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Simulation of applied principles of envelope functions for Fabry-Perot spectroscopy of plane wave for single-layer structures

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Abstract. The method to obtain analytical expressions for envelope functions in spectra of normal incidence light reflection and transmission by single-layer structures is proposed.

Keywords: Fabry-Perot interferometry, single-layer structure, reflection, transmission, envelope function method.

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1. Introduction

The principle of multibeam interferometry of light for plane parallel plate developed by Fabry-Perot [1] have been widely used for solving a large number of scientific and applied problems and have been discussed in many textbooks [2-5]. Neveretheless, a number of new regularities in the scope of the problem were recently found. It was shown that Fabry-Perot interferometry is still actual for non-destructive control of optical parameters of micro- and nano-size layers [6-8].

Recently, the so-called method of envelope functions as tangential to contours of amplitude spectra of light reflection and transmission by a single-layer Fabry-Perot structures [10-19] was developed. So, for example, on the basis of the method the approach for determining the variation parameters in thickness and influence of spectral distribution of light and light absorption [9-15] was grounded. The envelope function method was used for grounding the new approach for determination of the instrumental characteristics of interference bands [16, 17] and for reconstruction of the phase of a reflected wave by single-layer films [16, 17, 19]. In other our papers [20-26], we developed the envelope function method for arbitrary ratio between the indices of refraction $n_{1,2,3}$ of three media.

In the paper, authors generalized the basic principles of application of the envelope function method to the data analysis in amplitude-phase spectroscopy of light interference for single-layer structures of Fabry-Perot type (SLSFP) in the case of normal incidence of a beam onto the interfaces for transparent and absorptive structures. A method for obtaining the analytical expression for energetic coefficients $(R,T)_{\max,\min}$ and phase $\phi_{\max,\min}$ is proposed.

2. General relations

The SLSFP of a thickness d and of the complex refraction index $\tilde{n}_2 = n_2 - i\chi_2$ is bounded by the semi-infinite top transparent media with refraction index n_1 (interface 12) and bottom (transparent or absorbing) media with refraction index $\tilde{n}_3$ (interface 23). In the absorbing layer, the phase change of wave is equal to $\tilde{\delta} = \frac{4\pi d}{\lambda}\tilde{n}_2$. It is well known also that taking into account the multibeam interference the complex amplitude values of light reflection $\tilde{r}$ and transmission $\tilde{t}$ by SLSFP are determined from

$$\tilde{r} = \frac{\tilde{r}_{12} + \tilde{r}_{23}\exp\left(-i\,\tilde{\delta}\right)}{1+\tilde{r}_{12}\,\tilde{r}_{23}\exp\left(-i\,\tilde{\delta}\right)} \text{ and } \tilde{t} = \frac{\tilde{t}_{12}\tilde{t}_{23}\exp\left(-i\,\tilde{\delta}/2\right)}{1+\tilde{r}_{12}\tilde{r}_{23}\exp\left(-i\,\tilde{\delta}\right)}, \quad (1)$$

where $\tilde{r}_{12,23}$ and $\tilde{t}_{12,23}$ are the well-known amplitude Fresnel's coefficients [2] for the single interfaces with subscripts 12 and 23. The resonant dispersion of the media is modelled by one-oscillator function of permittivity [27]

$$\tilde{\varepsilon} = \varepsilon_0 + \frac{4\pi\lambda\,\omega_0^2}{\omega_0^2 - \omega^2 + i\omega\tau}. \quad (2)$$

Here ε_0 is the background permittivity, $4\pi\lambda$ is the oscillator force of transition into electron state with resonant frequency ω_0, τ is the damping factor.

According to Eq. (1), the energy coefficient of reflection R and transmission T, and the tangent of phases $\tan\phi$ and $\tan\Phi$ of light are defined as follows

$$R = \frac{\sigma_{12}^2 + \Theta^2 + 2\sigma_{12}\Theta\cos F_-}{1+\sigma_{12}^2\Theta^2 + 2\sigma_{12}\Theta\cos F_+},$$

$$T = \frac{n_3}{n_1}\frac{T_{12}T_{23}\Omega}{1+\sigma_{12}^2\Theta^2 + 2\sigma_{12}\Theta\cos F_+}, \tag{3}$$

and

$$\tan\phi = \frac{\operatorname{Im}\tilde{r}}{\operatorname{Re}\tilde{r}} = \\ = \frac{\sigma_{12}\left(1-\Theta^2\right)\sin\phi_{12} + \Theta\left(1-\sigma_{12}^2\right)\sin\left(\phi_{23} - \operatorname{Re}\tilde{\delta}\right)}{\sigma_{12}\left(1+\Theta^2\right)\cos\phi_{12} + \Theta\left(1+\sigma_{12}^2\right)\cos\left(\phi_{23} - \operatorname{Re}\tilde{\delta}\right)}, \tag{4}$$

