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ANALYSIS OF NONLINEAR STRESSES AND STRAINS IN A THIN CURRENT-CARRYING ELASTIC PLATE

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Abstract. The two-dimensional magnetoelastic problems of a thin current-carrying plate under the interaction of an unsteady electromagnetic field and a mechanical field are studied. The nonlinear magnetoelastic kinetic equations, the geometric equations, the physical equations, the electrodynamics equations, and the expressions of Lorentz force of a thin current-carrying plate under the action of a coupled field are given. The Normal Cauchy form nonlinear differential equations, which include ten basic unknown functions in all, are obtained by the variable replacement method. Using the difference and quasi-linearization methods, the nonlinear magnetoelastic equations are reduced to a sequence of quasilinear differential equations, which can be solved by the discrete-orthogonalization method. Numerical solutions for the magnetoelastic stresses and strains in a thin current-carrying elastic plate are obtained by considering a specific example. The results that the stresses and deformations in the thin current-carrying elastic plate change with variation of the electromagnetic parameters are discussed. The results show that the stress-strain state in thin plates can be controlled by changing the electromagnetic parameters. This provides a method of theoretical analysis and numerical calculation for changing the service conditions and intensity researches of thin plates of engineering structures in an electromagnetic field.

Key words: two-dimensional magnetoelasticity of thin current-carrying plate, electromagnetic and mechanical fields, nonlinearity, Lorentz force, quasi-linearization method, the discrete-orthogonalization method.

1. Introduction.

Applications of magnetoelastic theory are broad in the important departments of energy sources, traffic and national defense and so on. Knopoff [1] (1955) and Chadwick [2] (1972) applied originally the magnetoelastic theory to solve the problem of wave propagation in conductors. Subsequently, the propagation of a plane wave in an infinite elastic space with finite electric conductivity was analyzed, the fundamental kinetic equations of elastic and inelastic anisotropic bodies in an intense magnetic field were obtained, and the movement of elastic bodies with electric conduction correlated with the temperature field was studied. Before 1970, the research works completed by scholars were not elaborate. In the recent four decades, the research achievements have entered a completely new stage. These research achievements include: some basic theories and computation models based on nonlinear coupled electromagnetic theory and the research achievements by Pao and Yeh [3] (1973); Ambartsumyan [4] (1977); Moon [5] (1984); Van de Ven [6] (1986); Ulitko, Mol'chenko, Kovalchuk [7] (1994); Mol'chenko, Grigorenko [8] (2010); Zhou, Gao, Zheng [9] (2000); Hasanyan [10, 11] (2001, 2006); Wang [12, 13] (2003, 2008); Zheng, Zhang, Zhou [14] (2005); Wu [15] (2007); Ottenio, Destrade, Ogden [16] (2008); Kaloerov, Boronenko [17] (2005); Podil'chuk, Dashko [18] (2005), and others. These research achievements laid a good foundation for studies on magnetoelastic theory and its applications.

However, so far the studies of the nonlinear theory are not complete enough. Except for the problems about the vibration and stability of the structures such as rods, beams, plates, shells, etc. in electromagnetic fields, the state analysis of the stresses and strains in the problem is rare [19 – 26]. Two-or three-dimensional magnetoelastic problems are only in theoretical research phase for establishing the equations, no studies of the stresses and strains in current-carrying plates and shells in electromagnetic fields on two- and tree-dimensional problems have yet been reported, the achievements in practical applications are seen also rarely. Therefore, the researches for the stresses and strains in current-carrying plates and shells in electromagnetic fields on two-dimensional problems have recently become one of the most important topics for mechanics operators.

In this paper, the two-dimensional magnetoelastic problems of a thin current-carrying plate under the interaction of an unsteady electromagnetic field and a mechanical field are studied. The nonlinear magnetoelastic kinetic equations, the geometric equations, the physical equations, the electrodynamics equations, and the expressions of Lorentz force of a thin current-carrying plate under the action of a coupled field are given. Normal Cauchy form nonlinear differential equations, which include ten basic unknown functions in all, are obtained by the variable replacement method. Using the difference and quasi-linearization methods, the nonlinear magnetoelastic equations are reduced to a sequence of quasilinear differential equations, which can be solved by the discrete-orthogonalization method. Numerical solutions for the magnetoelastic stresses and deformations in the thin current-carrying elastic plate are obtained by considering an example. Computational results show that the states of the strains and stresses in thin plates can be controlled by changing the electromagnetic parameters. This lays the foundation for practical applications of two-dimensional magnetoelastic problems.

2. Fundamental Equations.

We consider a thin elastic plate moving in the applied magnetic field $\mathbf{B} = \{0, B_y, 0\}$. The plate, the mechanical load, and electric current distribution in rectangular coordinates $Oxyz$ are shown in Fig. 1. Current-carrying flexible bodies in a time-dependent electromagnetic field satisfy the hypothesis that the normal is straight and the Kirchhoff – Love hypothesis that no stretching occurs in vertical fibers. By satisfying the magnetoelastic supposition of the thin plate [27] and using the theory of elastic mechanics, Ohm's law and Maxwell equations in electromagnetic basic theories, we obtain the two-dimensional electrodynamics equations, the magnetoelastic kinetic equations, the expressions of Lorentz force, the geometric equations, and the physical equations for a thin current-carrying plate.

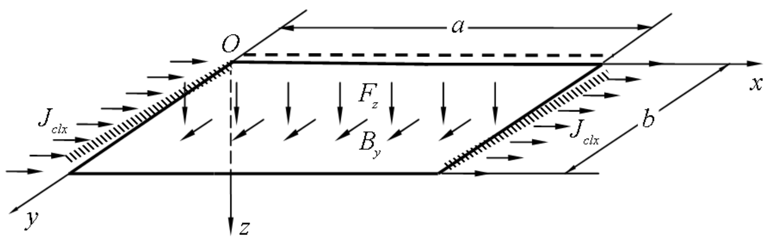

Fig. 1. A thin current-carrying elastic plate in a magnetic field.

