

## The baric changes of the refractive properties of $K_2SO_4$ crystals

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The dependencies of the birefringence  $\Delta n_i$  of  $K_2SO_4$  crystals in wide spectral (250–800 nm) ranges, as well as the effect of uniaxial mechanical stress on them have been studied. The values of  $n_i$  for  $K_2SO_4$  are found to be rather sensitive to the uniaxial stress. The spectral characteristics of the refractive indices  $n_i$  of free and mechanically clamped  $K_2SO_4$  crystals have been studied. The refractive indices turned out to increase under the effect of uniaxial stress. The variations in the refraction, electron polarizability and the position of the center of the effective ultraviolet absorption band have been calculated. It is shown that the positions of the isotropic points in these crystals can be controlled by thermal and spectral methods.

**Key words:** *birefringence, refractive indices, uniaxial pressure, phase transition, isotropic state, structure, energy band*

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### 1. Introduction

Crystals  $K_2SO_4$  (PS) – are classic ferroelastics. The phase transition (PT) at 860 K from paraelastic to ferroelastic phase of space group symmetry  $D_{2h}^{16}$  ( $c_0 = 7.48 \text{ \AA}$ ,  $b_0 = 10.07 \text{ \AA}$ ,  $a_0 = 5.76 \text{ \AA}$ ,  $Z = 4$ ) was revealed [1–4]. In spite of a certain interest in PTs taking place in  $K_2SO_4$  crystals [5,6], their optical properties have not been studied well enough. There are practically no data in the literature concerning the effect of uniaxial stress on the optical indicatrix of these crystals. Although this stress does not generally change the crystal symmetry without affecting the quantities under investigation, they make it possible to selectively modify certain bond groups and certain structural entities of the crystal.

The study of the temperature and spectral characteristics of the refractive indices  $n_i$  of the  $A_2BX_4$  [ $K_2SO_4$ ,  $Rb_2SO_4$ ,  $LiKSO_4$ , and  $(NH_4)_2BeF_4$ ] crystals made it possible to establish the isotropic points, i.e. the intersection of the dispersion curves of the refractive indices  $n_i$  at certain temperatures and wavelengths. The  $Rb_2SO_4$  crystals have three isotropic points. In the short wavelength range of the spectrum, the isotropic points slightly depend on temperature with  $|\frac{\partial \lambda_{0y}}{T}| < |\frac{\partial \lambda_{0x}}{T}|$ , where, like in the long wavelength range, the shifts of the isotropic points are quite pronounced, especially along the  $Y$  – direction [7]. The  $(NH_4)_2BeF_4$  crystals also have two isotropic points [8]. In the  $LiKSO_4$  crystals, a decrease of the temperature causes the shift of the point of the intersection of the  $n_x$  and  $n_z$  curves toward the long wavelength range of the spectrum [4]. The  $K_2SO_4$  crystals have two isotropic points in the spectral range 250–850 nm as well as demonstrate the property of *isospectrality* [8].

The aim of this work is to study the birefringence  $\Delta n_i$  and the principal refractive indices  $n_i$  of  $K_2SO_4$  crystals in a wide spectral (300–800 nm) range, as well as the effect of uniaxial mechanical stress applied along the main crystallophysical directions and the bisectors of the angles formed by these directions on the birefringence properties and on the behavior of the isotropic points.

## 2. Experiments

$\text{K}_2\text{SO}_4$  crystals were grown by a slow evaporation of an aqueous solution. The grown crystals were of good optical quality and were of an orthorhombic shape. The spectral dependencies of the  $\Delta n_i$  were measured using the interference technique.

