PECULIARITY OF THE CHARGED PARTICLES DYNAMICS AT THE CYCLOTRON RESONANCES

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In this work the analysis was provided of the discrepancy between thresholds for appearance of the chaotic regime in the conditions of cyclotron resonances, obtained by analytical consideration of the particle dynamics, on the one hand, and by numerical investigation, on the other hand. The explanation is given for these threshold discrepancies.

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INTRODUCTION

Cyclotron resonance (CR) is one of those which are most of all studied and widely used, e.g., in many high-frequency generators and amplifiers (see for example [1]), for plasma heating, etc. When studying CR, important is the question on the conditions of implementation of regular and chaotic dynamics of particles. In [2, 3] the criteria allowing to define the conditions of the transition from regular dynamics of particles in CR to the chaotic dynamics were obtained. These criteria have been analyzed numerically in many papers, besides they have been confirmed in some experimental investigations [4, 5]. Therefore it would seem that now the question about conditions of regular and chaotic dynamics of particles in the CR conditions has finally been solved. However, some results of our numerical studies [4, 5], as well as studies of autoresonance [6, 7] demonstrate that in some important cases the particle dynamics does not correspond to the known criteria. In particular, the autoresonance study showed: it is impossible experimentally to achieve a high efficiency of energy transfer between the charged particles and waves [6]. In this paper, we try to give answers to some of the arising contradictions.

In section 2 the known conditions of occurrence of the regimes with dynamic chaos in conditions of CR are studied. Possible contradictions of numerical results and analytical criteria are indicated. The explanation of these contradictions is given.

In section 3 it is shown that in conditions of autoresonance the dynamics of particles is anomalously sensitive to external fluctuations. It is possible that just this sensitivity does determine the considerable decrease of efficiency of the energy transfer between particles and waves at autoresonance.

1. CONDITIONS OF OVERLAPPING RESONANCES

Let us consider the motion of a charged particle in plane electromagnetic wave with arbitrary polarization in the presence of constant external magnetic field $\vec{H}_0$

$$\vec{E} = \mathrm{Re}\,\vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)},\ \vec{H} = \mathrm{Re}(c/\omega)\left[\vec{k}\vec{E}\right],$$

$$\vec{E}_0 \equiv \left\{E_0\left(\alpha_x, i\alpha_y, \alpha_z\right)\right\}, \qquad (1)$$

where E_0 is the amplitude of electric field and $\vec{\alpha} \equiv \left\{\alpha_x, i\alpha_y, \alpha_z\right\}$ is the vector of wave polarization.

The equations of motion of charged particle in this case is

$$\frac{d\vec{p}}{dt} \equiv \frac{d}{dt}\left(\gamma m\vec{v}\right) = e\vec{E} + \frac{e}{mc}\left[\vec{v}\vec{H}_0\right] + \frac{e}{mc}\left[\vec{v},\vec{H}\right],$$

$$d\vec{r}/dt = \vec{p}/\gamma. \qquad (2)$$

Similar to works [2, 3], we introduce the following dimensionless variables:

$\tau = \omega t$, $\vec{p} \to \vec{p}/mc$, $\vec{r} \to k\vec{r}$, $\vec{k} \to \vec{k}c/\omega$, $\varepsilon_0 = eE_0/mc\omega$, $\vec{h} = \vec{H}_0/\left|\vec{H}_0\right|$, $\omega_H = eH_0/mc\omega$.

The electric field of the wave can be written in the form $\vec{\varepsilon} = \mathrm{Re}\,\vec{\varepsilon}e^{i\Psi}$, $\Psi = \vec{k}\vec{r} - \tau$, $\vec{k} \equiv \left\{k_x, 0, k_z\right\}$.

In these variables, the equation (2) reduces to:

$$\dot{\vec{p}} = \left(1 - \frac{\vec{k}\vec{p}}{\gamma}\right)\mathrm{Re}\left(\vec{\varepsilon}e^{i\Psi}\right) + \frac{\vec{k}}{\gamma}\mathrm{Re}\left(\vec{p}\vec{\varepsilon}\right)e^{i\Psi} + \frac{\omega_H}{\gamma}\left[\vec{p},\vec{h}\right],$$

$$\dot{\vec{r}} = \vec{p}/\gamma,\ \dot{\Psi} = \vec{k}\vec{p}/\gamma - 1. \qquad (3)$$

Below we will consider the simplest structure of the field of electromagnetic waves, i.e.:

$\vec{E} = \mathrm{Re}\left\{0, iE_y, 0\right\}$, $\vec{H} = \left\{0, 0, H_z\right\}$, $\vec{k} = \left\{k_x, 0, 0\right\}$

With this, the criterion of resonance overlapping, obtained in [3] can be written as:

$$\varepsilon_0 \geq \omega_H^2 \Big/ \left(16\left|W_s\right|k_x^2\right), \qquad (4)$$

where $W_s \equiv -p_\perp J_s'(\mu)$, $\mu \equiv k_x p_\perp/\omega_H$, $p_x = p_\perp\cos\theta$, $p_y = p_\perp\sin\theta$.

Using the first cyclotron resonance for estimation it is possible to write $k_x = 1$, $s = 1$, $W_1 = -p_\perp J_1'$.

In Fig. 1 the dependence of amplitude of the field expressed by formula (4) is presented. The whole area, located below the curve is that where the resonance overlapping conditions are not satisfied and chaotic dynamics should not arise.

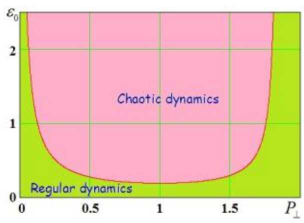

Fig. 1. Criterion of the resonance overlapping

For detail studying of the charged particle dynamics, numerical calculations of the system in the selected configuration (4) of the field of electromagnetic waves were carried out. The equations of particle motion for this case have the form:

$$\dot{x} = p_x/\gamma, \quad \dot{y} = p_y/\gamma,$$
$$\dot{p}_x = \left(p_y/\gamma\right)\left(-\varepsilon_0 \sin(\psi) + \omega_H\right), \tag{5}$$
$$\dot{p}_y = -\varepsilon_0 \sin(\psi) + \left(p_x/\gamma\right)\left(\varepsilon_0 \sin(\psi) - \omega_H\right),$$

In Fig. 2 the solutions of set of equations (5) are presented for $\varepsilon_0 = 5\cdot 10^{-4}$, $p_{x,y} = 0$, $p_z = 0$. As seen, in Fig. 2,a – momentum p_x, in Fig.2,b its spectrum and in Fig. 2,c – correlative function is represented. From these figures it is seen that for given values of the parameters dynamics of the particles is regular for all initial phases of the particle in the interval $0 \le x_0, y_0 < 2\pi$.

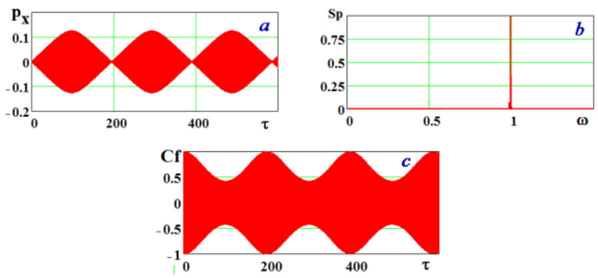

Fig. 2. Momentum of the particle p_x (a), its spectrum (b) and correlation function (c) at $\varepsilon_0 = 5\cdot 10^{-4}$, $p_{x,y} = 0$, $p_z = 0$, $x_0 = 0$, $y_0 = 0$

With increasing the field strength to values of the order of $\varepsilon_0 = 0.1$, more intensive beats occurs, however dynamics of particle remains regular. The spectrum is narrow, and the correlation function does not decrease (Fig. 3).

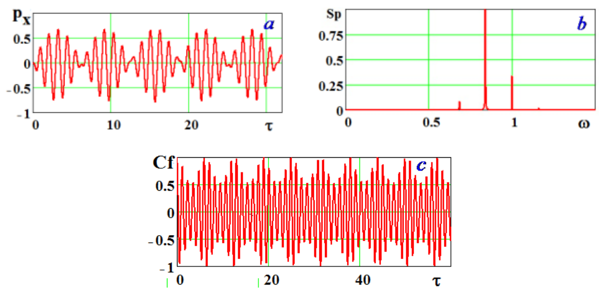

Fig. 3. Momentum of particle p_x (a), its spectrum(b) and correlation function (c) at $\varepsilon_0 = 0.1$, $p_{x,y} = 0$, $p_z = 0$, $x_0 = 0$, $y_0 = 0$

In this case the particle dynamics is also regular in the range of initial phases $0 \le \{x_0, y_0\} < 2\pi$.