2.1. Two-Dimensional Electrodynamics Equations of a Thin Plate.

$$-\frac{\partial B_z}{\partial t} = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}; \tag{1}$$

$$\sigma\left[E_x + \frac{\partial v}{\partial t}B_z - \frac{1}{2}\frac{\partial w}{\partial t}(B_y^+ + B_y^-)\right] = \frac{\partial H_z}{\partial y} - \frac{H_y^+ - H_y^-}{h}; \tag{2}$$

$$\sigma\left[E_y-\frac{\partial u}{\partial t}B_z+\frac{1}{2}\frac{\partial w}{\partial t}(B_x^+ + B_x^-)\right]=-\frac{\partial H_z}{\partial x}+\frac{H_x^+ - H_x^-}{h}, \qquad (3)$$

where u, v and w are the displacements along the x, y and z directions, respectively; E_x and E_y are the electric field intensities along the x and y directions, respectively; H_z is the magnetic field intensity along the z direction; B_z is the magnetic induction intensity along the z direction; h is the thickness of the plate; t is the time variable; σ is the electrical conductivity of the plate; $B_i^{\pm}$ and $H_i^{\pm}$ ($i=x,y$) are the values of B_i and H_i on the upper and lower surfaces of the plate, respectively.

2.2. Magnetoelastic Kinetic Equations of a Thin Plate

$$\frac{\partial N_x}{\partial x}+\frac{\partial S}{\partial y}+F_x+n_x+f_x=\rho h\frac{\partial^2 u}{\partial t^2}; \qquad (4)$$

$$\frac{\partial N_y}{\partial y}+\frac{\partial S}{\partial x}+F_y+n_y+f_y=\rho h\frac{\partial^2 v}{\partial t^2}; \qquad (5)$$

$$\frac{\partial(Q_x-N_x\theta_x-S\theta_y)}{\partial x}+\frac{\partial(Q_y-N_y\theta_y-S\theta_x)}{\partial y}+F_z+n_z+f_z=\rho h\frac{\partial^2 w}{\partial t^2}; \qquad (6)$$

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=\frac{\rho h^3}{12}\frac{\partial^2\theta_x}{\partial t^2}; \qquad (7)$$

$$\frac{\partial M_y}{\partial y}+\frac{\partial M_{xy}}{\partial x}-Q_y=\frac{\rho h^3}{12}\frac{\partial^2\theta_y}{\partial t^2}. \qquad (8)$$

$$\Bigg(f_x=J_{cly}hB_z+\sigma hE_yB_z-\sigma h\frac{\partial u}{\partial t}B_z^2+\frac{\sigma h}{2}\frac{\partial w}{\partial t}(B_x^+ + B_x^-)B_z, \qquad (9)$$

$$f_y=-J_{clx}hB_z-\sigma hE_xB_z-\sigma h\frac{\partial v}{\partial t}B_z^2+\frac{\sigma h}{2}\frac{\partial w}{\partial t}(B_y^+ + B_y^-)B_z, \qquad (10)$$

$$f_z=\frac{h}{2}\left[J_{clx}(B_y^+ + B_y^-)-J_{cly}(B_x^+ + B_x^-)\right]+\frac{\sigma h}{2}E_x(B_y^+ + B_y^-)-$$

$$-\frac{\sigma h}{2}E_y(B_x^+ + B_x^-)+\sigma h\left[\frac{1}{2}\frac{\partial v}{\partial t}(B_y^+ + B_y^-)+\frac{h}{12}\frac{\partial\theta_y}{\partial t}(B_y^+ - B_y^-)\right]B_z-$$

$$-\sigma h\frac{\partial w}{\partial t}\left[\frac{1}{4}(B_y^+ + B_y^-)^2+\frac{1}{12}(B_y^+ - B_y^-)^2+\frac{1}{4}(B_x^+ + B_x^-)^2+\frac{1}{12}(B_x^+ - B_x^-)^2\right]+$$

$$+\frac{\sigma h}{2}\frac{\partial u}{\partial t}B_z(B_x^+ + B_x^-)+\frac{\sigma h^2}{12}\frac{\partial\theta_x}{\partial t}B_z(B_x^+ - B_x^-)\Bigg), \qquad (11)$$

where N_x, N_y, Q_x, Q_y, S, M_x, M_y and M_{xy} are the internal forces and moments along the corresponding directions in the plate; F_x, F_y, and F_z are the mechanical loads; n_x, n_y, and n_z are the volume forces; f_x, f_y, and f_z are the Lorentz forces along the corresponding directions; θ_x and θ_y are the angles of rotation; J_{clx} and J_{cly} are the densities of side electric current along the corresponding directions; ρ is the mass density of the plate.

2.3. Geometric Equations of a Thin Plate.

Considering Von Karman's geometric relationship for the deformation of a thin plate, we can write the geometric equations describing nonlinear deformation as

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\theta_x^2 ; \; \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\theta_y^2 ; \qquad (12\text{ a, b})$$

$$\chi_x = \frac{\partial \theta_x}{\partial x} ; \; \chi_y = \frac{\partial \theta_y}{\partial y} ; \; \chi_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} ; \qquad (13\text{ a, b, c})$$

$$\Omega = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \theta_x \theta_y ; \;\; \theta_x = -\frac{\partial w}{\partial x} ; \;\; \theta_y = -\frac{\partial w}{\partial y} . \qquad (14\text{ a, b, c})$$

2.4. Physical Equations of a Thin Plate.

The physical equations of the thin current-carrying plate are given by

$$N_x = D_N(\varepsilon_x + \nu\varepsilon_y) ; \; N_y = D_N(\varepsilon_y + \nu\varepsilon_x) ; \qquad (15\text{ a, b})$$

$$M_x = D_M(\chi_x + \nu\chi_y) ; \; M_y = D_M(\chi_y + \nu\chi_x) ; \qquad (16\text{ a, b})$$

$$S = D_N \frac{1-\nu}{2}\Omega ; \; M_{xy} = M_{yx} = \frac{1}{2}D_M(1-\nu)\chi_{xy} , \qquad (17\text{ a, b})$$

where D_N and D_M are the tensile and bending rigidities, respectively,

$$D_N = \frac{Eh}{1-\nu^2} ; \; D_M = \frac{Eh^3}{12(1-\nu^2)} , \qquad (18\text{a, b})$$

where E is Young's modulus; ν is Poisson's ratio.