To define the birefringence, a photographic recording of the interference pattern in the focal plane of a DFS-8 spectograph was used, which ensured a spatial resolution of extrema of various orders, excluded “smeared” patterns, and therefore allowed for their independent registration. A transmittance of the polarizing system, composed of crossed polarizers, with a specimen being installed between them in a diagonal position and normally to the incident light beam, is determined according to the relation

$$I = I_0 \sin^2 \left( \frac{\pi(n_i - n_j)d}{\lambda} \right), \quad (1)$$

where  $I_0$  and  $I$  are intensities of the incident and transmitted light beams, respectively,  $\lambda$  is a wavelength, and  $n_i - n_j$  is the birefringence and  $d$  is a crystal thickness in the direction of the incident light beam. The extremum positions satisfy the relation

$$d(n_i - n_j) = k\lambda \quad (2)$$

where  $k$  is an interference order.

Under the variation of the temperature and due to the  $d(T)$ - and  $n_i(T)$ -dependencies, the positions of interference extrema will shift, and the birefringence will be determined by the expression

$$\Delta n_i(\lambda, T) = \frac{k\lambda}{d_i(T)}. \quad (3)$$

An uniaxial stress was developed making use of a special attachment to a nitrogen cryostat and an electric furnace, which allowed us to obtain a stress of about 200 bar. Assuming the mechanical stress, the birefringence is determined through the expression

$$\Delta(T, \lambda, \sigma) = \frac{k(T, \sigma)}{d(T)} \Delta n_i(\lambda, T) = \frac{k\lambda}{d_i(T)}. \quad (4)$$

The temperature variations of  $\Delta n$  and  $d$  were investigated independently for unstressed and stressed specimens. By varying either of the parameters ( $T$  or  $\sigma$ ), with another one being fixed, one can unambiguously determine the temperature or baric dependence of  $\Delta n$ .

The principal refractive indices were determined using the immersion Obreimov method at room temperature.

## 3. Results and discussion

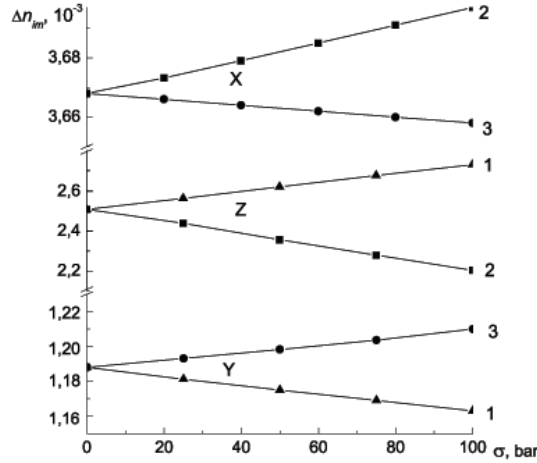
### 3.1. Effect of the uniaxial stress on the birefringence

The dispersion of the  $\Delta n$  is normal in the whole spectral interval of investigation. A comparison of  $\Delta n$  with the corresponding values for  $\text{Rb}_2\text{SO}_4$  and  $\text{LiKSO}_4$  isomorphous crystals showed that the cation substitution  $\text{Li}^+ \rightarrow \text{K}^+$  in the sublattice causes a birefringence growth by 0.008-0.010 on the average, while the  $\text{Li}^+ \rightarrow \text{K}^+$  and  $\text{Rb}^+ \rightarrow \text{Li}^+$  substitutions cause an average decrease of  $\Delta n_i$  by 0.02 and 0.03, respectively. The results are primarily due to the variations of the  $n_i$ , to the shifts of UV and IR absorption bands, as well as due to the changes of relevant additive refractions and electron polarizability.

The values of  $\Delta n_i$  for this crystal are sensitive to the uniaxial stress and change almost linearly with the stress as follows:  $\Delta n_z$ :  $\delta(\Delta n_z) = +0.15 \cdot 10^{-4}$  and  $-1.80 \cdot 10^{-4}$  (for  $\sigma_x$  and  $\sigma_y = 100$  bar, respectively). In general, although  $\sigma_m$  stress affects the  $\Delta n_i$  values, it does not substantially change their dispersion.

The analogous dependencies were observed for value  $\Delta n_x$  at pressures  $\sigma_y$  and  $\sigma_z$ :  $\delta(\Delta n_x) = 1.58 \cdot 10^{-4}$  and  $-1.92 \cdot 10^{-4}$  (for  $\sigma_y$  and  $\sigma_z = 100$  bar, respectively).

