EFFECT OF DUST PARTICLES ON ELECTRON ENERGYDISTRIBUTION IN GLOW AND AFTERGLOW PLASMAS

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Analytical expressions describing electron energy probability functions (EEPFs) in glow and afterglow dusty plasmas are obtained from the homogeneous Boltzmann equation for electrons. At large energies in a glow dusty plasma, the quasiclassical approach for calculation of the EEPF is applied. Considering the afterglow case, the analytical expressions are obtained assuming that the electron energy loss is mainly due to momentum-transfer electron-neutral collisions and due to deposition of electrons on dust particles. Effect of dust particles on the EEPF is analyzed.

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INTRODUCTION

Dusty plasmas have been extensively studied in the last three decades because these complex ionized gas systems are of great interest in different fields [1-5]. At theoretical description of dusty plasmas, one usually assumes that electrons are in Maxwellian equilibrium [1, 2]. However, for most of industrial and laboratory plasmas, the electron energy probability function (EEPF) often deviates from Maxwellian because of the many different electron collision processes [3]. The profile of electron energy probability function affects different plasma parameters. Therefore, determination of the EEPF profile is very important in studying of different plasmas. For calculation of the EEPF in dusty plasmas, usually different numerical approaches are used. Here, we present analytical expressions describing the EEPF in glow and afterglow dusty plasmas.

1. MAIN EQUATIONS AND ASSUMPTIONS

We consider an argon dusty plasma maintained by an electric field $E(t)$. Plasma consists of electrons, ions with number densities n_e and n_i, respectively, and of dust particles of submicron size with density n_d and radius a_d. The plasma is assumed to be quasineutral, i.e., $n_e + n_d|Z_d| = n_i$, where Z_d is the dust charge (in units of electron charge e). It is also supposed that the ions have Maxwellian distribution with temperature T_i (= 0.026eV), but the electron energy probability function F in general is not Maxwellian and satisfies to the homogeneous Boltzmann equation [6]:

$$\frac{\partial F(u,t)}{\partial t} - \frac{2e}{3m_e\sqrt{u}}\frac{\partial}{\partial u}\left[\frac{u^{3/2}}{\nu_m}E^2\frac{\partial F(u,t)}{\partial u}\right] \approx S(F), \quad (1)$$

where u is the electron energy (in eV), t is the time, m_e is the electron mass, $E^2 = \frac{E_p^2}{2}\frac{\nu_m^2(u)}{\nu_m^2(u)+\omega^2}$ for the RF case, and $E = E_p$ is the external electric field in the DC case. Here, E_p is the RF field amplitude, $\omega = 2\pi f_E$, and f_E is the RF frequency. $\nu_m = \nu_{ed}^e + \nu_{em}$, where ν_{ed}^e and ν_{em} are the frequencies for momentum-transfer electron-dust and electron-atom collisions. $S(F)=S_{ea}(F)+S_{ed}(F)$, where $S_{ea}(F)$ and $S_{ed}(F)$ are the terms describing electron-atom and electron-dust collisions, respectively [3, 4]. We consider low-ionized dusty plasma, therefore, electron-electron collisions are not accounted for in the model. The homogeneous Boltzmann equation (1) may be used if the energy relaxation length is small compared to the spatial inhomogeneity scale of the discharge. For the case of a glow dusty plasma and for the afterglow with low dust density, the EEPF can be presented in the form $F_0 = n_e f_0(u)$. The function f_0 is normalized by $\int_0^\infty f_0(u)\sqrt{u}du = 1$.

It is assumed that the electron (I_e) and ion (I_i) currents to a floating dust particle are in balance, or $I_e + I_i=0$. The electron and ion currents to a dust particle are calculated using the orbit-motion-limited (OML) theory, taking into account the ion-neutral collisions in the sheath around a dust particle [1, 2, 4]. In general, the EEPF from Eq. (1) may be found only numerically. However, as it will be shown below, approximate analytical solutions of Eq. (1) also exist.

2. THE APPROXIMATE ANALYTICAL SOLUTIONS FOR THE EEPF IN GLOW DUSTY PLASMA

Assuming that the number of electrons with energy larger than the first excitation energy threshold ($u^* \approx 11.5$ eV for Ar) is small, one can neglect by transformation of electrons with large energy into low-energetic electrons, and the term describing inelastic electron-atom collisions can be presented in the following form [6] $S_{ea}^{exc}(F) \approx -\sum_k \nu_{ea}^k(u)F(u)u^{1/2}$, where ν_{ea}^k is the collision frequency of the k-th inelastic process with a threshold energy V_k..