3. Magnetoelastic Nonlinear Equations of a Thin Current-Carrying Plate.

For obtaining normal Cauchy form nonlinear partial differential equations, let u, v, w, θ_y, N_y, $\hat{Q}_y$, S, M_y, E_x and B_z be the basic unknown functions. We obtain:

$$\frac{\partial u}{\partial y} = \frac{2S}{D_N(1-\nu)} - \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\theta_y ; \qquad (19)$$

$$\frac{\partial v}{\partial y} = \frac{N_y}{D_N} - \frac{1}{2}\theta_y{}^2 - \frac{\nu}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \nu\frac{\partial u}{\partial x} ; \qquad (20)$$

$$\frac{\partial w}{\partial y} = -\theta_y ; \qquad (21)$$

$$\frac{\partial \theta_y}{\partial y} = \frac{1}{D_M}M_y + \nu\frac{\partial^2 w}{\partial x^2} ; \qquad (22)$$

$$\frac{\partial N_y}{\partial y} = \rho h \frac{\partial^2 v}{\partial t^2} - \frac{\partial S}{\partial x} - (F_y + n_y + f_y)\,; \tag{23}$$

$$\frac{\partial \hat{Q}_y}{\partial y} = \frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2}\left(\frac{12}{h^2}w - \frac{\partial^2 w}{\partial x^2}\right) - (F_z + n_z + f_z) - \nu \frac{\partial N_y}{\partial x}\frac{\partial w}{\partial x} -$$

$$-\nu N_y \frac{\partial^2 w}{\partial x^2} - \frac{\partial w}{\partial x} Eh\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}\right) -$$

$$-\frac{\partial^2 w}{\partial x^2} Eh\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right] + S\frac{\partial \theta_y}{\partial x} + \theta_y \frac{\partial S}{\partial x} - \nu \frac{\partial^2 M_y}{\partial x^2} + \frac{Eh^3}{12}\frac{\partial^4 w}{\partial x^4}\,; \tag{24}$$

$$\frac{\partial S}{\partial y} = \rho h \frac{\partial^2 u}{\partial t^2} - Eh\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}\right) - \nu \frac{\partial N_y}{\partial x} - (F_x + n_x + f_x)\,; \tag{25}$$

$$\frac{\partial M_y}{\partial y} = \hat{Q}_y + N_y \theta_y + S\theta_x + \frac{\rho h^3}{12}\frac{\partial^2 \theta_y}{\partial t^2} - 2\frac{\partial}{\partial x}\left[D_M (1-\nu)\frac{\partial \theta_y}{\partial x}\right]; \tag{26}$$

$$\frac{\partial E_x}{\partial y} = -\frac{1}{\sigma\mu}\frac{\partial^2 B_z}{\partial x^2} + B_z \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right) + \frac{\partial u}{\partial t}\frac{\partial B_z}{\partial x} - \frac{(B_x^+ + B_x^-)}{2}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial x}\right) + \frac{\partial B_z}{\partial t}\,; \tag{27}$$

$$\frac{\partial B_z}{\partial y} = \sigma\mu\left[E_x + \frac{\partial v}{\partial t}B_z - \frac{\partial w}{\partial t}\frac{(B_y^+ + B_y^-)}{2}\right] + \frac{B_y^+ - B_y^-}{h}\,, \tag{28}$$

where μ is the permeability of the plate, and

$$\hat{Q}_y = Q_y - N_y \theta_y - S\theta_x + \frac{\partial M_{xy}}{\partial x}\,. \tag{29}$$

4. Computational Method.

Equations (19) – (28) can be written as boundary-value problems:

$$\frac{\partial \mathbf{N}}{\partial y} = \mathbf{F}(x,\ y,\ \mathbf{N})\,; \tag{30}$$

$$D_1\,\mathbf{N}\big|_{y=d_1} = \mathbf{d}_1\,;\ D_2\,\mathbf{N}\big|_{y=d_2} = \mathbf{d}_2\,, \tag{31 a, b}$$

where $\mathbf{N} = \{u,\ v,\ w,\ \theta_y,\ N_y,\ \hat{Q}_y,\ S,\ M_y,\ E_x,\ B_z\}^{\mathrm{T}}$; D_1 and D_2 are given orthogonal matrixes; $\mathbf{d}_1$ and $\mathbf{d}_2$ are given vectors.

4.1. Establishing of Difference Format.

For problems (30) and (31), the difference format is built along x direction, at the i node, by writing the partial derivatives $\frac{\partial N}{\partial x}$, $\frac{\partial^2 N}{\partial x^2}$, $\frac{\partial^3 N}{\partial x^3}$, $\frac{\partial^4 N}{\partial x^4}$ with respect to x as

$$\frac{\partial f^i}{\partial x} = \frac{f^{i+1} - f^{i-1}}{2\lambda}\,;\ \ \frac{\partial^2 f^i}{\partial x^2} = \frac{f^{i+1} - 2f^i + f^{i-1}}{\lambda^2}\,; \tag{32 a, b}$$

$$\frac{\partial^3 f^i}{\partial x^3} = \frac{f^{i+2} - 2f^{i+1} + 2f^{i-1} - f^{i-2}}{2\lambda^3}\,;\ \frac{\partial^4 f^i}{\partial x^4} = \frac{f^{i+2} - 4f^{i+1} + 6f^i - 4f^{i-1} + f^{i-2}}{\lambda^4}\,, \quad (33\text{ a, b})$$

where λ is the difference step length. At the same time, Newmark's stable finite equidifferent formulas [27] are used to find the derivatives with respect to time in Eqs. (19) – (28) for a time step length:

$$\ddot{u}^{t+\Delta t} = \frac{u^{t+\Delta t} - u^t}{\beta(\Delta t)^2} - \left[\frac{\dot{u}^t}{\Delta t} + \ddot{u}^t\left(\frac{1}{2} - \beta\right)\right]\frac{1}{\beta}\,;$$

$$\dot{u}^{t+\Delta t} = \dot{u}^t + \frac{\Delta t}{2}(\ddot{u}^t + \ddot{u}^{t+\Delta t})\,, \quad (34\text{ a, b})$$

where β is the parameter of the scheme, $\beta = 0,25$ is generally selected; Δt is time increment.

Thus, Eqs. (19) – (28) can be expressed as

$$\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}y} = \mathbf{F}(y,\ \mathbf{N})\,; \quad (35)$$

$$D_1\,\mathbf{N}\big|_{y=d_1} = \mathbf{d}_1\,;\ D_2\,\mathbf{N}\big|_{y=d_2} = \mathbf{d}_2\,. \quad (36\text{ a, b})$$

4.2. Derivation of Linearized Iteration Equations.

After establishing the difference format, the problems described by Eq. (35) are nonlinear. With an iteration method, nonlinear problems can be turned into a series of linear problems. The iteration equations are

$$\frac{\mathrm{d}\,\mathbf{N}^{(k+1)}}{\mathrm{d}\,y} = \mathbf{F}(y,\mathbf{N}^{(k)}) + \boldsymbol{\Gamma}(y,\mathbf{N}^{(k)})(\mathbf{N}^{(k+1)} - \mathbf{N}^{(k)})\,; \quad (37)$$

$$D_1\mathbf{N}^{(k+1)}(d_1) = \mathbf{d}_1\,;\quad D_2\mathbf{N}^{(k+1)}(d_2) = \mathbf{d}_2\,,\quad (k = 0, 1, 2, \cdots)\,, \quad (38\text{ a, b})$$

where $\boldsymbol{\Gamma}(y,\mathbf{N}^{(k)})$ is Jacobi's matrix.