$$\tan\Phi = \frac{\operatorname{Im}\tilde{t}}{\operatorname{Re}\tilde{t}} = \\ = -\frac{\sin\left(\frac{1}{2}\operatorname{Re}\tilde{\delta}\right) + \sigma_{12}\Theta\sin\left(F_+ - \frac{1}{2}\operatorname{Re}\tilde{\delta}\right)}{\cos\left(\frac{1}{2}\operatorname{Re}\tilde{\delta}\right) + \sigma_{12}\Theta\cos\left(F_+ - \frac{1}{2}\operatorname{Re}\tilde{\delta}\right)} \tag{5}$$

where $\Omega = \exp\left(-\operatorname{Im}\tilde{\delta}\right)$, $T_{12,23} = \tilde{t}_{12,23}\cdot\tilde{t}^*_{12,23}$, $\tilde{r}_{12,23} = \sigma_{12,23}\exp\left(i\,\phi_{12,23}\right)$, $F_\pm = \phi_{12} \pm \left(\phi_{23} - \operatorname{Re}\tilde{\delta}\right)$.

Then expression for energetic coefficients of reflection and transmission are defined as [21, 24-26]:

$$R = \frac{R_{\min} + b^2\cos^2\frac{F_-}{2}}{1+b^2\cos^2\frac{F_+}{2}} = \frac{R_{\max} - a^2\sin^2\frac{F_-}{2}}{1-a^2\sin^2\frac{F_+}{2}} \text{ and}$$

$$T = \frac{T_{\min}}{1-a^2\sin^2\frac{F_+}{2}} = \frac{T_{\max}}{1+b^2\cos^2\frac{F_+}{2}}, \tag{6}$$

where $a^2 = \dfrac{4\sigma_{12}\Theta}{\left(1+\sigma_{12}\Theta\right)^2}$ and $b^2 = \dfrac{4\sigma_{12}\Theta}{\left(1-\sigma_{12}\Theta\right)^2}$, $\Theta = \sigma_{23}\Omega$. The functions

$$R_{\max,\min} = \left(\frac{\sigma_{12} \pm \Theta}{1\pm\sigma_{12}\Theta}\right)^2 \text{ and } T_{\max,\min} = \frac{n_3}{n_1}\frac{T_{12}T_{23}\Omega}{\left(1\mp\sigma_{12}\Theta\right)^2} \tag{7}$$

are the envelope functions of Fabry-Perot spectra.

3. Discussion

1. SLSFP with resonant dispersion (2). In many practical problems, medium with refraction n_1 is air or vacuum and $n_1 = 1$. For a free layer with resonant dispersion of $\tilde{\varepsilon}(\omega)$, the calculated spectra $R(\omega)$, $T(\omega)$ $\phi(\omega)$, $\Phi(\omega)$ and their envelope functions $\left(R, T, \phi\right)_{\max,\min}$ are shown in Fig. 2. In the region of resonant absorption near the resonant frequency ω_0, it is possible to separate a frequency interval with width $\Delta\omega_p$, which is bounded by an interval with significant absorption where $\Omega \to 0$. Inside the interval $R_{\max} \approx R_{\min}$, and the spectra are formed as if the light wave is reflected from a semi-infinite medium with resonant dispersion (2) [21, 24-26]. The distance between envelope functions $\Delta R = R_{\max} - R_{\min}$ and $\Delta T = T_{\max} - T_{\min}$ are equal to

$$\Delta R = \frac{4\sigma_{12}\Theta}{\left(1-\sigma_{12}^2\Theta^2\right)^2}\left(1-\sigma_{12}^2\right)\left(1-\Theta^2\right) \text{ and}$$

$$\Delta T = \frac{4\sigma_{12}\Theta\Omega}{\left(1-\sigma_{12}^2\Theta^2\right)^2}\frac{n_3}{n_1}T_{12}T_{23}. \tag{8}$$

The absorption level increases with approaching to the resonant frequency ω_0, and the value $\Delta R \to 0$ because of damping factor approaching to null $\Omega \to 0$. Near the resonant region beyond the interval $\Delta\omega_p$, the functions (Fig. 1b)

$$\phi_{\max,\min} \cong 2\pi \pm \frac{\sigma_{12}\left(1-\sigma_{23}\Theta\right)\sin\phi_{12} + \sigma_{23}(1-\sigma_{12}^2)\Omega}{\sigma_{12}\left(1+\Theta\right)\cos\phi_{12}} \tag{9}$$

are the envelope functions of the phase spectrum of reflection. It is problematic to apply the envelope function method (Fig. 1b) to describe the phase spectrum $\Phi(\omega)$ for light transmitted through the layer.