When the field strength starts to exceed the value $\varepsilon_0 = 0.15$ with $p_{x,y} = 0$, $p_z = 0$, a qualitative change takes place in the dynamics of particle. Note that such field strength is still lower than the field necessary to fulfill the conditions of resonance overlapping. The criterion (4) is not satisfied.

At first glance, this result is in contradiction of the criteria (4) obtained above. The explanation of this result is as follows: initially and for a certain period of time the dynamics of particles is regular, they are in cyclotron resonance and gain the energy. However, after reaching a sufficient level of the transversal momentum an overlapping criterion (4) starts to hold.

Fig. 4 shows the solution of equations (5). Fig. 4,a shows momentum p_x, Fig. 4,b – its spectrum, and Fig. 4,c – correlation function. From these figures it is clear that for given values of the parameters of particle dynamics becomes nonregular. It should be noted that even with small changes in the initial coordinates of the particle changes its dynamics significantly.

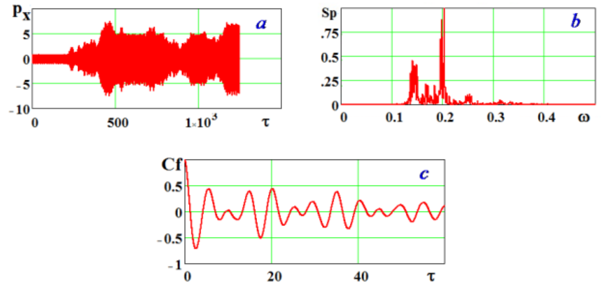

Fig. 4. Momentum of the particle p_x (a), its spectrum (b) and correlation function (c) at $\varepsilon_0 = 0.19$, $p_{x,y} = 0$, $p_z = 0$, $x_0 = 0$, $y_0 = 0$

As follows from [2. 3], in the general case the non-linear cyclotron resonances are all overlapping. As a result, the particles do continuously gain the energy (by diffusion law). With that, the particles have to fall into the area with the transverse momentum of the order $p_\perp = 1.8$. In accordance with the criterion (4), the field strength required for occurrence of the mode with dynamic chaos rises sharply. One would expect that with these values of transverse moment the energy which particles can get has to be stabilized. However, our numerical calculations demonstrate the lack of stabilization, i.e., the energy of particles is continuing to gain (see Fig. 4,a). The explanation of this result is that for these values of the parameters, the cyclotron resonances, remaining unaccounted in the criterion (4), start to play a determining role.

2. AND MULTIPLICATIVE FLUCTUATIONS

The second important case when one may expect the regimes with the regular dynamics to be realized is the case with conditions close to autoresonance conditions. Indeed, in these conditions, as was shown in [2, 3], with approaching autoresonance conditions the distance between cyclotron resonances is growing much faster than the width of the cyclotron resonance themselves. As a result, there is no nonlinear overlapping of resonances, and the regimes with dynamic chaos do not arise. However, as shown in [8], in the conditions of the autoresonance the dynamics of particles is anomalously sensitive to external fluctuations.

Below we consider this question for the simplest structure of the field of the electromagnetic wave propagating along the magnetic field $\vec{H}_0 \parallel z$,

$\vec{E} = \mathrm{Re}\{E_x, 0, 0\}$, $\vec{H} = \{0, H_y, 0\}$, $\vec{k} = \{0, 0, k_z = 1\}$.

It is assumed that autoresonant conditions are fulfilled, i.e., $R_s = k_z \upsilon_z + \left(s\omega_H/\gamma\right) - 1 \to 0$. Taking into account the small magnitude of the field amplitude ε_0, it is possible to linearize the system of equations (3) averaged over the fast-changing (non-resonant) phases [8]:

$$\frac{d\tilde{\gamma}}{d\tau} = -B\tilde{\theta}, \quad \frac{d\tilde{\theta}}{d\tau} = \alpha\tilde{\gamma} + f, \tag{6}$$

where $B \equiv (\varepsilon_0 W_1 / 2\gamma_0)\sin\theta_0$, $\theta_1 = \theta + \tilde{\theta}$, $\tilde{\theta} << 1$, $\tilde{\gamma} << 1$, $\alpha = (\partial R_{n0} / \partial\gamma)_{\gamma_0}$, $\gamma << \tilde{\gamma}$, $f = \tilde{\omega}_H$ – additive fluctuation force. At analytical approach it is considered that $f(\tau)$ is Gaussian, and is the δ – correlated random process with the mean value equals to zero:

$$\langle f(\tau) f(\tau') \rangle = 2D\delta(\tau - \tau'), \ \langle f \rangle = 0.$$

The magnitude of parameter α determines the closeness of conditions to autoresonance conditions. At exact autoresonance this parameter is equal to zero.