Thus, Eqs. (19) – (28) can be written as

$$\frac{\mathrm{d}(u^i)^{(k+1)}}{\mathrm{d}y} = \frac{2(S^i)^{(k+1)}}{D_N(1-\nu)} - \frac{(v^{i+1})^{(k+1)} - (v^{i-1})^{(k+1)}}{2\lambda} + \frac{1}{2\lambda}\{[(w^{i+1})^{(k)}(\theta_y{}^i)^{(k+1)} + (w^{i+1})^{(k+1)}(\theta_y^i)^{(k)};$$

$$-(w^{i+1})^{(k)}(\theta_y{}^i)^{(k)}] - [(w^{i-1})^{(k)}(\theta_y{}^i)^{(k+1)} + (w^{i-1})^{(k+1)}(\theta_y{}^i)^{(k)} - (w^{i-1})^{(k)}(\theta_y{}^i)^{(k)}]\}\,; \quad (39)$$

$$\frac{\mathrm{d}(v^i)^{(k+1)}}{\mathrm{d}\,y} = \frac{(N_y^i)^{(k+1)}}{D_N} - \frac{1}{2}[2(\theta_y^i)^{(k)}(\theta_y^i)^{(k+1)} - ((\theta_y^i)^{(k)})^2] - \frac{\nu}{8\lambda^2}\times$$

$$\times\{[(2w^{i+1})^{(k)}(w^{i+1})^{(k+1)} - ((w^{i+1})^{(k)})^2] -$$

$$-2[(w^{i+1})^{(k)}(w^{i-1})^{(k+1)} + (w^{i+1})^{(k+1)}(w^{i-1})^{(k)} - (w^{i+1})^{(k)}(w^{i-1})^{(k)}] +$$

$$+[(2w^{i-1})^{(k)}(w^{i-1})^{(k+1)} - ((w^{i-1})^{(k)})^2]\} - \frac{\nu}{2\lambda}((u^{i+1})^{(k+1)} - (u^{i-1})^{(k+1)})\,, \quad (40)$$

$$\frac{\mathrm{d}(\,w^i)^{(k+1)}}{\mathrm{d}\,y} = -(\theta_y^i)^{(k+1)}\,; \quad (41)$$

$$\frac{\mathrm{d}(\theta_y^i)^{(k+1)}}{\mathrm{d}\,y}=\frac{1}{D_M}(M_y^i)^{(k+1)}+\frac{\nu}{\lambda^2}[(w^{i+1})^{(k+1)}-2(w^i)^{(k+1)}+(w^{i-1})^{(k+1)}]\,; \qquad (42)$$

$$\frac{\mathrm{d}(N_y^i)^{(k+1)}}{\mathrm{d}\,y}=\rho h\frac{\partial^2(v^i)^{(k+1)}}{\partial t^2}-\frac{(S^{i+1})^{(k+1)}-(S^{i-1})^{(k+1)}}{2\lambda}-(F_y^i+n_y^i+f_y^i)^{(k+1)}\,; \qquad (43)$$

$$\frac{\mathrm{d}(\hat{Q}_y^i)^{(k+1)}}{\mathrm{d}y}=\frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2}\left[\frac{12}{h^2}(w^i)^{(k+1)}-\frac{(w^{i+1})^{(k+1)}-2(w^i)^{(k+1)}+(w^{i-1})^{(k+1)}}{\lambda^2}\right]-(F_z^i+n_z^i+f_z^i)^{(k+1)}-$$

$$-\frac{\nu}{4\lambda^2}\Big\{\Big[(N_y^{i+1})^{(k+1)}(w^{i+1})^{(k)}+(N_y^{i+1})^{(k)}(w^{i+1})^{(k+1)}-(N_y^{i+1})^{(k)}(w^{i+1})^{(k)}\Big]-$$

$$-\Big[(N_y^{i+1})^{(k+1)}(w^{i-1})^{(k)}+(N_y^{i+1})^{(k)}(w^{i-1})^{(k+1)}-(N_y^{i+1})^{(k)}(w^{i-1})^{(k)}\Big]+$$

$$+\Big[(N_y^{i-1})^{(k+1)}(w^{i-1})^{(k)}+(N_y^{i-1})^{(k)}(w^{i-1})^{(k+1)}-(N_y^{i-1})^{(k)}(w^{i-1})^{(k)}\Big]-$$

$$-\Big[(N_y^{i-1})^{(k+1)}(w^{i+1})^{(k)}+(N_y^{i-1})^{(k)}(w^{i+1})^{(k+1)}-(N_y^{i-1})^{(k)}(w^{i+1})^{(k)}\Big]\Big\}-$$

$$-\frac{\nu}{\lambda^2}\Big\{\Big[(N_y^i)^{(k+1)}(w^{i+1})^{(k)}+(N_y^i)^{(k)}(w^{i+1})^{(k+1)}-(N_y^i)^{(k)}(w^{i+1})^{(k)}\Big]-$$

$$-2\Big[(N_y^i)^{(k+1)}(w^i)^{(k)}+(N_y^i)^{(k)}(w^i)^{(k+1)}-(N_y^i)^{(k)}(w^i)^{(k)}\Big]+$$

$$+\Big[(N_y^i)^{(k+1)}(w^{i-1})^{(k)}+(N_y^i)^{(k)}(w^{i-1})^{(k+1)}-(N_y^i)^{(k)}(w^{i-1})^{(k)}\Big]\Big\}-$$

$$-\frac{Eh}{2\lambda^3}\Big[(w^{i+1}-w^{i-1})^{(k+1)}(u^{i+1}-2u^i+u^{i-1})^{(k)}+(w^{i+1}-w^{i-1})^{(k)}(u^{i+1}-2u^i+u^{i-1})^{(k+1)}-$$

$$-(w^{i+1}-w^{i-1})^{(k)}(u^{i+1}-2u^i+u^{i-1})^{(k)}\Big]-$$

$$-\frac{Eh}{4\lambda^4}\Big[2(w^{i+1}-w^{i-1})^{(k+1)}(w^{i+1}-w^{i-1})^{(k)}(w^{i+1}-2w^i+w^{i-1})^{(k)}+$$