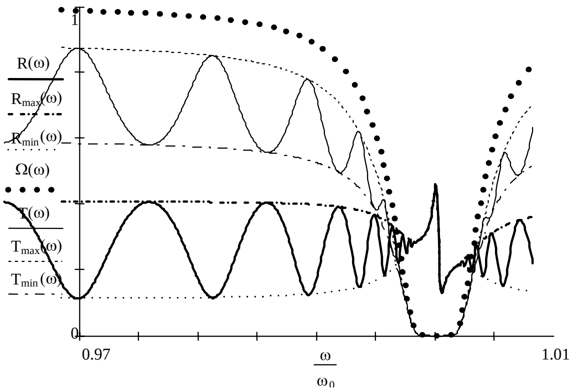
At the arbitrary frequency, we have the following expression:

$$\frac{R_{\max} - R}{R - R_{\min}} = \frac{T - T_{\min}}{T_{\max} - T} = \left(\frac{a}{b}\right)^2 \tan^2\frac{F_\pm}{2}. \tag{10}$$

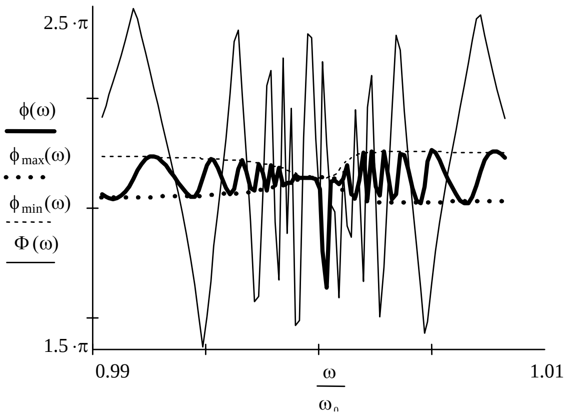
The ratios $\dfrac{R_{\max} - R}{R - R_{\min}}$ and $\dfrac{T_{\max} - T}{T - T_{\min}}$ vary from 0 to $+\infty$, and at the frequencies $\omega_{1,2}$ from both side of extreme contour peak the ratios are equal to 1. Hence, coefficients for reflected and transmitted light at the frequencies $\omega_{1,2}$ equal (Fig. 2)

$$\Sigma R = \frac{1}{2}\left[R_{\max} + R_{\min}\right] \text{ and } \Sigma T = \frac{1}{2}\left[T_{\max} + T_{\min}\right]. \tag{11}$$

It is the approach (11) that enables to ground the validity of interferogram apparatus characteristic determination [16, 17]. The extreme contour width $\Delta\omega$ under the condition (11) equals $\Delta\omega = \dfrac{c_0}{2n_2 d}\Delta\delta$, where $\Delta\delta$ is the width contour extreme in phase units (phase distance between frequencies $\omega_{1,2}$ on both sides from its peak).



a)



b)

Fig. 1. Calculated spectra of $R(\omega)$, $T(\omega)$ in the resonant region and their envelope functions $(R,T)_{\max,\min}$ (a) and $\phi(\omega)$, $\Phi(\omega)$ (b).

The phase width $\Delta\delta$ equals $\Delta\delta = 2\pi\dfrac{\Delta\omega}{\Delta\omega_{\min}}$, where $\Delta\omega_{\min}$ is the distance between the frequencies on both sides from the maximum.

The 2π-periodicity of Fabry-Perot spectra allows to determine the area restricted by the contour of a maximum minus the area restricted by envelope function of adjacent minima through the structure parameters as

$$S_R = 2\int_{\pi/2}^{3\pi/2}\left[R(\zeta) - R_{\min}\right]d\zeta\,, \qquad (12)$$

where $\zeta = \dfrac{\mathrm{Re}\,\tilde{\delta}}{2}$. For transparent structures the area S_R is equal

$$S_R = \left(1 - R_{\min}\right)\left[\pi - \int_{\pi/2}^{3\pi/2}\frac{d\zeta}{1 + b^2\cos^2\zeta}\right].$$

This integral is the standard integral [28].

In the region of absorption the integration procedure can be simplified by replacement $R_{\min}$ at the maximum frequency by the mean value of two adjacent minima $\frac{1}{2}\left(R_{\min,m-1} + R_{\min,m+1}\right)$ [22]. Clearly, this way can be applied to the transmitted waves.

