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Studying anisotropic properties of longitudinal inhomogeneous nondepolarizing media with elliptical phase anisotropy

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Abstract. In this paper, basing on the anisotropic properties of longitudinal inhomogeneous nondepolarizing media with linear phase anisotropy, more general type of media with elliptical phase anisotropy was studied. The features of propagation of light with privileged states of polarization were observed. Transformation of polarization states of eigenwaves along *z* axes of light propagation was studied. The orthogonalization properties inherent to this type of medium were obtained. Evolution of linear polarized light in this media was presented and discussed.

Keywords: Jones method, birefringence, spectral problem, longitudinally inhomogeneous medium.

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1. Introduction

The history of investigation of longitudinal inhomogeneous nondepolarizing media returns us to Ref. [1]. The study of anisotropic properties of this type of media in time of publication [1] was difficult as caused by the complexity of the mathematical apparatus used. This problem was solved using the Jones and Mueller matrix methods [2, 3]. For the first time, the Jones matrix of longitudinal inhomogeneous anisotropic medium was presented in [4]. R. Azzam made the next step in [5], where he considered the type of longitudinal inhomogeneous medium with linear phase anisotropy. In particular, in [6-8] the examples of these media were considered: cholesteric and twisted nematic liquid crystals. However, the studies of anisotropic properties of this type of media were made only partially because of complexity of these objects. Specifically, such properties as the solution of spectral problem and orthogonalization properties were missed.

Development of the modern display technology results in the fact that these types of liquid crystals are very widely used [9-11]. Optical anisotropy of these media is in the basis of using various liquid crystals in computer displays and indicator devices. Therefore, the study of anisotropic properties of longitudinal inhomogeneous anisotropic medium is very important problem for display technology. The case of longitudinal inhomogeneous nondepolarizing media with linear phase anisotropy, which in particular includes cholesterics and twisted nematics, was solved in [12, 13]. This paper generalizes the results obtained in [12, 13] for the case of the media characterized by both linear and circular phase anisotropy (elliptic phase anisotropy).

2. Polarimetric models of media

Nondepolarizing longitudinal inhomogeneous medium with elliptical phase anisotropy can be equivalently represented as a sequence of molecular planes that consist of elongated molecules, oriented parallel to each other. At the same time, such molecules possess inherent chirality. Each of these molecular planes can be present as a thin phase plate with elliptical phase anisotropy (elliptical birefringence). The fast (slow) axis of the plate is parallel (perpendicular) to the direction along which directed is the plane of the molecules that is considered, and lies entirely in this plane. In these media, the axis of each following molecular layer is rotated relative to the previous one by some angle α_0 [7]:

$$\alpha_0 = \frac{2\pi}{p}, \tag{1}$$

where p is the step of helical structure in this medium (the shortest distance between the planes with the same orientation of molecules). Then, the molecular orientation of the plane in distance z from the input can be defined as:

$$\alpha = \alpha_0 z . \tag{2}$$

Anisotropic properties of one molecular layer in this medium are described by the differential Jones matrix (in circular basis) [6]:

$$N = \begin{bmatrix} -i\varphi_0 & i\frac{1}{2}\delta_0 e^{2i\alpha_0 z} \\ i\frac{1}{2}\delta_0 e^{-2i\alpha_0 z} & i\varphi_0 \end{bmatrix}. \tag{3}$$

And the anisotropic properties of this longitudinally inhomogeneous medium consisting of a sequence of molecular planes with a longitudinal size z are described by the integral Jones matrix (in linear basis):

$$T = \frac{1}{A}\begin{bmatrix} AC(\alpha_0 z)C\left(\frac{zA}{2}\right) + (i\delta_0 C(\alpha_0 z) + 2(\alpha_0+\varphi_0)S(\alpha_0 z))S\left(\frac{zA}{2}\right) & -AC\left(\frac{zA}{2}\right)S(\alpha_0 z) + (i\delta_0 S(\alpha_0 z) + 2(\alpha_0+\varphi_0)C(\alpha_0 z))S\left(\frac{zA}{2}\right) \\ AC\left(\frac{zA}{2}\right)S(\alpha_0 z) + (i\delta_0 S(\alpha_0 z) - 2(\alpha_0+\varphi_0)C(\alpha_0 z))S\left(\frac{zA}{2}\right) & AC(\alpha_0 z)C\left(\frac{zA}{2}\right) + (-i\delta_0 C(\alpha_0 z) + 2(\alpha_0+\varphi_0)S(\alpha_0 z))S\left(\frac{zA}{2}\right) \end{bmatrix}. \tag{4}$$

In Eqs (3) and (4), δ_0, φ_0 are magnitude of linear and circular birefringence per unit thickness in the direction of light propagation, respectively, and the following replacement is used: $A = \sqrt{\delta_0^2 + 4(\alpha_0 + \varphi_0)^2}$, $C(x) = \cos(x)$, $S(x) = \sin(x)$. Note that the integral matrix (4) can be obtained from the differential (3) by using the vector transfer equation and technique presented in [14].