$$+((w^{i+1}-w^{i-1})^{(k)})^2(w^{i+1}-2w^i+w^{i-1})^{(k+1)}-2((w^{i+1}-w^{i-1})^{(k)})^2(w^{i+1}-2w^i+w^{i-1})^{(k)}\Big]-$$

$$-\frac{Eh}{2\lambda^3}\Big[(w^{i+1}-2w^i+w^{i-1})^{(k+1)}(u^{i+1}-u^{i-1})^{(k)}+(w^{i+1}-2w^i+w^{i-1})^{(k)}(u^{i+1}-u^{i-1})^{(k+1)}-$$

$$-(w^{i+1}-2w^i+w^{i-1})^{(k)}(u^{i+1}-u^{i-1})^{(k)}\Big]-\frac{Eh}{8\lambda^4}\Big[(w^{i+1}-2w^i+w^{i-1})^{(k+1)}\left((w^{i+1}-w^{i-1})^{(k)}\right)^2+$$

$$+2(w^{i+1}-2w^i+w^{i-1})^{(k)}(w^{i+1}-w^{i-1})^{(k)}(w^{i+1}-w^{i-1})^{(k+1)}-$$

$$-2(w^{i+1}-2w^{i}+w^{i-1})^{(k)}((w^{i+1}-w^{i-1})^{(k)})^{2}\Big]+$$

$$+\frac{1}{2\lambda}\Big[(S^{i})^{(k+1)}(\theta_{y}^{i+1}-\theta_{y}^{i-1})^{(k)}+(S^{i})^{(k)}(\theta_{y}^{i+1}-\theta_{y}^{i-1})^{(k+1)}-(S^{i})^{(k)}(\theta_{y}^{i+1}-\theta_{y}^{i-1})^{(k)}\Big]+$$

$$+\frac{1}{2\lambda}\Big[(\theta_{y}^{i})^{(k+1)}(S^{i+1}-S^{i-1})^{(k)}+(\theta_{y}^{i})^{(k)}(S^{i+1}-S^{i-1})^{(k+1)}-(\theta_{y}^{i})^{(k)}(S^{i+1}-S^{i-1})^{(k)}\Big]-$$

$$-\frac{\nu}{\lambda^{2}}(M_{y}^{i+1}-2M_{y}^{i}+M_{y}^{i-1})^{(k+1)}+\frac{Eh^{3}}{12\lambda^{4}}(w^{i+2}-4w^{i+1}+6w^{i}-4w^{i-1}+w^{i-2})^{(k+1)};\qquad(44)$$

$$\frac{\mathrm{d}(S^{i})^{(k+1)}}{\mathrm{d}\,y}=\rho h\frac{\partial^{2}(u^{i})^{(k+1)}}{\partial t^{2}}-(F_{x}^{i}+n_{x}^{i}+f_{x}^{i})^{(k+1)}-$$

$$-\frac{Eh}{\lambda^{2}}(u^{i+1}-2u^{i}+u^{i-1})^{(k+1)}-\frac{\nu}{2\lambda}(N_{y}^{i+1}-N_{y}^{i-1})^{(k+1)}-$$

$$-\frac{Eh}{2\lambda^{3}}\Big\{\Big[2(w^{i+1})^{(k+1)}(w^{i+1})^{(k)}-((w^{i+1})^{(k)})^{2}\Big]-2\Big[(w^{i})^{(k)}(w^{i+1})^{(k+1)}+(w^{i+1})^{(k)}(w^{i})^{(k+1)}-$$

$$-(w^{i+1})^{(k)}(w^{i})^{(k)}\Big]+2\Big[(w^{i})^{(k)}(w^{i-1})^{(k+1)}+(w^{i-1})^{(k)}(w^{i})^{(k+1)}-(w^{i-1})^{(k)}(w^{i})^{(k)}\Big]-$$

$$-\Big[2(w^{i-1})^{(k+1)}(w^{i-1})^{(k)}-((w^{i-1})^{(k)})^{2}\Big]\Big\};\qquad(45)$$

$$\frac{\mathrm{d}(M_{y}^{i})^{(k+1)}}{\mathrm{d}\,y}=(\hat{Q}_{y}^{i})^{(k+1)}+\Big[(N_{y}^{i})^{(k+1)}(\theta_{y}^{i})^{(k)}+(N_{y}^{i})^{(k)}(\theta_{y}^{i})^{(k+1)}-(N_{y}^{i})^{(k)}(\theta_{y}^{i})^{(k)}\Big]-$$

$$-\frac{1}{2\lambda}\Big[(S^{i})^{(k+1)}(w^{i+1})^{(k)}+(S^{i})^{(k)}(w^{i+1})^{(k+1)}-(S^{i})^{(k)}(w^{i+1})^{(k)}\Big]+$$

$$+\frac{1}{2\lambda}\Big[(S^{i})^{(k+1)}(w^{i-1})^{(k)}+(S^{i})^{(k)}(w^{i-1})^{(k+1)}-(S^{i})^{(k)}(w^{i-1})^{(k)}\Big]-$$

$$-\frac{2D_{M}(1-\nu)}{\lambda^{2}}\Big[(\theta_{y}^{i+1})^{(k+1)}-2(\theta_{y}^{i})^{(k+1)}+(\theta_{y}^{i-1})^{(k+1)}\Big]+\frac{\rho h^{3}}{12}\frac{\partial^{2}(\theta_{y}^{i})^{(k+1)}}{\partial t^{2}};\qquad(46)$$

$$\frac{\mathrm{d}(E_{x}^{i})^{(k+1)}}{\mathrm{d}\,y}=\frac{\partial(B_{z}^{i})^{(k+1)}}{\partial t}-\frac{1}{\sigma\mu\lambda^{2}}\Big[(B_{z}^{i+1})^{(k+1)}-2(B_{z}^{i})^{(k+1)}+(B_{z}^{i-1})^{(k+1)}\Big]+$$

$$+\frac{1}{2\lambda}\left[\frac{\partial(u^{i+1})^{(k+1)}}{\partial t}(B_{z}^{i})^{(k)}-\frac{\partial(u^{i-1})^{(k+1)}}{\partial t}(B_{z}^{i})^{(k)}+\frac{\partial(u^{i+1})^{(k)}}{\partial t}(B_{z}^{i})^{(k+1)}\right]-$$

