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Stochastic Model Predictive Control for Hybrid Energy Systems

Microgrids are a promising approach for the integration of renewable energy sources in existing networks and the energy supply of rural areas. A cost effective option for a microgrid is given by a hybrid energy system, which combines e.g. diesel generators, photovoltaic panels and batteries as considered in this paper. However, the interaction of the components and uncertainties in the load demand and photovoltaic power make the controller design challenging. This paper discusses a Stochastic Model Predictive Control approach which yields promising results regarding effectiveness and reliability as shown in a simulation study.

Использование электроэнергетических микрогрид-систем является перспективным подходом к интеграции возобновляемых источников в существующие сети и энергообеспечение сельской местности. Экономическая эффективность электроэнергетических микрогрид-систем зависит от гибридной энергосистемы, объединяющей дизельные генераторы, фотоэлектрические панели и батареи. Однако взаимодействие составляющих и неопределенность графика нагрузки и фотоэлектрической энергии обусловливают необходимость создания блока управления. Рассмотрено применение стохастической модели для интеллектуального управления, что позволит обеспечить эффективность и надежность энергосистемы.

K e y w o r d s : microgrid, hybrid energy system, optimal energy dispatch, Stochastic Model Predictive Control.

Introduction. Global warming, a rising energy demand in developing countries and the energy supply of rural areas represent challenges for the energy supply of the future. To reduce carbon dioxide emission against global warming, the integration of renewable energy sources into the energy grid is essential. However, renewable energy sources like photovoltaic energy only provide temporary and fluctuating power affecting the reliability of the energy supply. In order to meet this challenge, decentralized energy generation with microgrids gained popularity. Furthermore, they provide the possibility to supply rural areas with electricity, which are not connected to the electricity grid. A cost effective option for microgrids are hybrid solutions, which use for example photovoltaic panels besides a diesel generator for energy generation and a battery for energy storage.

For the calculation of an optimal dispatch strategy for a hybrid energy systems, future values of the load demand and photovoltaic power are necessary and predicted by forecast algorithms. Due to the fluctuations in the load demand and photovoltaic power resulting from varying weather conditions, the forecast algorithms are not able to predict the future data accurately. This yields uncertainties, which affect the reliable and cost effective operation of the microgrid. For the consideration of these uncertainties in the control strategy of energy systems exist robust control [1, 2] or stochastic control approaches [3—5]. Robust control approaches guarantee an operation for worst case scenarios, but may result in conservative and more expensive operational strategies. In comparison, stochastic control approaches provide less conservative operational strategies since they consider the probability of a scenario and calculate expected values. The stochastic approaches include Stochastic Model Predictive Control (SMPC), which recently gained popularity for the control of energy systems, see [6—11]. The SMPC approaches mainly use two-stage stochastic programming [8—10], scenario based solution methods [11], dynamic programming with empirical mean [6] or the evaluation of the chance constraints with the cumulative distribution function [7].

This paper discusses an effective SMPC approach presented in a previous work [12], which is based on an analytical formulation of the chance constraints. This formulation is used for the evaluation of the cumulative distribution function of normal distributed photovoltaic power and load demand, in which the physical limits are considered by a set-based approach. This method is applied to a hybrid energy system, which is represented by a nonlinear mixed-integer model.

In the following, the model of the hybrid energy system as well as the forecast of the load demand and photovoltaic power is presented. Afterwards the stochastic optimization problem and the analytical formulation including the set-based approach are formulated, before the SMPC scheme is presented. Finally the effectiveness of the SMPC is illustrated in a simulation study with a real world scenario.

System modeling. The hybrid energy system consists of photovoltaic (PV) panels, lead-acid batteries and a diesel generator as described in [13]. The schematic diagram is illustrated in Fig. 1.