It was found that if the uniaxial stress is X-directed, then  $\Delta n_z$  increases and  $\Delta n_y$  decreases, for Y- directed, then  $\Delta n_x$  increases and  $\Delta n_z$  decreases (figure 1). Since the relations  $n_x > n_z > n_y$  and  $\Delta n_x = n_z - n_y$ ,  $\Delta n_y = n_x - n_z$  and  $\Delta n_z = n_x - n_y$  of  $K_2SO_4$  crystals were established, the increase of the anisotropy of optical indicatrix under  $\sigma_m$  uniaxial stress action was observed.



**Figure 1.** The stress dependencies of birefringence of  $K_2SO_4$  crystal at room temperature for  $\lambda = 500$  nm: 1 –  $\sigma_x$ , 2 –  $\sigma_y$ , 3 –  $\sigma_z$ .

By extrapolation of lines  $\Delta n_i = f(\sigma_m)$  and by solving the solution  $\Delta n_{z(\sigma=0)} + b_1\sigma_x = \Delta n_{x(\sigma=0)} - b_2\sigma_z$  it was determined that  $\Delta n_z \sim \Delta n_x \sim 3.49 \cdot 10^{-3}$  at  $\sigma_z \sim \sigma_x \sim 1.66$  kbar. It means that the application of the uniaxial stress leads to the appearance of the pseudo-isotropic point of  $K_2SO_4$  crystal.

### 3.2. Refractive indices

The variations in the refractive indices of the  $K_2SO_4$  crystals were studied through analyzing the effect of uniaxial  $\sigma_m$  on  $\Delta n_i$ .

It has been established that the dispersion of the  $n_i$  has no anomalies and is well described by the single-oscillator Sellmeier formula

$$n_i^2(\lambda, T) = 1 + \frac{B_i(T)\lambda_{0i}^2(T)\lambda^2}{\lambda^2 - \lambda_{0i}^2(T)}, \quad (5)$$

where  $B_i$  is the parameter related to the strength, effective mass, and charge of UV-oscillators and  $\lambda_{0i}$  is the position of the effective center of the UV absorption band. Using the experimental dependencies  $n_i(\lambda)$  and the well-known Lorenz-Lorentz formula,

$$\frac{(n_i^2 - 1)}{(n_i^2 + 2)} = \frac{4}{3}\pi N_0\alpha_i = (\rho/\mu)R_i \quad (6)$$

we calculated the effective parameters of the Sellmeier formula, electron polarizability  $\alpha_i$ , and specific refractivity  $R_i$  of the crystals (table 1) ( $\rho$  and  $\mu$  are the density and the molar mass of the crystal and  $N_0$  is the Avogadro number). The comparison with the analogous characteristics of the isomorphous  $Rb_2SO_4$  crystal showed that the cation substitution  $K^+ \rightarrow Rb^+$  causes an increase of the  $n_i$  by 0.004–0.007, the  $\alpha_i$  by  $(2 - 4) \cdot 10^{-25} \text{ cm}^3$ , and  $R_i$  by 0.8–1.0  $\text{cm}^3$ .

Using the data on the spectral variations of  $\Delta n_i$  under the pressure  $\sigma_m$ , we calculated the spectral dependencies of the piezooptical constants of the  $K_2SO_4$  crystals in the spectral ranges of 300–700 nm, respectively, by the formula

$$\pi_{im}^0 = 2\delta\Delta n_i/\sigma_m + 2\Delta n_i s_{im}, \quad (7)$$

where  $\delta n_i$  is the induced change in birefringence for the light propagating along the  $i$ -axis,  $s_{im}$  are the coefficients of elastic compliance. The second term in formula (7) describing the variations in the geometric dimensions of the crystal under pressure was ignored, because its contribution to the change of  $\pi_{im}$  did not exceed 2–5%, i.e., it was comparable with the experimental accuracy. Using the well-known expressions for piezobirefringence of orthorhombic crystals (the traditional Pockels method [9]) we calculated the spectral characteristics of the absolute piezooptical constants of the  $K_2SO_4$  crystals by solving the systems of equations with nine unknowns  $\pi_{im}$

$$\delta \Delta n_i^m = \frac{1}{2} (n_m^3 \pi_{mm} - n_k^3 \pi_{km}), \quad (i, k, m = 1, 2, 3), \quad (8)$$