As a result, the homogeneous Boltzmann equation (1) can be written as

$$\frac{\partial}{\partial u}\left[u^{3/2}(\delta_D\nu_{ed}^e(u) + \delta\nu_{em}(u))\left\{f_0(u) + \frac{1}{\beta}\frac{\partial f_0(u)}{\partial u}\right\}\right] \approx$$

$$\approx (\nu_{ed}^c(u) + \nu_{1\Sigma}(u))f_0(u)\sqrt{u}, \quad (2)$$

where δ =2m_e/m_a, δ_D =2m_e/m_d, m_a and m_d are the masses of neutral gas atoms and dust particles, respectively $\beta^{-1} = T_g + \frac{2e}{3m_e}\frac{E^2}{\nu_m(\delta_D\nu^e_{ed} + \delta\nu_{em})}$, $\nu_{1\Sigma}(u) = \sum_k \nu^k_{ea}(u)$ is the total frequency for inelastic electron-atom collisions including processes of excitation and ionization, ν^c_{ed} is the frequency describing deposition of electrons on dust particles [4].

At large electron energies ($u \geq u_1^* > u^* = 11.5$ eV, and by taking u_1^* = 20 eV), the electron energy probability function decreases rapidly with an increase of u. Therefore, the quasiclassical approach can be applied for calculation of the EEPF at large electron energies, and [6]:

$$f_0(u) \approx C_2 \exp(S(u)), \tag{3}$$

where C_2 is a constant, and $S(u) = -\int_{u_1^*}^{u} \beta_1(u')du'$ with

$$\beta_1(u) = \frac{1}{\sqrt{u}}\sqrt{\frac{\beta[\nu^c_{ed}(u) + \nu_{1\Sigma}(u)]}{\delta_D\nu^e_{ed}(u) + \delta\nu_{em}(u)}}.$$

At large electron energies the EEPF in a dusty plasma decreases faster than in a dust-free plasma because the ratio of β_1 in a dusty plasma to that in a dust-free plasma for the same electric field sustaining the plasmas is approximately $\sqrt{\frac{\nu^c_{ed}(u) + \nu_{1\Sigma}(u)}{\nu_{1\Sigma}(u)}}$. The decrease increases with an increase of n_d or a_d.

To calculate the EEPF at moderate and low energies $(u < u_1^*)$, we move from u to the new variable $y = u_1^* - u$. In this case, Eq. (3) can be presented in the following form

$$-\frac{\partial}{\partial y}\left[\alpha(y)\left\{f_0(y) - \frac{1}{\beta}\frac{\partial f_0(y)}{\partial y}\right\}\right] \approx$$
$$\approx [\nu^c_{ed}(y) + \nu_{1\Sigma}(y)]\, f_0(y)\sqrt{u_1^* - y} \tag{4}$$

where $\alpha(y) = u^{3/2}(\delta_D\nu^e_{ed}(u) + \delta\nu_{em}(u))$ with $u = u_1^* - y$. It follows from Eq. (4) that

$$f_0(y) = \left[C_2 - \int_0^y \beta(y')\chi(y')\exp\left(-\int_0^{y'}\beta(y'')dy''\right)dy'\right]\times$$
$$\times \exp\left(\int_0^y \beta(y')dy'\right), \tag{5}$$

where

$$\chi(y) = \frac{A - \int_0^y [\nu^c_{ed}(y') + \nu_{1\Sigma}(y')]\, f_0(y')\sqrt{u_1^* - y'}\,dy'}{\alpha(y)}.$$

To obtain Eq. (5), we assumed that the EEPF and it's derivative on energy are continious at $u = u_1^*$. The constant C_2 in Eqs. (3) and (5) can be found from the normalization condition $\left(\int_0^\infty f_0(u)\sqrt{u}du = 1\right)$. Dusty plasma parameters (the EEPF, effective electron temperature and dust charge) obtained using Eqs. (3) and (5) were compared with those calculated numerically by a finite-difference method with accounting for electron-electron collisions and transformation of high-energy electrons into low-energetic electrons at inelastic electron-neutral collisions. It was found that the analytical expressions (3) and (5) can be used for calculation of the EEPF and dusty plasma parameters at typical experimental conditions [7], in particular, in the positive column of a direct-current glow discharge and in the case of an RF plasma maintained by an electric field with frequency f=13.56 MHz. Moreover, in a 13.56 MHz plasma, the EEPF may become close to the Maxwellian distribution at an increase of dust density. That is in a qualitative agreement with numerical and experimental results of previous authors [4, 7].