The load demand is covered by a diesel generator, PV panels and batteries. The load demand and diesel generator are connected to the PV panels and batteries through an AC/DC converter. The batteries are equipped with a battery

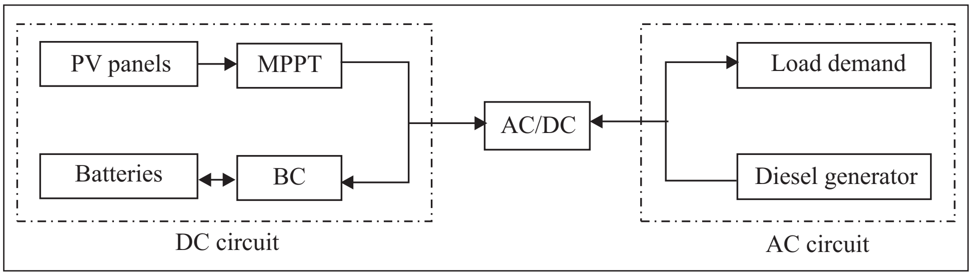

Fig. 1. Schematic diagram of the hybrid energy system according to [1]

charger (BC) and the PV panels are equipped with a converter, which includes a maximum power point tracker (MPPT). By the principle of conservation of energy, the power of the PV panels $P_{PV,k}$, the power of the diesel generator $P_{DG,k}$ and the power of the battery $P_{b,k}$ at time step k are related according to

$$\eta_{Load} P_{Load,k} + \eta_{bc} P_{b,k} - \eta_{AC/DC} P_{DG,k} - \eta_{MPPT} P_{PV,k} = 0, \qquad (1)$$

where the efficiencies of the load η_{Load}, the battery charger η_{bc}, the AC/DC converter $\eta_{AC/DC}$ and the converter with MPPT η_{MPPT} are considered. The efficiencies of the power electronics for the different energy flows are represented by nonlinear efficiency curves $\eta(P_{in})$ dependent on the input power of the component P_{in}.

The dynamics of the different components are explained in [13]. The electrical power generated by the PV panels depends on the type of mounting, the number of modules and strings as well as the irradiation data according to [14]. The dynamical behavior of the battery is described by the state of charge (SOC), which is described by the differential equation

$$\frac{\partial x_{SOC,k}}{\partial t} = \frac{-V_{OC} + \sqrt{V_{OC}^2 + 4P_{b,k} R_{Cell}}}{2R_{Cell} C_{Cell}}, \qquad (2)$$

where C_{Cell} is the nominal cell capacity. The open circuit voltage $V_{OC} = V_{OC}(x_{SOC})$ and internal resistance $R_{Cell} = R_{Cell}(x_{SOC})$ are given as nonlinear functions of the SOC, see [15]. The diesel generator is represented by a simplified, static model using continuous and discrete states. The operational state (on/off) of the diesel generator is described by $\delta_{st} \in \{0,1\}$ and the off time of the diesel generator by the discrete counter variable $\delta_{toff} \in \{0,1,\ldots,\delta_{toff,\max}\}$. The off time is necessary for the consideration of the switch on/shut down time of the

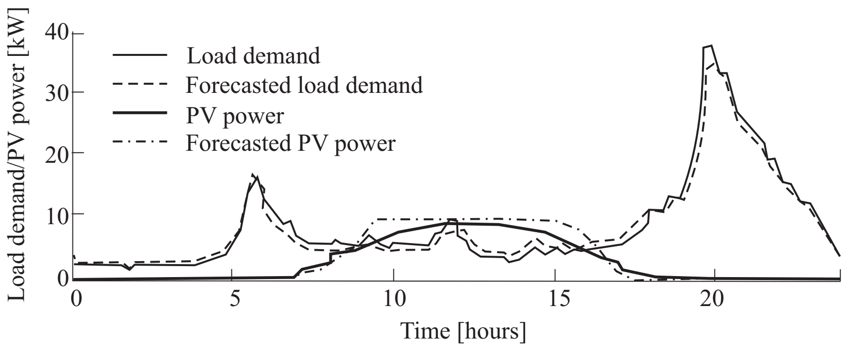

Fig. 2. Real and forecasted load demand and PV power for one day

generator. The generated power is described by the continuous variable $P_{DG,k} \in [0, P_{DG,\max}]$, which is used to define the off time at time step $k+1$ as