Numerical study of the temporal dynamics of the charged particles was carried out in the cases close to autoresonance. For calculations the value of parameter B was chosen as $B \sim 0.033$. The parameters α and B can be obtained with appropriate values of initial conditions for particles and parameter of the external field $\varepsilon_0 = 0.1$. The value of α is varied in the range from $\alpha = 5 \cdot 10^{-4}$ to $\alpha = 10^{-7}$. As the fluctuations, the random variable with uniform law of distribution in the range $-\Delta\omega_H, \Delta\omega_H$ was chosen with $\Delta\omega_H = 0.1$. The initial conditions for energy additives and phases: $\tilde{\gamma}(0) = 0, \ \tilde{\theta}(0) = \pi / 60$.

To find the increment of the mean square energy $\langle \tilde{\gamma}^2 \rangle$ the averaging over the ensemble of 40 realizations was carried out. In each realization the sequence of random numbers from the interval ($-\Delta\omega_H, \Delta\omega_H$) was generated by means of a random-number generator. The dimensionless time is measured in periods $\tau = t / T$.

The results of numerical analysis regarding the time dependence of the mean value of particle energy when parameter α is changed are given in Figs. 5, 6. Here the solid curves present the results of numerical calculations of the time dependence of the mean energy square for ensemble after 40 realizations. The dots indicate their approximation by a power function of the form $F(\tau) = D_{ad} \cdot \tau^{\nu}$ (D_{ad} - constant). In this case the values of parameter (for the fields and particles, as well as initial conditions for the temporal additives to energy and phase) remain constants.

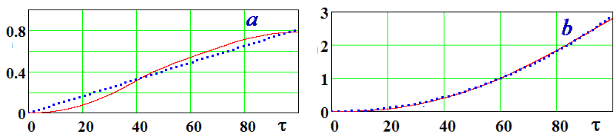

Fig. 5. The time dependence of the particle mean energy square at $\alpha = 5 \cdot 10^{-4}$ (a), $\alpha = 1 \cdot 10^{-4}$ (b)

As follows from graphs of Fig.5,a, the time dependence of the mean square of energy is close to linear: $D_{ad} \cdot \tau^1$. This corresponds to the well known law of energy diffusion of the type $D_{diff} \cdot \tau^{1/2}$. With decreasing α to $\alpha = 10^{-4}$, the time dependence of the mean square energy of particles changes qualitatively (it becomes quadratic, $D_{ad} \cdot \tau^2$ (see Fig. 5,b)), and an increase of the diffusion coefficient occurs.

Further reduction of the parameter α - $\alpha = 10^{-5}$ leads to change in the dependence of the mean square of the particle energy on the time. Parameter $\nu \approx 2.4$ and mean square of the energy is proportional to $D_{ad} \cdot \tau^{2.4}$. The iIllustration of such change of energy is shown in the graph of Fig. 6,a.

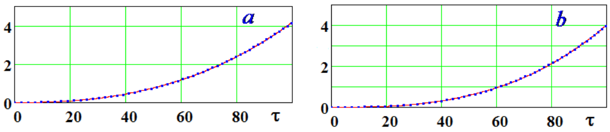

Fig. 6. Dependence of the mean square energy of the particle on time at a) $\alpha = 10^{-5}$, b) $\alpha = 10^{-7}$

A subsequent decrease of parameter α, from $\alpha = 10^{-5}$ to $\alpha = 10^{-7}$, is characterized by only minor change of the diffusion coefficient: the exponent grows to $\nu = 2.8$. The data for $\alpha = 10^{-7}$ are shown in Fig. 6,b.

As follows from these graphs, there is a rather reasonable agreement between results of numerical calculations and results obtained in approximation with the power function $D_{ad} \cdot \tau^{2.8}$.

These results demonstrate that appearance of the additive fluctuations, even very low amplitude, leads to initiation of a superdiffusion.