3. Propagation of radiation with privileged states of polarization

One of the anisotropic properties of longitudinally inhomogeneous media that does not exist in a longitudinally homogeneous media is availability of the privileged polarization states. The term "privileged" was at first introduced in [7]. It means the state of polarization of eigenwaves in longitudinal homogeneous (untwisted) media of this type.

Evolution of polarization states along the z axis of light propagation in anisotropic medium can be described by the first order differential equation for polarization complex variable [6]. For the case of longitudinally inhomogeneous medium with elliptical phase anisotropy, this equation is:

$$d\chi / dz = -i\frac{1}{2}\delta_0 e^{2i\alpha_0 z}\chi^2 + i2\varphi_0\chi + i\frac{1}{2}\delta_0 e^{-2i\alpha_0 z}, \tag{5}$$

where χ is the complex variable that determines the light polarization state. Eq. (5) has a partial solution in the form:

$$\chi_{1,2} = K_{1,2}\, e^{-2i\alpha_0 z}, \tag{6}$$

where $K_{1,2}$ are privileged polarization states of light. Azimuth of orientation coincides with orientation of input molecular plane, and the angle of ellipticity of privileged states is determined as:

$$e_{1,2} = \\ = \arctan\left(\frac{(\pm 2(\alpha_0+\varphi_0)/\delta_0) - 1 + \sqrt{(2(\alpha_0+\varphi_0)/\delta_0)^2 + 1}}{(\pm 2(\alpha_0+\varphi_0)/\delta_0) + 1 + \sqrt{(2(\alpha_0+\varphi_0)/\delta_0)^2 + 1}}\right). \tag{7}$$

These two privileged polarization states $K_{1,2}$ are orthogonal and elliptical in a general case, and transformation into two elliptical eigenpolarization for longitudinally inhomogeneous medium with elliptical phase anisotropy takes place when $\alpha_0, \varphi_0 = 0$.

Eq. (6) describes the change of privileged polarization states of light propagation in the medium along the z axis. Since Eq. (5) is written for the differential Jones matrix in the circular basis (3), the azimuth and angle of ellipticity of polarization $\chi_{p(1,2)}$ can be determined as:

$$\theta_p = \frac{1}{2}\arg\left(\chi_{p(1,2)}\right),\ e_p = \arctan\left(\frac{\chi_{p(1,2)} - 1}{\chi_{p(1,2)} + 1}\right). \tag{8}$$

Graphical representation of (8) is shown in Fig. 1.

From Eq. (6) and Fig. 1, we can see that the azimuths of privileged polarization states are changed by the law $\theta_{p(1,2)} = \alpha_0$, and for each molecular plane they coincide with their principal axes, while the angle of ellipticity remains unchanged and is defined by Eq. (7). As a result of propagation of light with the privileged state of polarization along the z axis, this type of medium is characterized by efficiency of circular phase anisotropy with the magnitude of the relative angle $\varphi_0 = \alpha_0$ (rad/mm).