$$\delta_{toff,k+1} = \begin{cases} \delta_{toff,k} + 1 & \text{if } P_{DG,k+1} = 0, \\ 0 & \text{if } P_{DG,k+1} > 0. \end{cases} \quad (3)$$

The integer variables $\delta_{u,k}$ and $\delta_{d,k}$ are introduced for the consideration of the start up/shut down of the generators. They are given by

$$\begin{aligned} \delta_{u,k} &= \delta_{st,k}(1 - \delta_{st,k-1}), \\ \delta_{d,k} &= \delta_{st,k-1}(1 - \delta_{st,k}). \end{aligned} \quad (4)$$

Load and PV power forecast. The calculation of an optimal dispatch strategy is based on future values of the load demand and PV power. For real world applications, this future data is uncertain and needs to be predicted online. Therefore a combination of a seasonal auto regression integrated moving average model (SARIMA) and exponential smoothing is used, see [16, 17]. The forecast algorithm uses a linear combination of exponentially weighted past load and PV data to predict future values.

Fig. 2 shows an example of the real load demand and PV power as well as forecasted values resulting from the forecast algorithm for one day.

The quality of the forecast depends on the fluctuations in the PV power and load demand of different days. Due to similar load demand every night and zero energy output of the PV panels, the forecast values in the night are accurate. During the day, different weather conditions on different days influence the PV power and energy consumption making them difficult to predict with past data. Hence, it follows a deviation between the real and forecasted data especially for the fluctuating PV power.

Uncertainty description. For the consideration of the forecast error in the operational strategy, the uncertainty in the forecast needs to be quantified. For this reason, the forecasted variables of load demand and PV power are summarized to the net power, which represents a disturbance on the system and is defined as $w_k = P_{net,k} = P_{Load,k} - P_{PV,k}$.

The disturbance is assumed as a normal distributed random variable $w_k \sim N(w_k^{fc}, \sigma_{w,k}^2)$ with forecasted value w_k^{fc} and variance $\sigma_{w,k}^2$. The variance at time step k is calculated by an exponentially weighted sum of the squared forecast error of past days

$$\sigma_{w,k}^2 = \frac{1}{d}\sum_{i=1}^{d} q_i (w_{k-1440i}^{fc} - P_{net,\,k-1440i}^{real})^2,$$

where d is the number of past days, $P_{net,\,k-1440i}^{real}$ the real net power i days ago, $w_{k-1440i}^{fc}$ the forecasted disturbance i days ago and q_i are normalized weighting factors given by

$$q_i = e^{i-d} / \sum_{i=1}^{d} e^{i-d}.$$

The variance is calculated for every time step of the forecasted data yielding a time dependent function of the variance for the forecast horizon.

Stochastic optimization problem. The control strategy aims to minimize the expected operational cost of the hybrid energy system to cover the load demand in compliance with the operational constraints. The optimization variable is the trajectory of the diesel generator power represented by $u = [P_{DG,1}, \ldots$ $\ldots, P_{DG,N-1}]^T$ with time horizon N. As presented in [12], the stochastic optimization problem with the expected operational cost E_w is given by

$$\min_u E_w\left[J_N(x_N) + \sum_{k=0}^{N-1} (\delta J_k(x_k, u_k, w_k)) \right],$$

s.t. (1)—(4)

$$\delta_{toff,k}^{\min} \le \delta_{toff,k}, \quad 0 \le P_{DG,k} \le P_{DG}^{\max}, \tag{5}$$

$$P_{b,k}^{\min} \le P_{b,k} \le P_{b,k}^{\max}, \quad C_{rate,k}^{\min} \le \frac{P_{b,k}}{V_{bat} C_N} \le C_{rate,k}^{\max},$$

$$\Pr(x_{SOC,k} \le x_{SOC}^{\min}) \le \underline{p}_k, \quad \Pr(x_{SOC,k} \ge x_{SOC}^{\max}) \le \overline{p}_k.$$