$$-\frac{1}{2\lambda}\left[\frac{\partial(u^{i-1})^{(k)}}{\partial t}(B_{z}^{i})^{(k+1)}+\frac{\partial(u^{i+1})^{(k)}}{\partial t}(B_{z}^{i})^{(k)}-\frac{\partial(u^{i-1})^{(k)}}{\partial t}(B_{z}^{i})^{(k)}\right]+$$

$$+\frac{1}{2\lambda}\left[\frac{\partial(u^i)^{(k+1)}}{\partial t}(B_z^{i+1})^{(k)}-\frac{\partial(u^i)^{(k+1)}}{\partial t}(B_z^{i-1})^{(k)}+\frac{\partial(u^i)^{(k)}}{\partial t}(B_z^{i+1})^{(k+1)}\right]-$$

$$-\frac{1}{2\lambda}\left[\frac{\partial(u^i)^{(k)}}{\partial t}(B_z^{i-1})^{(k+1)}+\frac{\partial(u^i)^{(k)}}{\partial t}(B_z^{i+1})^{(k)}-\frac{\partial(u^i)^{(k)}}{\partial t}(B_z^{i-1})^{(k)}\right]-$$

$$-\frac{B_x^+ + B_x^-}{2}\frac{\partial}{\partial t}\left(\frac{w^{i+1}-w^{i-1}}{2\lambda}\right)^{(k+1)}; \quad (47)$$

$$\frac{\mathrm{d}(B_z^i)^{(k+1)}}{\mathrm{d}y}=\sigma\mu(E_x^i)^{(k+1)}+\sigma\mu\left[\frac{\partial(v^i)^{(k)}}{\partial t}(B_z^i)^{(k+1)}+\frac{\partial(v^i)^{(k+1)}}{\partial t}(B_z^i)^{(k)}-\frac{\partial(v^i)^{(k)}}{\partial t}(B_z^i)^{(k)}\right]-$$

$$-\frac{\sigma\mu}{2}\left(B_y^+ + B_y^-\right)\frac{\partial(w^i)^{(k+1)}}{\partial t}+\frac{B_y^+ - B_y^-}{h}. \quad (48)$$

The Lorentz forces in the equations can be linearized by the same method. Thus, we obtain a set of linear ordinary differential equations. All unknown variables can be found by the discrete-orthogonalization method in numerical computations.

5. Analysis of Numerical Results.

Figure 1 shows a thin rectangular plate made of aluminium in a magnetic field $\mathbf{B}=\{0, B_y, 0\}$. It bears the mechanical load $\mathbf{F}=\{0, 0, F_z\}$, alternating electric current that the density is $\mathbf{J_{cl}}=\{J_{clx}, 0, 0\}$ is exteriorly imported the plate. Let $E=7{,}1\times10^{10}\ \mathrm{N/m^2}$, $\nu=0{,}3$, $\rho=2670\ \mathrm{kg/m^3}$, $\sigma=3{,}63\times10^7(\Omega\cdot\mathrm{m})^{-1}$, $\mu=1{,}256\times10^{-6}\ \mathrm{H/m}$, $J_{clx}=J_x\sin\omega t\ \mathrm{A/m^2}$, $\omega=\pi\times10^2\sec^{-1}$, $F_z=80\ \mathrm{N/m^2}$, the length of the plate is $a=1$ m, the width of the plate is $b=0{,}5$ m, $h=2\times10^{-3}$ m.

The boundary conditions are

$$x=0:\ u=v=w=0,\ \theta_x=0; \quad (49\ \text{a-d})$$

$$x=1\ \mathrm{m}:\ u=v=w=0,\ \theta_x=0; \quad (50\ \text{a-d})$$

$$y=0:\ B_z=0.1\sin\omega t\,\mathrm{T},\ u=v=w=0,\ M_y=0; \quad (51\ \text{a-e})$$

$$y=0.5\ \mathrm{m}:\ B_z=0,\ N_y=0,\ \hat{Q}_y=0,\ S=0,\ M_y=0. \quad (52\ \text{a-e})$$

The initial conditions are

$$\mathbf{N}(x,y,t)\big|_{t=0}=0;\ \dot{u}(x,y,t)\big|_{t=0}=\dot{v}(x,y,t)\big|_{t=0}=\dot{w}(x,y,t)\big|_{t=0}=\dot{\theta}_y(x,y,t)\big|_{t=0}=0. \quad (53\ \text{a-e})$$

Programming Eqs. (39) – (48) and conducting computations for the known data and initial and boundary conditions yield the ten basic functions u, v, w, θ_y, N_y, $\hat{Q}_y$, S, M_y, E_x and B_z. Then the relations and variation laws between the mechanical and electromagnetic variables can be determined by changing the relevant parameters.

5.1. The Effect on the Deformation of Thin Plate by the Electromagnetic Parameters.

Figure 2 shows the deflection w distribution in the plate along the width direction (y direction) under the action of $F_z = 80\,\mathrm{N/m^2}$; $J_x = 0{,}5\,\mathrm{MA/m^2}$, $B_y = 0{,}2\,\mathrm{T}$; and a coupled field $(t = 14\,\mathrm{ms})$.

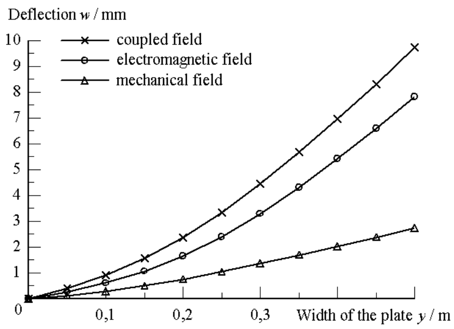

Fig. 2. Contrast diagrams of the deflection.

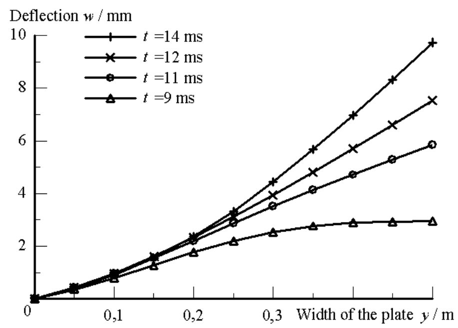

Fig. 3. Curves of the deflection distribution for $J_x = 0{,}5\,\mathrm{MA/m^2}$, $B_y = 0{,}2\,\mathrm{T}$, and different moment.