3. THE APPROXIMATE ANALYTICAL SOLUTION FOR THE EEPF IN DUSTY PLASMA AFTERGLOW

Now consider a dusty plasma afterglow. The afterglow has the two characteristic times [6]: $\tau_1 = <\delta\nu_m(u)>^{-1}$ and $\tau_2 = <\nu_{1\Sigma}(u) + \nu^c_{ed}(u)>^{-1}$, where $<\ldots>$ denotes the averaging on time. The time τ_2 determines the energy relaxation of electrons with large energy ($u > u^*$). τ_1 is the time for energy relaxation of electrons in the EEPF core (with $u < u^*$). For $u > u^*$, $\nu_{1\Sigma}(u) >> \delta\nu_m(u)$, and, therefore, $\tau_2 << \tau_1$ [6]. In the afterglow, the number of electrons decreases first in the tail of the EEPF, and essential changes in the EEPF core take place at larger afterglow times ($t > \tau_2$).

Here, we will consider the afterglow times larger than τ_2. For these times, the effect of inelastic electron-atom collisions on the EEPF is small, and

$$S_{ea}(F) \approx \frac{1}{\sqrt{u}}\frac{\partial}{\partial u}\left[\frac{2m_e}{m_a}u^{3/2}\nu_m(u)\left(F + T_g\frac{\partial F}{\partial u}\right)\right], \tag{6}$$

where T_g is the neutral gas temperature, which is assumed to be equal to 300 K (0.026 eV).

It is supposed that the average electron energy is larger than the neutral gas temperature. We also assume that for the most of electron energies considered here the electron-atom elastic collisions dominate over the electron-dust momentum-transfer collisions. Therefore, the term in Eq. (1) describing electron-dust collisions simplifies to $S_{ed}(F) \approx -\nu^c_{ed}(u,t)F$, where $\nu^c_{ed}(u,t) = n_d\sigma^c_{ed}(u,t)\sqrt{2eu/m_e}$ is the frequency describing collection of electrons by dust particles with the cross-section $\sigma^c_{ed}(u) = \pi a^2_d(1 - \varphi_s(t)/u)$ for $u \geq \varphi_s(t)$ and 0 for $u < \varphi_s(t)$. Here, $\varphi_s(t)$ is the absolute value of dust surface potential. Taking into account these assumptions, Eq. (1) simplifies to the following equation:

$$\frac{\partial F(u,t)}{\partial t} - \frac{1}{\sqrt{u}}\frac{\partial}{\partial u}\left[\delta\nu_m(u)u^{3/2}F(u,t)\right] + \\ + F(u,t)\nu_{ed}^{c}(u,t) = 0. \qquad (7)$$

Here, we assume that the time-dependencies for dust charge $Z_d(t)$ and dust surface potential $\varphi_s(t)$ are exponential: $Z_d(t) = Z_{d0}\exp(-t/\gamma)$, $\varphi_s(t) = \varphi_{s0}\exp(-t/\gamma)$, where $\varphi_{s0} = \varphi_s(t=0)$, $Z_{d0} = Z_d(t=0)$, and γ is the time characterizing the dust charge decrease in afterglow. In this case, $\nu_{ed}^{c}(u,t) = \alpha(1-\varphi_{s0}\exp(-t/\gamma)/u)\sqrt{u}$ for $u > \varphi_s(t)$, where $\alpha = n_d \pi a_d^2 \sqrt{2/m_e}$. In this study, it is also assumed that the frequency for electron-atom collisions does not depend on electron energy ($\delta\nu_m \equiv \delta\nu = const$) and the EEPF for $t = 0$ can be presented in the following form [8]

$$F(u,0) = A_1\exp(-A_2 u^x)\,, \qquad (8)$$

where x is a number. For Maxwellian and Druyvesteyn EEPFs, $x = 1$ and $x = 2$, correspondingly.