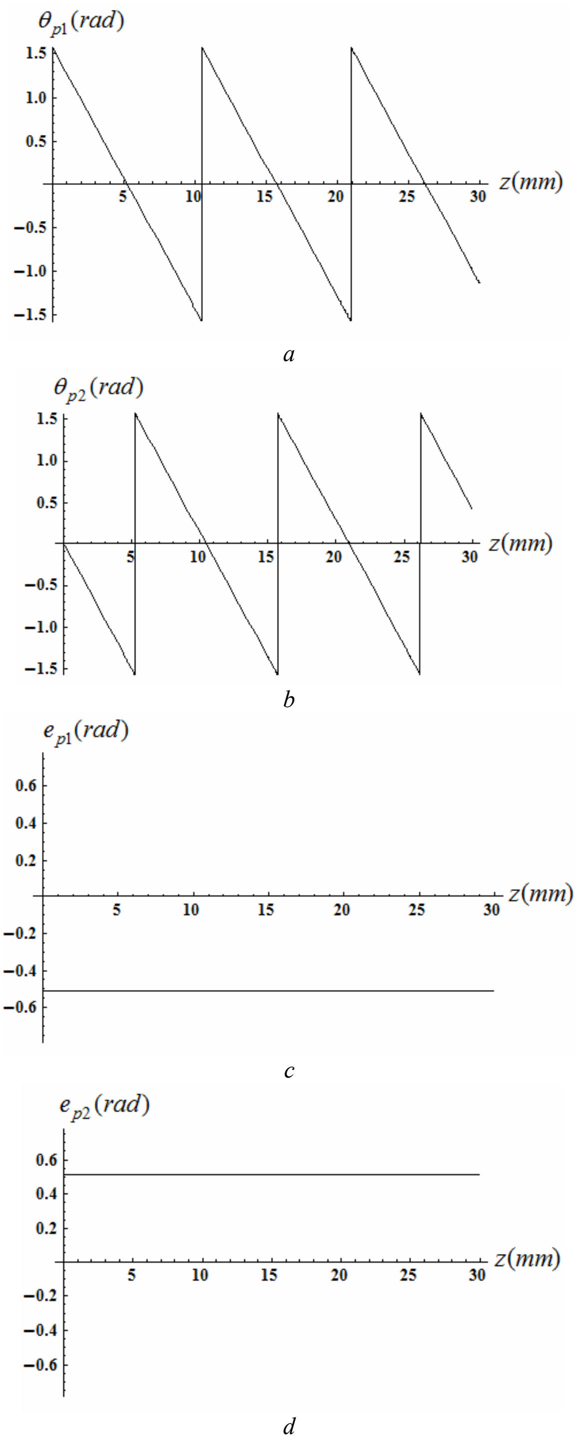


Fig. 1. Evolution of the azimuth and angle of ellipticity in longitudinally inhomogeneous medium with elliptical phase anisotropy and parameters: $\delta_0 = 1.22$ rad/mm, $\alpha_0 = 0.3$ rad/mm, $\varphi_0 = 0.7$ rad/mm; (a), (b) azimuth, (c), (d) angle of ellipticity for polarizations χ_{p1}, χ_{p2}, respectively.

4. The solution of the spectral problem

To analyze the basic anisotropic properties for the medium of this type, we need to find properties of their eigenwaves, i.e. to solve the spectral problem. For this, we use the relations represented in [14], which gives the solutions of the spectral problem for the integral (3) and differential (4) Jones matrices. As a result, eigenpolarization and eigenvalues for these matrices are as follows:

$$V_{e1,2T} = \frac{R_4 \mp \sqrt{A^2 R_2 + R_3 + A R_3^2}}{A},$$
$$V_{e1,2N} = \mp\frac{1}{2}\left(i\sqrt{\delta_0^2 + 4\varphi_0^2}\right); \tag{9}$$

$$\chi_{e1,2T} = \frac{R_1 \pm \sqrt{A^2 R_2 + R_3 + A R_3^2}}{Q},$$
$$\chi_{e1,2N} = \frac{\pm i\sqrt{-\delta_0^2 - 4\varphi_0^2} + \delta_0\cos(2\alpha_0 z)}{2i\varphi_0 + \delta_0 \sin(2\alpha_0 z)}; \tag{10}$$

where the following substitutions are used:

$$R_1 = -i\delta_0 C(\alpha_0 z) S\left(\frac{zA}{2}\right),$$
$$R_2 = \left(C(\alpha_0 z)^2 C\left(\frac{zA}{2}\right)^2 - 1\right),$$
$$R_3 = 4(\alpha_0 + \varphi_0)^2 S(\alpha_0 z)^2 S\left(\frac{zA}{2}\right)^2,$$
$$R_4 = \left(\begin{array}{l} AC(\alpha_0 z) C\left(\frac{zA}{2}\right) + \\ + 2(\alpha_0 + \varphi_0) S(2\alpha_0 z) S\left(\frac{zA}{2}\right) \end{array}\right), \tag{11}$$
$$Q = -AC\left(\frac{zA}{2}\right) S(\alpha_0 z) +$$
$$+ \left(2(\alpha_0 + \varphi_0) C(\alpha_0 z) - i\delta_0 S(\alpha_0 z)\right) S\left(\frac{zA}{2}\right).$$

It follows from relations (9) that, as in the case of longitudinally inhomogeneous medium with linear phase anisotropy [13], the relation $V_{eT} = e^{V_{eN} z}$ (the relation occurs in homogeneous medium [14]) is no longer satisfied. Also, there is the dependence of the eigenvalues of the integral matrix on the angle α_0. However, as in the case of longitudinally homogeneous medium, these eigenvalues are the phase factors, and therefore only change the absolute phase of eigenwaves (their amplitude remains unchanged).