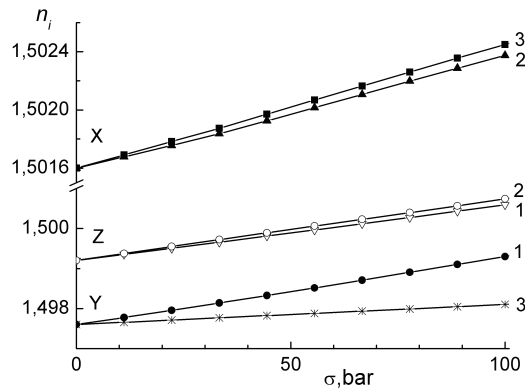
where  $\delta \Delta n_i^m$  is the change in birefringence along the  $i$ -direction under uniaxial stress applied along the  $m$ -axis, and  $n_k$  and  $n_m$  are the absolute values of the refractive indices of a free  $K_2SO_4$  crystal.

The analysis of the variations of the principal  $n_i$  of  $K_2SO_4$  crystals was performed using the piezooptic constants and the formula

$$n_i(\lambda, \sigma) = n_{i0}(\lambda) - \frac{1}{2} \pi_{im}(\lambda) \sigma_m n_{i0}^3(\lambda), \quad (9)$$

where  $n_{oi}(\lambda)$  are the spectral dependencies of the refractive indices for a mechanically free crystal.

Figure 2 shows the stress dependence of the principal  $n_i$  for  $\lambda = 500$  nm and various directions of the applied mechanical stresses at room temperature. The  $n_i$  increase with stresses application. In the UV spectrum range, the  $n_i$  are very sensitive to pressures, which is explained by a considerable dispersion of the absolute piezooptic constants.



**Figure 2.** The stress dependencies of the refractive indices of  $K_2SO_4$  crystal at  $T_{room}$  and  $\lambda = 500$  nm: 1 –  $\sigma_x$ , 2 –  $\sigma_y$ , 3 –  $\sigma_z$ .

For the light propagating along the X-, Y-, and Z-axes, the dependencies  $n_i(\lambda, \sigma)$  and formula (6) provide the calculation of the changes in  $\alpha_i$  and  $R_i$  of the crystal (table 1).

It is established that the pressures up to  $\sigma_m \sim 200$  bar raise the  $R_i$  (on the average by  $\sim 1E2 \cdot 10^{-25} \text{ cm}^3$ ) as well as the  $\alpha_i$  of the  $K_2SO_4$  crystal.

Probably, uniaxial pressures change the degree of ordering of the crystal and lead to a slight deformation and rotation of the framework formed by  $SO_4^{2-}$ -tetrahedra and to displacements of  $K^+$  ions, which can be seen from the changes in  $\alpha_i$  and  $R_i$ . Using the dependencies  $n_i(\lambda, \sigma)$  and formula (5), we also calculated the baric dependencies of the parameters of ultraviolet ( $\lambda_{0i}$ ,  $B_i$ ) oscillators (table 1).

It has been established that an increase of the  $n_i$  ( $\partial n / \partial \sigma \cong 5 \cdot 10^{-6} \text{ bar}^{-1}$ ) under pressure is caused by a decrease of the strength of the effective UV oscillator and by the displacement of the center of the effective UV absorption band to the long wavelength range ( $\partial \lambda_{0i} / \partial \sigma \cong 0.5E2 \cdot 10^{-2} \text{ \AA} / \text{bar} \approx 15 \cdot 10^{-5} \text{ eV} / \text{bar}$ ).