$$A_2 = \frac{1}{<u>^x}\left[\frac{\Gamma(\xi_2)}{\Gamma(\xi_1)}\right]^x,$$

where $\xi_1 = 3/(2x)$, $\xi_2 = 5/(2x)$, $\Gamma(\xi) = \int_0^\infty t^{\xi-1}\exp(-t)dt$ with ξ >0 is the gamma function, and $<u>$ is the mean energy of electrons, which is connected with the effective electron temperature by expression $<u> = \frac{3}{2}T_{eff}$. If $<u> \ll u^*$, then

$$A_1 \approx \frac{x}{<u>^{3/2}}\frac{[\Gamma(\xi_2)]^{3/2}}{[\Gamma(\xi_1)]^{5/2}}.$$

Taking into account these assumptions and using the method of characteristics [9], one gets from Eq. (7) for $u < \varphi_s(t)$ the following expression for the EEPF:

$$F(u,t) = A_1\exp(3\delta\nu t/2 - A_2 u^x e^{x\delta\nu t})\,. \qquad (9)$$

If the initial EEPF is Maxwellian, the previous expression coincides with that presented in [6]:

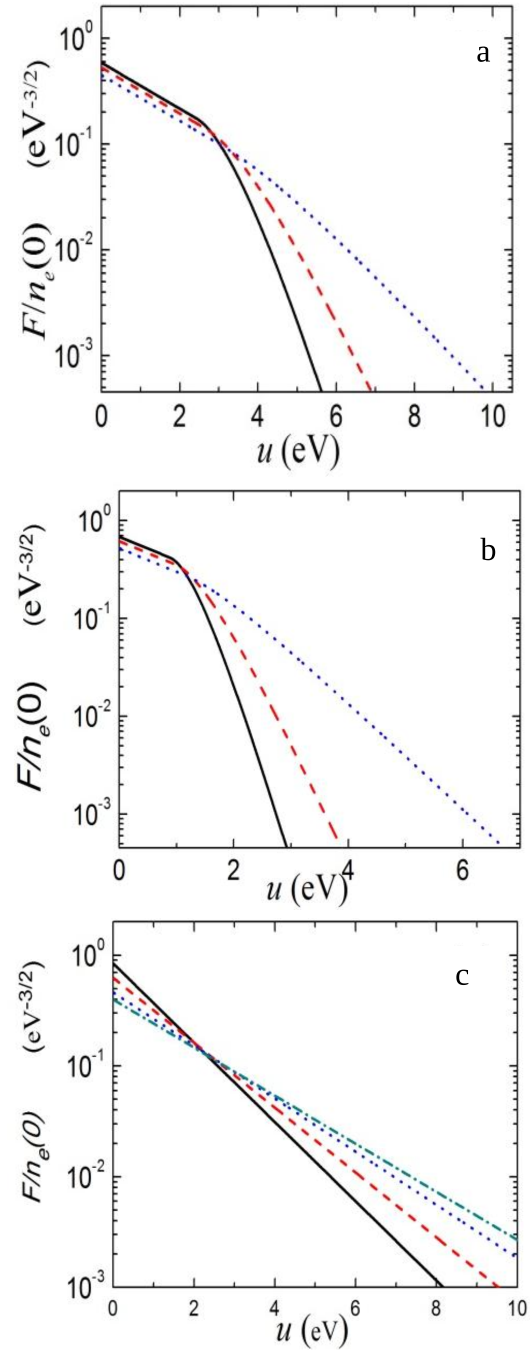
$$F(u,t) = C\exp\left\{-\frac{u}{T_{eff}}\exp(\delta\nu t)\right\}\exp(3\delta\nu t/2)\,. \qquad (10)$$

To find the EEPF at $u > \varphi_s(t)$, we also apply the method of characteristics and get from Eq. (7):

$$F(u,t) = A_1\exp\Bigg(\frac{3\delta\nu t}{2} - A_2 u^x e^{x\delta\nu t} - \frac{2\alpha_1\gamma}{2-\beta}\sqrt{\frac{\varphi_{s0}}{u}}e^{-t/\gamma} + \frac{2\alpha_1\gamma}{\beta}\sqrt{\frac{u}{\varphi_{s0}}} + \frac{4\alpha_1\gamma(\beta-1)}{\beta(2-\beta)}\left(\frac{u}{\varphi_{s0}}\right)^{\frac{1}{2(1-\beta)}} e^{\frac{\beta t}{2\gamma(1-\beta)}}\Bigg), \qquad (11)$$

where $\beta = \delta\nu\gamma$, and $\alpha_1 = n_d\pi a_d^2\sqrt{2\varphi_{s0}/m_e}$.