Fig. 3 shows the deflection w distribution in the plate along the width direction (y direction) for $J_x = 0{,}5\,\mathrm{MA/m^2}$, $B_y = 0{,}2\,\mathrm{T}$, and different moment. According to Fig. 3, the effect on the deformations of structures by electromagnetic loads is very obvious under the interaction of the mechanical loads and electromagnetic loads, it can’t be ignored.

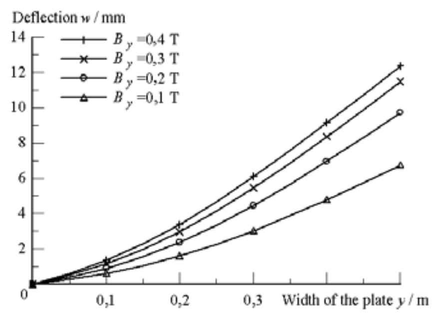

Fig.4. Curves of the deflection distribution for $J_x = 0{,}5\,\mathrm{MA/m^2}$, $t = 14\,\mathrm{ms}$, and different values of B_y.

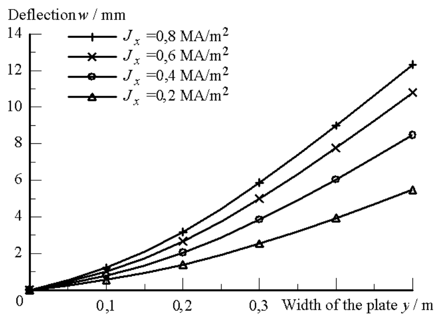

Fig. 5. Curves of the deflection distribution for $B_y = 0{,}2\,\mathrm{T}$, $t = 14\,\mathrm{ms}$, and different values of J_x.

Fig. 4 shows the deflection w distribution in the plate along the width direction (y direction) for $J_x = 0,5\ \text{MA}/\text{m}^2$, $t = 14\ \text{ms}$, and different magnetic induction intensity. According to Fig. 4, the deflection in the plate increases with increase in the magnetic induction intensity.

Fig. 5 shows the deflection w distribution in the plate along the width direction (y direction) for $B_y = 0,2\ \text{T}$, $t = 14\ \text{ms}$, and different electric current density. According to Fig. 5, the deflection in the plate increases with increase in the electric current density.

Fig. 6 shows the variation of the deflection in middle point of the free edge with time for $J_x = 0,5\ \text{MA}/\text{m}^2$ and different magnetic induction intensity, the other parameters being the same as above. Initially, the deflection varies a little with increase in the magnetic induction intensity. As time goes on, the deflection rapidly increases with the magnetic induction intensity, and peaking earlier. The deflection peaks somewhere between $t = 13\ \text{ms}$ and $t = 15\ \text{ms}$. It can be shown that the kinetic behavior of the plate can be changed by changing the magnetic induction intensity.

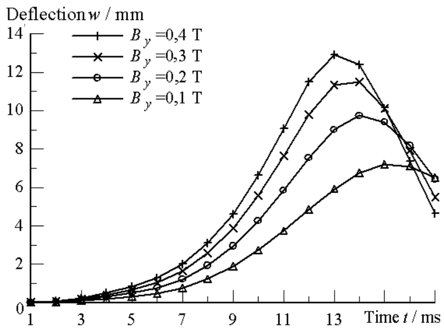

Fig. 6. The deflection in middle point of the free edge versus t for $J_x = 0,5\ \text{MA/m}^2$ and different values of B_y.

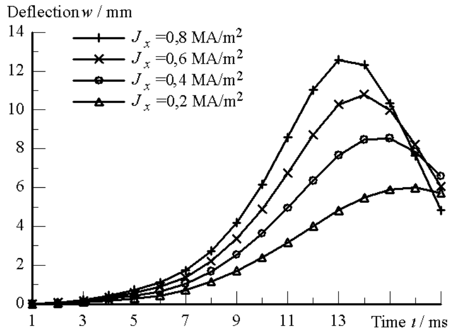

Fig. 7. The deflection in middle point of the free edge versus t for $B_y = 0,2\ \text{T}$ and different values of J_x.

Fig. 7 shows the variation of the deflection in middle point of the free edge with time for $B_y = 0,2\ \text{T}$, the same other parameters, and different electric current density. Initially, the deflection varies a little with the electric current density. As time goes on, the deflection rapidly increases with increase in the electric current density, and peaking earlier. The deflection peaks somewhere between $t = 13\ \text{ms}$ and $t = 16\ \text{ms}$. It can be shown that the kinetic behavior of the plate can be changed by changing the electric current density.

Fig. 8 shows the variation of the deflection in middle point of the free edge with the magnetic induction intensity for $J_x = 0,2\,\mathrm{MA}/\mathrm{m}^2$ and $t = 14$ ms, the other parameters being the same as above. According to Fig. 8, the deflection nonlinearly increases with the magnetic induction intensity.

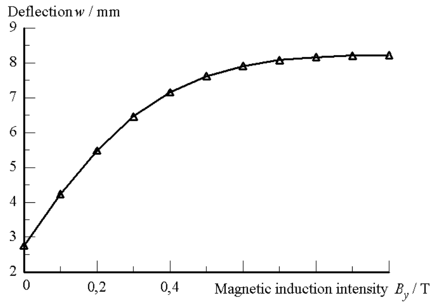

Fig. 8. The deflection in middle point of the free edge versus B_y for $J_x = 0,2$ MA/m^2 and $t = 14$ ms.

Fig. 9 shows the variation of the deflection in middle point of the free edge with the electric current density for $B_y = 0,2$ T and $t = 14$ ms, the other parameters being the same as above. According to Fig. 9, the deflection nonlinearly increases with the electric current density.

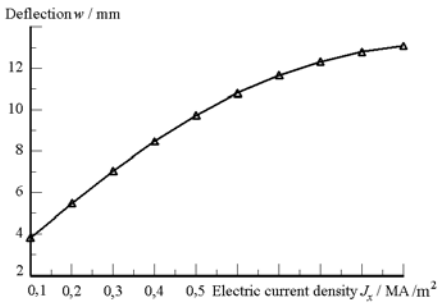

Fig. 9. The deflection in middle point of the free edge versus J_x for $B_y = 0,2$ T and $t = 14$ ms.