Indeed, upon differentiation of (5), we arrive at

$$\frac{dn}{d\sigma_m} \approx \frac{1}{2n} \frac{\lambda^2 \lambda_{0i}^2}{\lambda^2 - \lambda_{0i}^2} \left( \frac{dB_i}{d\sigma_m} + \frac{B_i}{\lambda_{0i}} \frac{d\lambda_{0i}}{d\sigma_m} \right). \quad (10)$$

**Table 1.** The stress dependencies of the electron polarizability  $\alpha_i$ , specific refractivity  $R_i$  ( $\lambda = 500$  nm) and effective parameters of Sellmeier formula for K<sub>2</sub>SO<sub>4</sub> crystals at  $T = 294$  K.

Parameters	$\sigma_i = 0$ bar	$\sigma_x = 200$ bar	$\sigma_y = 200$ bar	$\sigma_z = 200$ bar
$\alpha_x \times 10^{24}, \text{nm}^3$	7.642	7.662	7.682	7.683
$\alpha_y \times 10^{24}, \text{nm}^3$	7.588	7.620	7.592	7.612
$\alpha_z \times 10^{24}, \text{nm}^3$	7.608	7.612	7.632	7.611
$R_x, \text{nm}^3$	19.402	19.421	19.423	19.451
$R_y, \text{nm}^3$	19.275	19.298	19.279	19.289
$R_z, \text{nm}^3$	19.302	19.321	19.339	19.330
$\lambda_{0x}, \text{nm}$	91.130	91.185	91.235	91.256
$\lambda_{0y}, \text{nm}$	90.329	91.392	90.521	91.026
$\lambda_{0z}, \text{nm}$	90.332	90.832	91.523	91.421
$B_x \times 10^6, \text{nm}^{-2}$	146.175	146.021	146.002	145.992
$B_y \times 10^6, \text{nm}^{-2}$	147.527	147.532	147.529	148.023
$B_z \times 10^6, \text{nm}^{-2}$	148.033	149.001	148.563	148.365

The baric displacement of the effective UV oscillator is very large. It showed that UV oscillator is situated in a high energy spectrum region.

Thus, studying the spectral dependencies of the refractive indices of free K<sub>2</sub>SO<sub>4</sub> crystals mechanically clamped by the stresses along the main crystallophysical directions, we established that the refractive indices  $n_i$  increase because the applied pressure effects the electronic subsystem of the crystal.

We also calculated the variations in the electron polarizability, refraction and the parameters of the UV oscillators of free and mechanically clamped K<sub>2</sub>SO<sub>4</sub> crystals. It is seen that uniaxial pressures shift the center of the effective absorption band to the visible range of the spectrum, which seems to be associated with a decrease of the band gap.

It has been found that the birefringence is rather sensitive to the uniaxial stresses applied along the principal crystallophysical axes. Those stresses, applied along mutually perpendicular crystallophysical axes,  $\sigma_{i,m}$ , result in the changes  $\Delta n_j$  different in signs and values ( $i, j, m = 1, 2, 3$ ).

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## Баричні зміни рефрактивних параметрів кристалів $K_2SO_4$

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Досліджено залежності двопронезаломлення  $\Delta n_i$  кристалів  $K_2SO_4$  в широкому спектральному (250–800 нм) діапазоні під впливом одновісних тисків. Виявлено, що величини  $n_i$  кристалів  $K_2SO_4$  є досить чутливими до впливу одновісних тисків. Досліджено спектральні залежності показників заломлення  $n_i$  вільних та механічно затиснутих кристалів  $K_2SO_4$ . Встановлено, що показники заломлення зростають під дією одновісних тисків. Розраховано зміни рефракцій, електронної поляризованості та положень центрів ефективних ультрафіолетових смуг поглинання. Показано, що положення ізоотропних точок в цих кристалах можна змінювати температурним та спектральним методами.

**Ключові слова:** двопронезаломлення, показник заломлення, фазовий перехід, ізоотропний стан, енергетична смуга

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