As presented in [16], the operational cost is represented by the transition cost

$$\delta J_k(x_k, u_k, w_k) =$$

$$= c_{fuel}(u_k) + c_u \delta_{u,k} + c_d \delta_{d,k} + c_{<30\%}(u_k) + c_{om} \delta_{st,k} + c_{bat} |P_{b,k}|,$$

where $c_{fuel}(u_k)$ is the fuel cost as a function of the diesel generator power, $c_{u/d}$ are the start up/shut down costs of the diesel generator, $c_{<30\%}$ is a cost function for a diesel generator operation below 30%, c_{om} is the operation and maintenance cost and c_{bat} is the battery usage cost. Moreover, a terminal stage cost is defined to penalize a deviation of the SOC between the initial and end value

$$J_N = |x_{SOC,N} - x_{SOC,0}| V_{OC,DC} C_{Cell} c_{bat},$$

where $V_{OC,DC}$ is the open circuit voltage of the DC circuit. The constraints of the optimization problem (5) consider the energy balance by equation (1), the battery dynamics of equation (2) as well as the integer variables of the diesel generator given by equations (3) and (4). Furthermore, a minimum off time of the diesel generator $\delta_{toff,k}^{\min}$ needs to be fulfilled due to the start up and shut down time of the diesel generator. The operation of the diesel generator is limited by the maximum power $P_{DG,k}^{\max}$. The battery needs to fulfill the charge/discharge power limits $P_{b,k}^{\min/\max}$ and charging rate $C_{rate,k}^{\min/\max}$. The chance constraints consider the probability of a violation of an lower/upper limit of the SOC $x_{SOC}^{\min/\max}$, which needs to be below a specified probability $\underline{p}/\overline{p}$.

Analytical reformulation of chance constraints. For the formulation of the chance constraints, the relationship between the uncertainty $P_{net,k}$ and the SOC is determined analytically as presented in [12]. A linear differential equation is used to described the battery dynamics given by

$$\frac{\partial x_{SOC}(t)}{\partial t} = \frac{\eta_{bat}}{Q}(P_{DG,k} - P_{net,k}), \tag{6}$$

where Q is the battery capacity. Using the explicit Euler method, the SOC of the next time step is given by the linear relationship

$$x_{SOC,k+1} = \frac{\eta_{bat}}{Q}(P_{DG,k} - P_{net,k})\, t_s + x_{SOC,k}, \tag{7}$$

which represents an affine and invertible function $x_{SOC,k+1} = g(P_{net,k})$ of the SOC respect to the random normal distributed variable $P_{net,k}$ with probability density function (pdf) $f_{P_{net}}$. Under these conditions, the pdf of the SOC follows from the integral substitution rule as

$$f_{X_{k+1}}(x_{SOC,k+1}) = f_{P_{net,k}}(g^{-1}(x_{SOC,k+1}))\frac{1}{|g'(g^{-1}(x_{SOC,k+1}))|}.$$