5.2. The Effect on the Stress of Thin Plate by the Electromagnetic Parameters.

Fig. 10 shows the variation of the stresses in middle point of the plate with time for $J_x = 0,5$ MA / m^2 and $B_y = 0,4$ T, the other parameters being the same as above. Curves a and b are the variation of normal stresses $\sigma_x^{\pm}$ along the x direction on the upper, lower surfaces of the plate with time, respectively; curves c and d are the variation of normal stresses $\sigma_y^{\pm}$ along the y direction on the upper, lower surfaces of the plate with time, respectively.

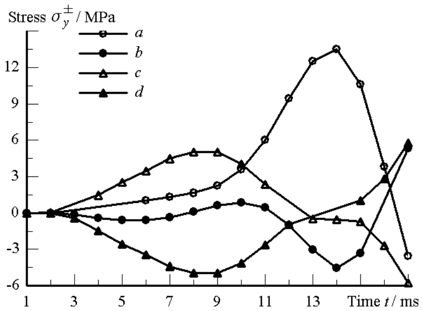

Fig. 10. The stress in middle point of the plate versus t for $J_x = 0,5$ MA/m^2 and $B_y = 0,4$ T.

Fig. 11 shows the stress distribution along the width direction (y direction) for $J_x = 1\,\mathrm{MA/m^2}$, $B_y = 0{,}4\ \mathrm{T}$, and $t = 17\ \mathrm{ms}$, the other parameters being the same as above. Curves a and b are the normal stresses $\sigma_x^{\pm}$ along the x direction on the upper, lower surfaces of the plate, respectively; curves c and d are the normal stresses $\sigma_y^{\pm}$ along the y direction on the upper, lower surfaces of the plate, respectively.

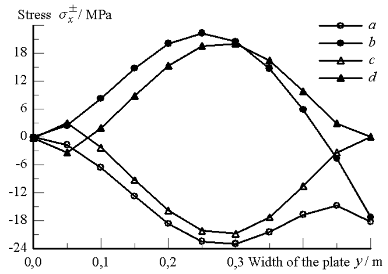

Fig. 11. Curves of the stress distribution for $J_x = 1\ \mathrm{MA/m^2}$, $B_y = 0{,}4\ \mathrm{T}$, and $t = 17\ \mathrm{ms}$.

Fig. 12 shows the variation of the normal stress σ_y^{+} along y direction on the upper surface in middle point of the plate with time for $J_x = 1\,\mathrm{MA/m^2}$ and different magnetic induction intensity, the other parameters being the same as above. Initially, the stress varies a little with increase in the magnetic induction intensity. As time goes on, the stress rapidly increases with the magnetic induction intensity, peaking generally at $t = 9\ \mathrm{ms}$.

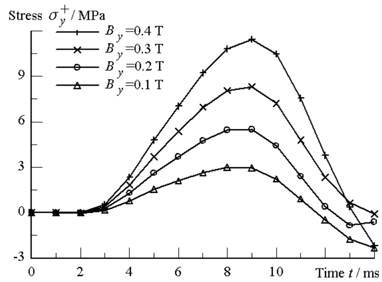

Fig. 12. The normal stress σ_y^{+} on the upper surface in middle point of the plate versus t for $J_x = 1\ \mathrm{MA/m^2}$ and different values of B_y .

Fig. 13 shows the variation of the normal stress σ_y^{+} along y direction on the upper surface in middle point of the plate with time for $B_y = 0{,}1\ \mathrm{T}$, the same other parameters, and different electric current density. Initially, the stress varies a little with the electric current density. After $3\ \mathrm{ms}$, the stress rapidly increases with increase in the electric current density, peaks at $t = 9\ \mathrm{ms}$, and then begins to decrease.

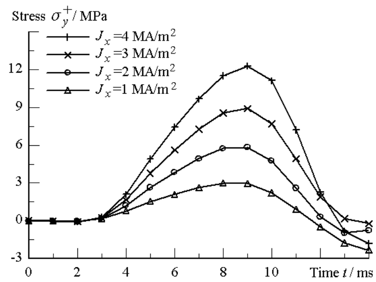

Fig. 13. The normal stress σ_y^{+} on the upper surface in middle point of the plate versus t for $B_y = 0{,}1\ \mathrm{T}$ and different values of J_x .

6. Conclusions.

Very important and complex problems in correlative mechanics domain are studied in this paper. Using the difference and quasilinearization methods, we have reduced nonlinear partial differential equations with ten basic unknown functions to a sequence of quasilinear differential equations, which can be solved by the discrete-orthogonalization method. Numerical solutions for two-dimensional magnetoelastic stresses and deformations in a thin current-carrying plate have been obtained. The relations between the stresses and deformations in a thin current-carrying plate under the interaction of an unsteady electromagnetic field and a mechanical load and the electromagnetic parameters have been discussed. This provides a method of theoretical analysis and numerical calculation for changing the service conditions and intensity researches of thin plates of engineering structures in an electromagnetic field. For example, we now know that:

(1) the effect of an electromagnetic field of low intensity on the displacements and stresses of structures and components is weak, this effect becoming stronger with increase in the electromagnetic field intensity; it is shown that the magnetoelastic analysis of structures and components in an electromagnetic field is necessary and very important;

(2) the stresses and deformations in a thin current-carrying plate nonlinearly increase with the magnetic induction intensity or the electric current density;

(3) the kinetic behavior of the plate can be changed by the magnetic induction intensity or the electric current density;

(4) the strain and stress states in a thin current-carrying plate can be controlled by changing the electromagnetic parameters;

(5) based on the work of this paper, some further studies for multidimensional magnetoelastic problems of elastic structures in complex electromagnetic fields can be carried out.

Acknowledgements.

This research was partially supported by a grant from the National Natural Science Foundation of China, the Foundation of Key Laboratory of Nonlinear Continuum Mechanics, Institute of Mechanics of Chinese Academy of Sciences. The author gratefully acknowledges these supports.

Р Е З Ю М Е . Вивчено двовимірні задачі для тонкої струмопровідної пластинки, що перебуває під дією нестаціонарних електромагнітного та механічного полів. Отримано нелінійні диференціальні рівняння типу Коші, що включають десять основних невідомих функцій. За допомогою різницевого методу і методу квазілінеаризації нелінійні рівняння магнітопружності зведено до послідовності квазілінійних диференціальних рівнянь, які можуть бути розв'язані методом квазілінеаризації. Для спеціального випадку отримано чисельний розв'язок щодо напружень і деформацій. Виявлено, що вони змінюються зі зміною електромагнітних параметрів. Показано, що напружено-деформований стан у тонкій пластинці може бути регульований зміною електромагнітних параметрів.

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Поступила 10.11.11 Утверждена в печать 30.09.14