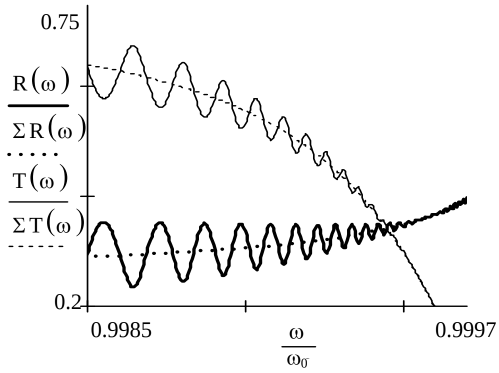


Fig. 2. Calculated spectra of $R(\omega)$, $T(\omega)$ in the resonant region and contours $\Sigma R(\omega)$ and $\Sigma T(\omega)$.

2. Free layer and the fixed one on substrate surfaces with constant absorption $\chi_2 = \mathrm{const}$ **.** The values of energetic coefficients $(R,T)_{\max,\min}$ in the absorptive films are a function of λ. Even for absorptive layers with constant absorption, energetic coefficients $(R,T)_{\max,\min}$ vary with variation of λ. According to the definition given by Michelson the visibility of the interferogram $V = \dfrac{R_{\max} - R_{\min}}{R_{\max} + R_{\min}}$. Then, according to the expression (7) we have

$$V = 2\left[\frac{\sigma_{12}\left(1-\Theta^2\right)}{\left(1-\sigma_{12}{}^2\right)\Theta} + \frac{\left(1-\sigma_{12}{}^2\right)\Theta}{\sigma_{12}\left(1-\Theta^2\right)}\right]^{-1}. \qquad (15)$$

Using the transformation

$$2\,\frac{x+y}{x-y} = \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} + \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\,, \qquad (16)$$

we obtained that $V = 2\left[W^{-1} + W\right]^{-1}$, where $W = \dfrac{\sqrt{R_{\max}} - \sqrt{R_{\min}}}{\sqrt{R_{\max}} + \sqrt{R_{\min}}} = \dfrac{\Theta}{\sigma_{12}}\dfrac{1-\sigma_{12}^2}{1-\Theta^2}$. Therefore, when the absorption index χ_2 does not depend on frequency, the relative slope between linear dependences $\ln W^{-1}$ and $\mathrm{Im}\,\tilde{\delta}$ does not depend on frequency for an arbitrary ratio between the refraction indices $n_{1,2,3}$ (Fig. 3) and the relation

$$\ln W^{-1} = \mathrm{const} + \exp\left(\mathrm{Im}\,\tilde{\delta}\right) \qquad (17)$$

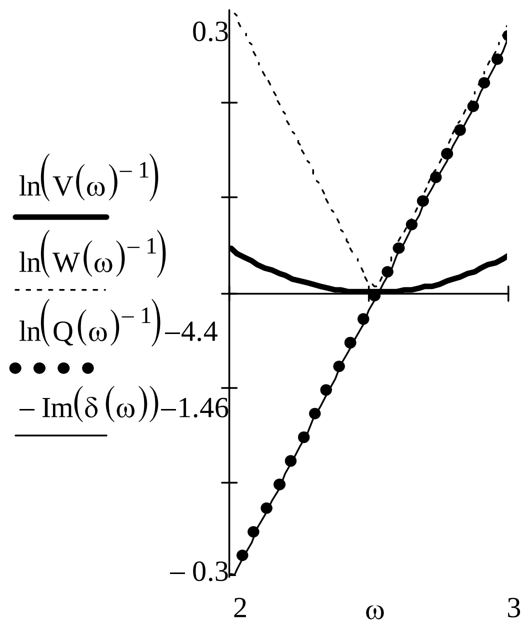


Fig. 3. Calculated dependences of $\ln W^{-1}$, $\ln V^{-1}$ and $\operatorname{Im}\tilde{\delta}$ for parameters: $n_1 = 1$, $n_2 = 1.25$, $n_3 = 3.5$, $\chi_2 = 0.0038$, $d = 15\ \mu\text{m}$.

is valid with accuracy to constant. Slope of $\ln V$ depends on the ratio between the refraction indices $n_{1,2,3}$.

4. Principal conclusions

Amplitude-phase Fabry-Perot spectra for single-layer structures at the normal can be described by the envelope functions $(R,T)_{\max,\min}$.

The width of the interference band as a phase separation between the points, for which the reflectance are $\frac{1}{2}\left[(R,T)_{\max} + (R,T)_{\min}\right]$, is determined through instrumental characteristics of the Fabry-Perot interferogram of single-layer structures.

In the region of constant light absorption in film, the slope of $\ln\left(\frac{\sqrt{R_{\max}} - \sqrt{R_{\min}}}{\sqrt{R_{\max}} + \sqrt{R_{\min}}}\right)$ equals to that of $\operatorname{Im}\tilde{\delta}$.

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