It follows from relation (10) that, like to the case of longitudinally inhomogeneous medium with linear phase anisotropy, the eigenpolarization depends on the value of z. Unlike homogeneous media, the eigenpolarization of the integral and differential Jones matrices are not equal. This leads to the fact that those values change with the coordinate z when light propagates from one to another molecular plane. Changes in the azimuth and angle of ellipticity for eigenpolarizations (basing on equation for azimuth and angle of ellipticity that are presented in [6]) with z values are shown in Fig. 2.

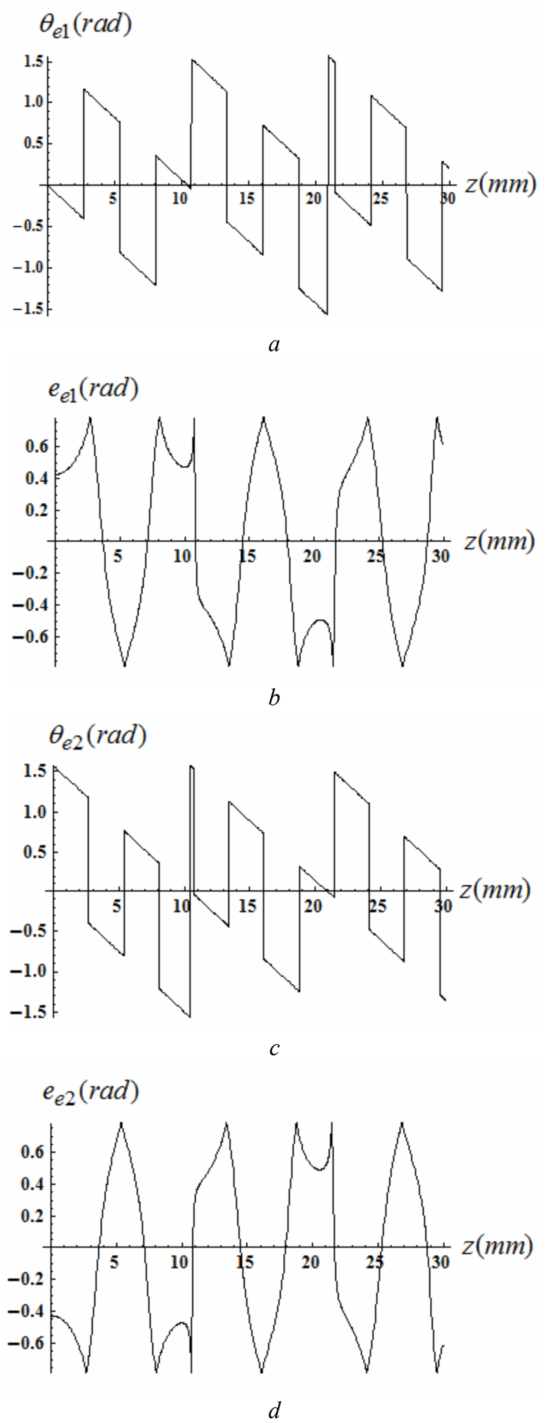


Fig. 2. Changes in the azimuth and angle of ellipticity with coordinate z along the axis of light propagation in longitudinally inhomogeneous medium with elliptical phase anisotropy and parameters δ_0 = 1.22 rad/mm, α_0 = 0.3 rad/mm, φ_0 = 0.7 rad/mm for (a), (b) χ_{e1T} and (c), (d) χ_{e2T} – eigenpolarizations, respectively.

As it follows from Figs 2a and 2c, the azimuths of eigenpolarizations are linear functions of z that are described by the equations: $\theta_{e1} = -\frac{\alpha_0 z}{2}$, $\theta_{e2} = -\frac{\alpha_0 z}{2} + \frac{\pi}{2}$. These linear relationships are broken in two cases. 1) When the angle of ellipticity of eigenpolarization takes the values $\pm\pi/4$ that correspond to the circular polarization for which the concept of the azimuth is degenerated. 2) A jump of azimuth from the maximum (minimum) to the minimum (maximum) value, which corresponds to the mathematical properties of functions arctg and has no physical meaning.