If the EEPF at $u < \varphi_s(t)$ is Maxwellian, it follows from Eq. (11) the following expression for the EEPF at $u > \varphi_s(t)$:



Normalized EEPF for t = 0 (a) and $t = 0.1/(\delta\nu)$ *(b); a_d=50 nm, n_e=10^9 cm^{-3} and different dust densities: n_d =5.0×10^7 cm^{-3} (solid curve), 3.0×10^7 cm^{-3} (dashed curve), 10^7 cm^{-3} (dotted curve). For $\varepsilon < \varphi_s(t)$, the EEPF is Maxwellian with T_{eff} =2 eB for t=0. Here, $\gamma = 0.1/(\delta\nu)$. The same for n_d =0 (c) and different afterglow times: t = 0 (dash-dotted curve); 0.1 /($\delta\nu$) (dotted curve); 0.3 /($\delta\nu$) (dashed curve); 0.5 /($\delta\nu$) (solid curve)*

$$F(u,t) = A_1 \exp\left(\frac{3\delta \nu t}{2} - \frac{u e^{\delta \nu t}}{T_{eff}} - \right.$$
$$\frac{2\alpha_1 \gamma}{2-\beta} \sqrt{\frac{\varphi_{s0}}{u}} e^{-t/\gamma} + \frac{2\alpha_1 \gamma}{\beta} \sqrt{\frac{u}{\varphi_{s0}}} + \quad (12)$$
$$\left. \frac{4\alpha_1 \gamma(\beta-1)}{\beta(2-\beta)} \left(\frac{u}{\varphi_{s0}} \right)^{\frac{1}{2(1-\beta)}} e^{\frac{\beta t}{2\gamma(1-\beta)}} \right).$$

Using Eqs. (10) and (12), the EEPF was calculated for different dust densities (see Figs.a,b). The case n_d =0 was also considered (Figure c).

One can see in Figure that the EEPFs for $\varepsilon > \varphi_s(t)$ in a dusty plasma differ essentially from those in a dust-free plasma. With an increase of dust density, the difference on Maxwellian distribution in the region increases which is accompanied by a decrease of a number of electrons at large energies ($\varepsilon > \varphi_s(t)$) and their increase at small energies ($\varepsilon < \varphi_s(t)$).

CONCLUSIONS

Thus, we have shown that the EEPF in dusty glow and afterglow plasmas can be described analytically. The results of analytical studies presented here are in a good agreement with numerical and experimental results on dusty plasma of previous authors. It has been shown that dust particles affect essentially the EEPF in glow and afterglow plasmas, decreasing the number of electrons at energies larger than the dust surface potential, while increasing their number at energies smaller than the dust surface potential.

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ВЛИЯНИЕ ПЫЛЕВЫХ ЧАСТИЦ НА РАСПРЕДЕЛЕНИЕ ЭЛЕКТРОНОВ ПО ЭНЕРГИИ В ПЛАЗМЕ В РЕЖИМАХ СВЕЧЕНИЯ И ПОСЛЕСВЕЧЕНИЯ

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Из однородного уравнения Больцмана получены аналитические выражения для функции распределения электронов по энергии (ФРЭЭ) в пылевой плазме в режимах свечения и послесвечения. Для расчёта ФРЭЭ в режиме свечения при больших электронных энергиях применён квазиклассический подход. Для плазмы в режиме послесвечения аналитические выражения получены в предположении, что потери энергии электронов происходят в основном за счёт упругих электрон-нейтральных столкновений и благодаря осаждению электронов на пылевые частицы. Проанализировано влияние пылевых частиц на ФРЭЭ.

ВПЛИВ ПИЛОВИХ ЧАСТИНОК НА РОЗПОДІЛ ЕЛЕКТРОНІВ ЗА ЕНЕРГІЄЮ В ПЛАЗМІ В РЕЖИМАХ СВІТІННЯ ТА ПІСЛЯСВІТІННЯ

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З однорідного рівняння Больцмана отримано аналітичні вирази для функції розподілу електронів за енергією (ФРЕЕ) в запорошеної плазмі в режимах світіння та післясвітіння. Для розрахунку ФРЕЕ в режимі світіння при великих електронних енергіях застосовано квазікласичний підхід. Для плазми в режимі післясвітіння аналітичні вирази отримані в припущенні, що втрати енергії електронів відбуваються в основному за рахунок пружних електрон-нейтральних зіткнень та завдяки осадженню електронів на пилові частинки. Проаналізовано вплив пилових частинок на ФРЕЕ.