Hence, the pdf of the SOC is given by

$$f_{X_{k+1}}(x_{SOC,k+1}) = \frac{1}{\sqrt{2\pi\sigma_{x,k}^2}}\exp\left(-\frac{(x_{SOC,k+1} - \mu_{k+1})^2}{2\sigma_{x,k}^2}\right)$$

with the variance of the SOC $\sigma^2_{x,k} = \sigma^2_{w,k} t_s^2 / Q^2$ and mean $\mu_{k+1} = (w_k^{fc} - P_{DG,k})\, t_S / Q + x_k$. For increasing the robustness of the method, these results are transferred for a longer time horizon. The SOC trajectory for a time horizon h is represented by

$$\bar{x}_{SOC} = [x_{SOC,k+1}, x_{SOC,k+2}, \ldots, x_{SOC,k+h}]^T.$$

According to equation (7), the mean value of the SOC trajectory follows as

$$\bar{x}_{SOC} = \gamma \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 1 & \cdots & 1 & 1 \end{bmatrix} (\bar{P}_{DG} - \bar{P}_{net}^{fc}) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} x_{SOC,k} \tag{8}$$

with $\gamma = \eta_{bat} t_s / Q$. The diesel generator power trajectory and the trajectory of the forecasted net power is defined by

$$\bar{P}_{DG} = [P_{DG,k}, P_{DG,k+1}, \ldots, P_{DG,k+h-1}]^T,\ \bar{P}_{net} = [P_{net,k}, P_{net,k+1}, \ldots, P_{net,k+h-1}]^T.$$

Based on the pdf of the SOC, the probability of SOC limit violations at time step $k+m$ with $m \in \{1, 2, \ldots, h\}$ are evaluated by the cumulative density function (cdf)

$$\Pr(x_{SOC,k+m} \le x_{SOC}^{\min}) = \int_{-\infty}^{x_{SOC}^{\min}} f_{X_{k+m}}(\bar{x}_{SOC})\, dx_{SOC,k+m}, \tag{9}$$

$$\Pr(x_{SOC,k+m} \ge x_{SOC}^{\max}) = \int_{x_{SOC}^{\max}}^{\infty} f_{X_{k+m}}(\bar{x}_{SOC})\, dx_{SOC,k+m}. \tag{10}$$

For the investigated system, the values of the load demand and PV power are limited which is not considered in the probabilities of equations (9) and (10). These limits are given by physical restrictions and the dimensioning of the PV panels. For the consideration of the limits, the formulation is extended by a set based approach as described in [12]. Therefore, the limits in the load demand and PV power are transferred to the net power, which is assumed to be in the set $P_{net,k} \in [P_{net,k}^{\min}, P_{net,k}^{\max}]$, where $P_{net,k}^{\min/\max}$ is the minimum/maximum limit of the net power. From equation (6) is concluded, that the SOC $x_{SOC,k+1}(t, P_{net,k})$ is a monotonically decreasing function of the net power and the cdf is monotonically decreasing/increasing with respect to the lower/upper bound. Hence, the probabilities for the set-based approach follow as

$$\Pr{}^*(x_{SOC,k+m} \le x_{SOC}^{\min}) = \int_{x_{SOC,k+m}(\bar{P}_{net}^{\max})}^{x_{SOC}^{\min}} f_{X_{k+m}}(\bar{x}_{SOC})\, dx_{SOC,k+m}, \tag{11}$$

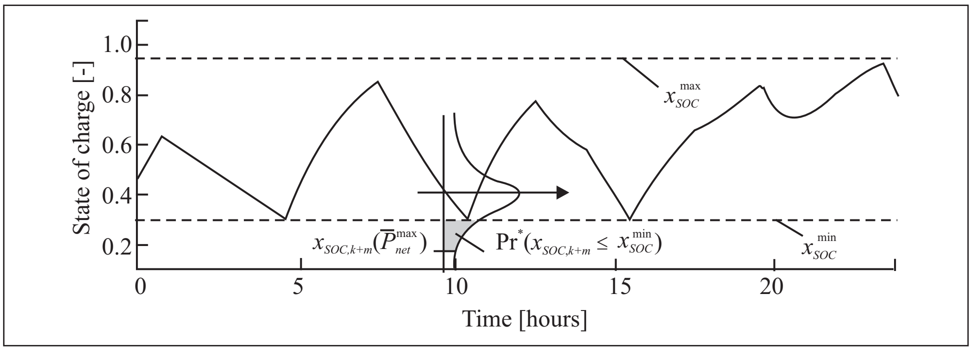

Fig. 3. Illustration of the set-based probability

$$\Pr^*(x_{SOC,k+m} \ge x_{SOC}^{max}) = \int_{x_{SOC}^{max}}^{x_{SOC,k+m}(\overline{P}_{net}^{min})} f_{X_{k+m}}(\overline{x}_{SOC})\, dx_{SOC,k+m},$$