Dependences of the angles of ellipticity for both eigenpolarizations on z are not linear and periodic. It should be also noted that there are z values, at which the angle of ellipticity corresponds to linearly polarized waves. As a result, the integral Jones matrix of inhomogeneous medium coincides with the matrix of the linear phase plate with the value of the linear birefringence [15]: $\delta = \arccos(M_{44})$, and orientation of the axis of birefringence: $\alpha = -\arctan(M_{42}/M_{43})$, which coincides with the azimuth of one of eigenpolarizations. In addition, there are the values for which the angle of ellipticity corresponds to circularly polarized waves. That is, the integral Jones matrix coincides with the matrix of circular phase plate with a value of circular birefringence: $\varphi = \alpha_0 z$.

From the condition $\arccos(M_{44}) = 0$ [15] (which ensures the absence of linear phase anisotropy), we obtain that in this type of medium eigenpolarization becomes circular in the carrying value:

$$z = \frac{2\pi k}{\sqrt{(\alpha_0 + \varphi_0)^2 + \delta_0^2}}, k \in Z, k \neq 0. \quad (12)$$

Similarly, the condition $\frac{1}{2}\arctan\left(\frac{M_{23} - M_{32}}{M_{22} + M_{33}}\right) = 0$ (absence of circular phase anisotropy) yields the following relation that provides linear eigenpolarization:

$$-B_1\left(\delta_0^2 + B_2 \cos B_1\right)\sin(2\alpha_0 z) + \\ +4(\alpha_0 + \varphi_0)B_1 \cos(2\alpha_0 z)\sin(B_1 z) = 0, \quad (13)$$

where $B_1 = \sqrt{4(\alpha_0 + \varphi_0)^2 + \delta_0^2}$, $B_2 = 8(\alpha_0 + \varphi_0)^2 + \delta_0^2$. So, compared to longitudinally homogeneous media, the content of eigenvectors of Jones matrices in longitudinally inhomogeneous case changes. In particular, they describe the polarization of light waves in the input of medium that are reproduced at its output, if it's specific thickness. As a result, these waves cannot be considered as eigenvectors in generally acceptable understanding. But specific monomolecular plane is homogeneous medium, so the eigenvectors of Jones matrices, of course, describe the state of polarization of its eigenwaves.

It also follows from Fig. 2 that eigenpolarization in this type of medium is always orthogonal, which can be checked by direct substitution of Eq. (10) into the condition $\chi\chi^* = -1$ that is transformed into an identity at any values of parameters: δ_0, α_0, φ_0, z. When checking the possibility of degeneration of anisotropy in this type of inhomogeneous medium and substituting elements of the Jones matrix (4) into the condition $\sqrt{(T_{22}-T_{11})^2+4T_{12}T_{21}}=0$, we find that there is no parameter values (δ_0, α_0, φ_0, z), in which the anisotropy becomes degenerate.

5. Orthogonalization properties

Using the notion "orthogonalization properties of media", we mean that for given polarization of input light the polarization of output one is orthogonal.

Implementation of orthogonalization properties of nondepolarizing media that are described by some Jones matrix was considered in [16]. Further, in work [17] it was obtained the relation on anisotropic parameters that provide existence of orthogonalization properties in specific types of media. As a result, for the existence of orthogonalization properties the inequality $F > 0$ should be valid, where F is as follows:

$$F=\left(a_1^2+b_1^2-4c_1\right)\left(a_2^2+b_2^2\right)-\left(a_1a_2+b_1b_2-2c_2\right)^2 . \quad (14)$$

And

$$a_1=\frac{X_{22}(X_{12}+X_{21})+Y_{22}(Y_{12}+Y_{21})}{X_{22}^2+Y_{22}^2},$$

$$b_1=\frac{X_{22}(Y_{21}-Y_{12})+Y_{22}(X_{12}-X_{21})}{X_{22}^2+Y_{22}^2},$$

$$c_1=\frac{X_{11}X_{22}+Y_{11}Y_{22}}{X_{22}^2+Y_{22}^2}, \quad (15)$$

$$a_2=X_{22}(Y_{12}+Y_{21})-Y_{22}(X_{12}+X_{21}),$$

$$b_2=X_{22}(X_{12}-X_{21})+Y_{22}(Y_{12}-Y_{21}),$$

$$c_2=X_{22}Y_{11}-Y_{22}X_{11}.$$

In Eqs. (15), X_{ij}, Y_{ij} are real and imaginary parts of the Jones matrix elements. If $F = 0$, then, for the existence of orthogonalization properties, an additional condition should be satisfied in elements of the Jones matrix. This condition has the form:

$$R_0^2=\frac{a_1^2+b_1^2}{4}-c_1\ge 0 . \quad (16)$$

But in the case when $R_1^2=0$, the intensity of light, the polarization state of which in output becomes orthogonal, equals zero. So, we have to check this feature.

