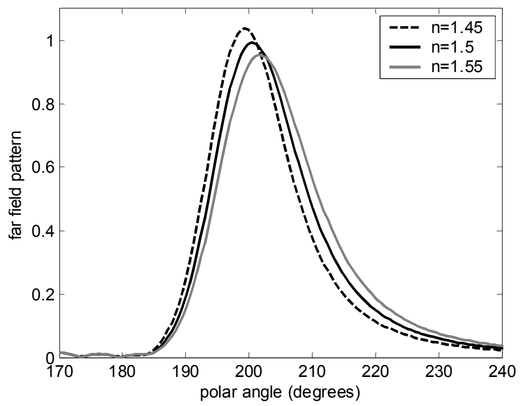

Fig. 4. Dependence of the deflection angle on the value of the refractive index inside the cylinder.

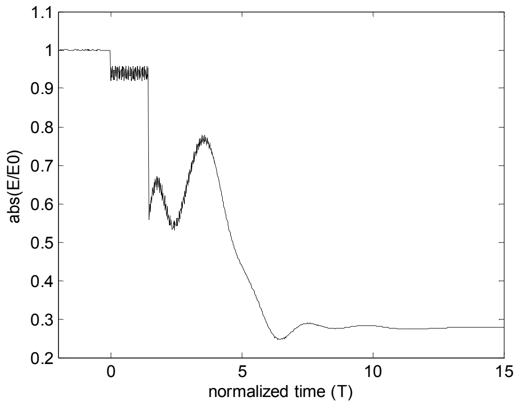

Fig. 5. Time representation of the electric field inside the cylinder. The refractive index changes from the value 1.4 up to 1.45.

that the widest deflection angle is observable for the value of h/a close to 0.5. An assumption that the media parameters are tuned in time makes it possible to control the angle of deflection. Thus, Fig. 4 represents the far-field distribution for different values of the refractive index of the lens. It is seen that the rise of the refractive index increases the angle of deflection and vice versa. To estimate the duration of the transient period, the time domain representation of the field was obtained. Fig. 5 shows the dependence of the absolute value of the total electric field normalized by amplitude of the incident beam on the normalized time ($T = tc/a$). The coordinates of the observation point are: $x/a = -0.5$, $y/a = 0$. All the beam parameters are the same as in Fig. 2. The initial value of the refractive index is 1.4 that at zero moment of time changes to the value 1.45. It should be noted that although this large jump of permittivity has not been yet attained in practice, it is used here to trace the dynamics of the phenomena in a clearer manner. Before zero moment of time, the incident field is presented. After zero moment of time, the field inside the cylinder is represented by the first term in Eq. (14). The initial wave is split into two waves: the time-transmitted and time-reflected ones with the shifted frequency $\omega_0 v_2/v_1$. Within the time interval $0 < T < n_2(1-\rho/a)$, these two waves are observable. The second term in (14) describes the influence of the transient boundary and demonstrates some time delay (for more information see e.g. [9–11]). Fig. 5 illustrates the moment of time $T > n_2(1-\rho/a)$ that from the physical viewpoint corresponds to the moment when the wavefront from the transient boundary reaches the point of observation. The transformed field involves the whole spectrum, which is given by the poles of the function (22). However, it is seen that, after short period of transients, the steady-state regime is achieved, which corresponds to the new position of the deflected angle.

4. Conclusions

The temporal analysis of the electromagnetic field transformed by the time change of the permittivity in the dielectric cylinder illuminated by a harmonic CSP beam is carried out. This analysis is based on the exact formulas for the interior and exterior fields that are obtained as the solutions of the initial-boundary value problem.

The theory is based on the eigenfunction expansion in the Laplace transform domain and the solution inversion into the time domain through evaluation of residues. The obtained results indicate a possibility of using this simple lens model for very fast beam control.

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