where $\overline{P}_{net,k}^{min/\,max}$ is the trajectory of the minimum/maximum limit of the net power.

Fig. 3 shows an example of a SOC trajectory, where the probability of the set-based approach in equation (11) is illustrated for one time step.

Stochastic Model Predictive Control. Similar to common Model Predictive Control (MPC) approaches, the idea of the Stochastic Model Predictive Control (SMPC) is to repeatedly solve the stochastic optimization problem (5). The calculated optimal solution of the input trajectories for the first time steps are used to control the hybrid energy system. The battery SOC is measured after each iteration and considered in the initial value of the stochastic optimization problem. Furthermore, the forecast is updated in every iteration, which limits the influence of the forecast error.

In this paper, the stochastic optimization problem (5) is solved with discrete dynamic programming [18]. The sampling time of the load demand, the PV power as well as the optimal input trajectories is one minute. The horizon of the stochastic optimization problem is 24 hours and it is repeatedly solved every ten minutes. Due to the update interval of ten minutes, the normal distribution of the SOC in equation (8) is considered after ten time steps in which the diesel generators power is assumed to be constant.

To guarantee feasibility of the SMPC, the chance constraints are considered in the cost function by an additional transition cost

$$\delta J_{exc} = J_{pen}(\Pr^*(x_{SOC,k+10} \le x_{SOC}^{min}) + (\Pr^*(x_{SOC,k+10} \ge x_{SOC}^{max}))$$

with the penalty cost parameter J_{pen}.