Substituting the elements of Jones matrix (4) for this type of longitudinally inhomogeneous media with elliptical phase anisotropy, as a result we get the value of F as a function of the anisotropy parameters δ_0, α_0, φ_0 and thickness z. This dependence is shown in Fig. 3a for the thickness z = 2 mm and value of parameter of anisotropy $\varphi_0 = \pi/2$ rad/mm. Presented in Fig. 3b is the dependence of F on the parameter of anisotropy α_0 for some values of parameters δ_0, φ_0.

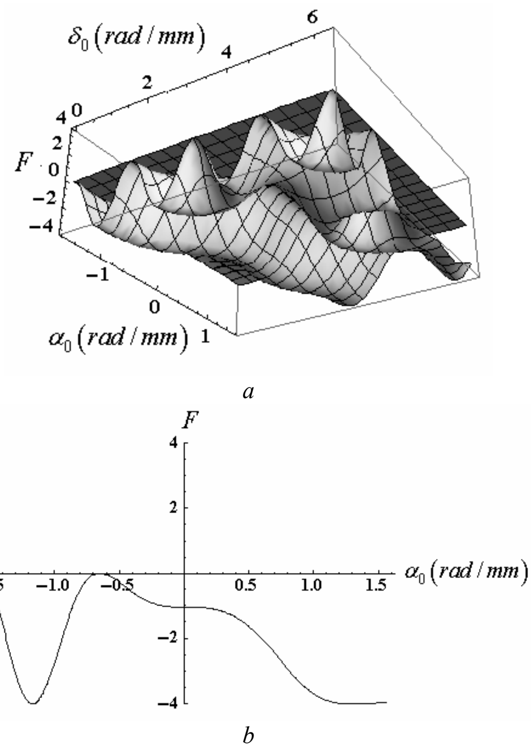


Fig. 3. Dependence of the function F on the anisotropy parameters (a) δ_0, α_0 and curve $F = 0$, (b) α_0 at $\delta_0 = \pi$ rad/mm for a medium with inhomogeneous elliptical phase anisotropy and parameters: $\varphi_0 = \pi/2$ rad/mm, z = 2 mm.

Like to the case of an inhomogeneous medium with linear phase anisotropy, in this type of medium with parameters: $\varphi_0 = \pi/2$ rad/mm, z = 2 mm, there exist several parameters of anisotropy δ_0, α_0 for which the medium might have orthogonalization properties. As an example, Fig. 3b demonstrates the section of curves in Fig. 3b at $\delta_0 = \pi$ rad/mm, where we can find such value $\varphi_0 = \pi/2$ rad/mm that gives $F = 0$. Substituting the above values of anisotropy δ_0, α_0, φ_0 and values z = 2 mm in Eq. (16), we obtain $R_1^2 = 83.3$, and, therefore, at a given set of parameters δ_0, α_0, φ_0, z, the corresponding type of medium characterized by orthogonalization properties.

We can find these states of polarization. To fulfil it, we shall consider the most general case when the input state of polarization is elliptical and described by the Jones vector [6]:

$$E_{inp}=\begin{bmatrix}\cos(\theta_{inp})\cos(e_{inp})-i\sin(\theta_{inp})\sin(e_{inp})\\ \sin(\theta_{inp})\cos(e_{inp})+i\cos(\theta_{inp})\sin(e_{inp})\end{bmatrix}, \quad (17)$$

where θ_{inp}, e_{inp} are the azimuth and angle of ellipticity for the polarization ellipse of input light. Now, apply the basic equation of the Jones matrix method:

$$E_{out}=TE_{inp}, \quad (18)$$

where E_{out} are the Jones vectors of output light. Also, pass from Jones vectors to corresponding complex variables based on the relations $\chi_{inp} = E_{y(inp)} / E_{x(inp)}$, $\chi_{out} = E_{y(out)} / E_{x(out)}$. As a result, we obtain the equation in the form:

$$\chi_{inp}\chi^{*}_{out} = f\left(\delta_0, \alpha_0, z, \theta_{inp}, e_{inp}\right), \qquad (19)$$

where orthogonalization properties sold at

$$\chi_{inp}\chi^{*}_{out} = -1 . \qquad (20)$$

Substituting the above values of parameters of anisotropy and thickness of the medium in Eqs (17) to (20), we find that according to the classification presented in [14] this type of medium in a general case is characterized by continuum of polarization states that are orthogonalized. Examples of these polarizations (solutions of Eq. (5)) are elliptical and linear polarization states with parameters: $\theta_{inp} = 16.81°$, $e_{inp} = 4.04°$, and $\theta_{inp} = 7.7°$, respectively. It should be noted that these states of polarization coincide with the corresponding polarization states for longitudinal inhomogeneous medium with linear phase anisotropy that was described in [12].