Now we consider the result of the presence of multiplicative fluctuations. Such fluctuations arise, for example, when there are fluctuations of the amplitude of the wave in which the particle moves. The most interest is the dynamics of the particle which is not located in the vicinity of the singular point of the "saddle", but in the vicinity of the "center". This is due to the fact that from the vicinity of the ‘saddle point’ the particles are removed exponentially rapidly. The equations for finding the temporal dynamics of small additives $\tilde{\gamma}$ and $\tilde{\theta}$ of particles that are near the points of the "center" of the mathematical pendulum in this case can be conveniently represented in the form [8];

$$\frac{du}{d\tau} = -(1 + f(\tau))\tilde{\theta}, \ \frac{d\tilde{\theta}}{d\tau} = u. \tag{7}$$

Note that in the system of equations (7) a new dimensionless time $\tau \equiv \omega \cdot t \cdot \sqrt{|\alpha \cdot B|}$ is introduced, and the connection between particle energy $\tilde{\gamma}$ and angle $\tilde{\theta}$ takes the form: $\tilde{\gamma} = \dot{\tilde{\theta}} \cdot \sqrt{|B / \alpha|}$. The numerical analysis of the equation (7) was carried out with the entry conditions: $u(0) = 0$, $\tilde{\theta}(0) = \pi / 60$, the amplitude of fluctuations $\Delta\omega_H = 0.1$. In Fig. 7 shown is the dependence of the mean square energy: the solid line is the results of numerical calculation, and the points indicate an approximation of the curve $F_{\exp}(\tau) = D_{mult} \cdot \exp(\delta\tau)$.

The exponential dependence of the mean square energy on time is clearly seen from data of Fig. 7.

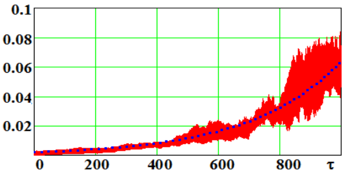

Fig. 7. Dependence of the mean square energy of the particle on time

CONCLUSIONS

Thus, the examples shown above demonstrate that the existing contradictions between numerical results and those obtained by the use of analytical formulas describing the dynamics of particles at cyclotron resonance, can be explained. The most important part of these explanations is the fact that in most analytical studies *the "non-resonant" cyclotron resonances are not taken into account*, meanwhile they can play a significant role. Indeed, as we saw in the 2nd part, exactly they determine the chaotic dynamics of particles in the region, for which an analytical criterion requires the regular dynamics to be applied. This is because the analytical criterion was obtained under conditions when only two adjacent cyclotron resonances are playing determinative role in the dynamics of particles. When the width of one of these nonlinear cyclotron resonances tends to zero, the obtained analytical criterion becomes not working. Conclusive in this case are the other "non-resonant" cyclotron resonances.

Even more significant role of "non-resonant" cyclotron resonances is becoming apparent in conditions close to the autoresonance. According to results of the 3rd part, under these conditions the particle dynamics is anomalously sensitive to both the additive and multiplicative fluctuations. It can be shown that the situation with large number of "non-resonant" cyclotron resonances can be modeled by influence of external fluctuations. Thus, it looks like the results of unsuccessful experiments in conditions similar to the autoresonance conditions can be explained. The discrepancy between the threshold of the mode with the dynamic chaos, defined by the analytical formula (4), and the numerical results for zero values of the initial particle pulse can be easily explained by the fact that being in the cyclotron resonance the particles are gaining the pulse (with a quite high rate) and are getting into the region corresponding to the regime with dynamic chaos (formula (4) becomes to be satisfied).

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ОСОБЕННОСТИ ДИНАМИКИ ЗАРЯЖЕННЫХ ЧАСТИЦ ПРИ ЦИКЛОТРОННЫХ РЕЗОНАНСАХ

В.А. Буц, В.В. Кузьмин, А.П. Толстолужский

Обсуждается расхождение в порогах появления хаотических режимов, которые получены при аналитическом рассмотрении и при численных исследованиях динамики частиц в условиях циклотронных резонансов. Дано объяснение этим расхождениям.

ОСОБЛИВОСТІ ДИНАМІКИ ЗАРЯДЖЕНИХ ЧАСТИНОК ПРИ ЦИКЛОТРОННИХ РЕЗОНАНСАХ

В.О. Буц, В.В. Кузьмін, О.П. Толстолужський

Обговорюється розбіжність у порогах появи хаотичних режимів, які отримані при аналітичному розгляді і при численних дослідженнях динаміки частинок в умовах циклотронних резонансів. Дано пояснення цим розбіжностям.