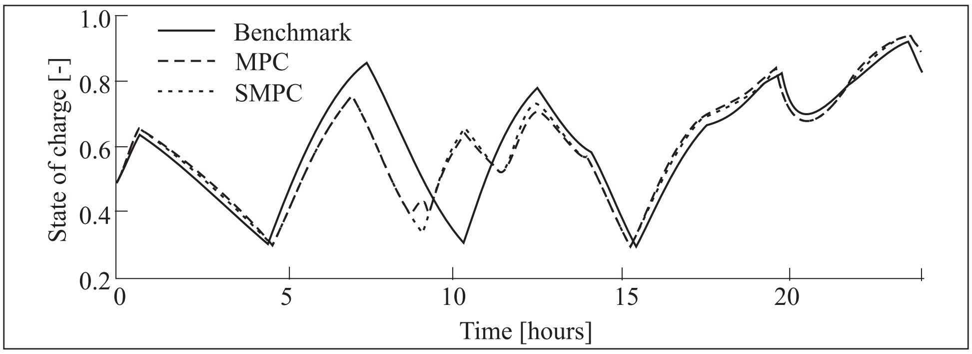

Fig. 4. SOC trajectory of the benchmark, MPC and SMPC for a horizon of one day

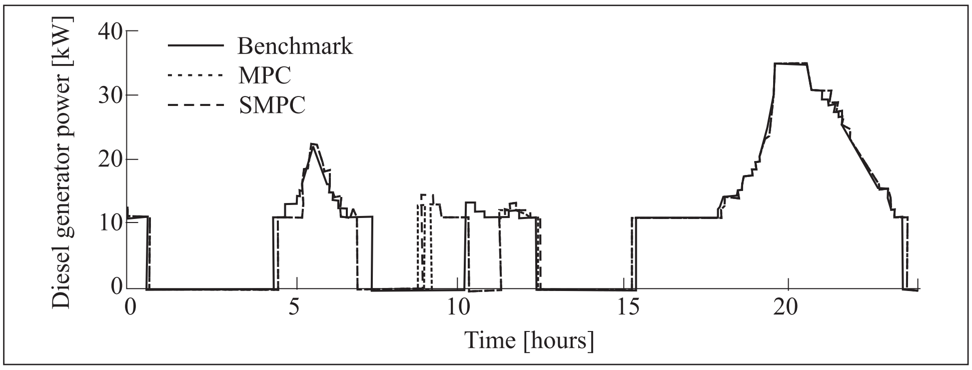

Fig. 5. Diesel generator power trajectory of the benchmark, MPC and SMPC for a horizon of one day

Simulation results. The effectiveness of the presented SMPC approach is illustrated by a simulation study with real world data from a village in Asia.

Fig. 4 shows an extract of the simulation results for the SOC over one day with a penalty cost parameter of $J_{pen} = 0.04$. The results include a benchmark solution, which uses a MPC with perfect knowledge of future data and represents the optimal solution, the solution with a regular MPC without chance constraints and the SMPC, which allows constraint violations of the SOC.

The MPC and SMPC are based on forecasted load and PV data from the SARIMA algorithm. In the beginning of the SOC trajectory, the uncertainties are small but increase in the afternoon due to the uncertain PV power and load demand in the evening. This yields differences in the SOC trajectory between the MPC and the SMPC since the latter considers the probability distribution of the forecasted PV power and load demand. As a result, the SOC of the SMPC approach moves closer to the optimal solution of the benchmark.

In Fig. 5 the corresponding trajectory of the diesel generator power is illustrated.

Comparison of control strategies

Parameter	Benchmark	MPC	SMPC
Fuel consumption [l/year]	40.894	42.878	41.348
LCOE [$/kWh]	0.5735	0.5809	0.5785
Computation time [s]	16.72	16.84	22.73

The trajectories of the diesel generator power are similar for the MPC and SMPC if the SOC is not in a critical region. If critical situations in the SOC occur, the MPC needs to utilize the diesel generator to avoid the violation of the SOC constraints. At around 9 and 15 hours the MPC switches on the diesel generator to avoid constraint violations. In comparison, the SMPC utilizes the diesel generator at around 9 hours less frequent. Furthermore, it activates the diesel generator at 15 hours slightly later since it takes the probability of violating the SOC constraints into account.

The Table shows the corresponding operational costs and computation time for the different control strategies. The costs include the fuel consumption of the diesel generator and the levelized cost of energy (LCOE). The illustrated costs are projected on a period of one year.

The SMPC approach yields cost reductions but still provides robustness for the illustrated example. The cost reductions are caused by a significant decrease in the fuel consumption due to the less frequent use of the diesel generator, which also affects the LCOE. In general, the reduction of the cost and robustness are contradictory objectives, which can be adjusted by the penalty cost parameter J_{pen}. Due to the higher computational complexity of the SMPC approach, its computation time exceeds the computation time of the MPC by 35 %. Since the computation time is still far below the update time of ten minutes and below the sampling time of one minute, the SMPC approach guarantees real-time capability.

Conclusion. This paper discusses an effective SMPC approach for the control of hybrid energy systems considering stochastic and set-based uncertainties. The approach is based on an analytical relationship between the uncertain load demand as well as PV power and battery SOC, which is used to formulate chance constraints. The chance constraints are further limited by set-based constraints given by physical limitations of the load demand and PV power. The approach is used for the calculation of the optimal power dispatch strategy of a hybrid energy system using discrete dynamic programming. It is shown that the SMPC approach yields better approximations of the optimal SOC trajectory for uncertain forecast data compared to a common MPC approach. This leads to significant reductions in the fuel consumption of the diesel generator but still provides robustness towards uncertainties, which is adjusted by a penalty cost parameter.

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