6. Evolution of the states of polarization

Now we study the evolution of linear state of polarization along the z axis in a longitudinally inhomogeneous medium with elliptic phase anisotropy. In this case, we use the Mueller matrix method. In this method, an arbitrary linear state of polarization with azimuth θ_{inp} is described by the Stokes vectors:

$$S_{inp} = \begin{pmatrix} 1 & \cos 2\theta_{inp} & \sin 2\theta_{inp} & 0 \end{pmatrix}^{T} . \qquad (21)$$

At the same time, the anisotropic properties of the medium are described by the integral Mueller matrix Eq. (22), which is obtained from the Jones matrix (4) using the method presented in [6].

Anisotropic properties of molecular layer in this method are described by the differential Mueller matrix:

$$m = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2\varphi_0 & -\delta_0 \sin(2\alpha_0 z) \\ 0 & -2\varphi_0 & 0 & \delta_0 \cos(2\alpha_0 z) \\ 0 & \delta_0 \sin(2\alpha_0 z) & -\delta_0 \cos(2\alpha_0 z) & 0 \end{bmatrix} . \qquad (23)$$

Matrix (23) can be obtained from the differentiated integral matrix (22) by using the equation $m = \left(\frac{dM}{dz}\right) M^{-1}$ or the relation between the differential Jones matrix (3) and the differential Mueller matrix [5]. It should be noted that the differential matrix (23) differs from the corresponding matrix for a homogeneous medium with elliptical phase anisotropy [18] only by its dependence α on z. The state of polarization of output light can be found using the equation:

$$S_{out} = M S_{inp} . \qquad (24)$$

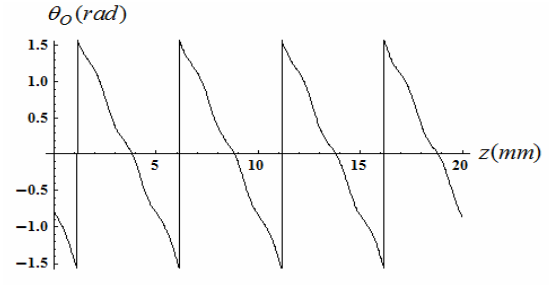
Substituting (21) and (22) into (24), we get:

$$S_{out}(z) = f\left(\delta_0, \alpha_0, \varphi_0, \theta_{inp}, z\right) \qquad (25)$$

This equation describes the evolution of Stokes vectors, which present the input state of polarization with azimuth θ_{inp} along the axis z in a medium with anisotropy parameters $\delta_0, \alpha_0, \varphi_0$. If polarized light propagates in the medium, it is very informative to consider evolution of the azimuth θ_O and angle of ellipticity e_O with z coordinates in direction of light propagation, which are defined as follows:

$$\theta_O = \frac{1}{2}\arctan\left[\frac{S_{out(3)}}{S_{out(2)}}\right], \qquad e_O = \frac{1}{2}\arctan\left[\frac{S_{out(4)}}{\sqrt{S^2_{out(2)} + S^2_{out(3)}}}\right]. \qquad (26)$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{C(\alpha_0 z)^2\left(\delta_0^2 + 4(\alpha_0+\varphi_0)^2 C(zA)\right) - \left(\delta_0^2 + 4(\alpha_0+\varphi_0)^2 C(zA)\right) S(\alpha_0 z)^2 + 2(\alpha_0+\varphi_0) A S(2\alpha_0 z) S(zA)}{A^2} & -C(zA) S(2\alpha_0 z) + \frac{2(\alpha_0+\varphi_0) S(zA) C(2\alpha_0 z)}{A} & \frac{4(\alpha_0+\varphi_0)\delta_0 C(2\alpha_0 z) S\left(\frac{zA}{2}\right)^2}{A^2} - \frac{\delta_0 S(2\alpha_0 z) S\left(\frac{zA}{2}\right)}{A} \\ 0 & \frac{A\left(\delta_0^2 + 4(\alpha_0+\varphi_0)^2 C(zA)\right) S(2\alpha_0 z) - 2(\alpha_0+\varphi_0) A^2 C(2\alpha_0 z) S(zA)}{A^3} & C(zA) C(2\alpha_0 z) + \frac{2(\alpha_0+\varphi_0) S(zA) S(2\alpha_0 z)}{A} & 2\delta_0 S\left(\frac{zA}{2}\right)\left(\frac{C(2\alpha_0 z) C\left(\frac{zA}{2}\right)}{A} + \frac{2(\alpha_0+\varphi_0) S(2\alpha_0 z) S\left(\frac{zA}{2}\right)}{A^2}\right) \\ 0 & -\frac{2(\alpha_0+\varphi_0)\delta_0 (C(zA) - 1)}{A^2} & -\frac{\delta_0 S(zA)}{A} & \frac{4(\alpha_0+\varphi_0)^2 + \delta_0^2 C(zA)}{A^2} \end{bmatrix} \qquad (22)$$



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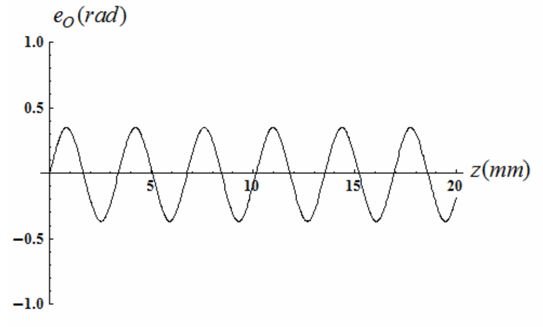

b

Fig. 4. Evolution of linearly polarized light with the azimuth $\theta_{inp} = -0.8$ rad in longitudinally inhomogeneous medium with elliptical phase anisotropy and parameter: $\delta_0 = 1.22$ rad/mm, $\alpha_0 = 0.3$ rad/mm, $\varphi_0 = 0.4$ rad/mm. a) Evolution of azimuth, b) evolution of angle of ellipticity.

Fig. 4 presents the evolution of polarization parameters θ_O and e_O for the cases of longitudinally inhomogeneous medium with elliptical phase anisotropy.

From Fig. 4, we have that in this case the azimuth is aperiodic function of z coordinates in contrast to that in homogeneous medium of this type that have been studied in [17]. The angle of ellipticity is a periodic function of z coordinates as in the case of homogeneous media of this type [17]. The period of ellipticity angle in this type of media should be equal to the thickness z of the medium in which eigenpolarization becomes circular (see Eq. (12)) at $k = 1$. Similar results were obtained in [5, 6] for longitudinally inhomogeneous medium with linear phase anisotropy.

7. Conclusions

Based on the Jones and Mueller matrix methods, the anisotropic properties of longitudinally inhomogeneous nondepolarizing media with elliptical phase anisotropy have been analyzed.

We have shown that the vector transfer equation for this type of media contains the partial solutions that describe evolution of privileged states of polarizations. In particular, it appears that these privileged states of polarization propagate in this type of media in such a way like that in the medium characterized by circular phase anisotropy. Namely, azimuths of polarization coincide with the principal axes of the specific molecular plane and the angles of ellipticity remain unchanged.

By solving the spectral problem, we have found that the exponential relationship between the eigenvalues of differential and integral Jones matrices derived in [4] is no longer held. However, the eigenvalues in this case remain phase factors. This results in the fact that the concept of eigenpolarizations in the case of longitudinally inhomogeneous nondepolarizing media is transformed in comparison with the case of homogeneous media. This is particularly evident in their dependence on z coordinates. This means that the traditional concept of eigenpolarizations is valid only for definite thickness of the medium.

Next, we have shown that the infinite numbers of input states of polarization can be orthogonalized by the medium of this class.

Studying evolution of linear polarization in this type of medium, it is shown that in comparison with the same type of homogeneous media the azimuth of polarization becomes a nonperiodic function of z coordinates. However, the angle of ellipticity is periodic with a definite period.

Summarizing, it has been found that the anisotropic properties of this type of media are qualitatively equivalent to the similar properties of longitudinally inhomogeneous media with linear phase anisotropy. This is due to the fact that the circular phase anisotropy is always a longitudinally homogeneous. In fact, this is natural generalization of the first equivalence theorem [2] for the case of longitudinally inhomogeneous media.

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