# ION MOTION IN CROSSED FIELDS AND SEPARATION MECHANISM IN "ARCHIMEDES PLASMA MASS FILTER"

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Trajectories of ions produced due to ionization of neutral particles (atoms, molecules) by electron impact in plasma are determined. The plasma is placed in crossed fields with positive radial electric field. It is shown that the light ions produced at the plasma periphery leave the plasma volume. Therefore, in the 'Archimedes Plasma Mass Filter' device, not only heavy but also light ions can hit a radial collector, what have to reduce essentially the degree of separation of light and heavy components of nuclear waste. The dependence of currents of light and heavy ions along radius (to the radial collector) and along device axis (to the end collector) are calculated. The conclusion is made about impossibility of complete separation of light and heavy components of nuclear waste in their single pass through the 'Archimedes Plasma Mass Filter' device.

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## INTRODUCTION

The shape of ion trajectory and its dependence on the ion mass define the efficiency of separation process for ions of different masses in devices with the crossed fields. One of such devices – Archimedes Plasma Mass Filter (APMF) - was suggested [1], constructed and investigated [2] to solve the problem of nuclear waste processing. It was reported about high efficiency of the device for separation of light (low radioactive waste) and heavy (high-level radioactive waste) components of waste, and about possibility of single-pass separation. Designs of plants for nuclear waste processing [3-5] and for recycling of spent nuclear fuel [3, 4] were considered with utilization of the APMF as a separating module.

The idea of separation of different mass ions in crossed fields, expressed in [1], appears to be very attractive, but, it seems that the separation process in APMF occurs not in a quite accordance with what was declared by its founders [1]. Not all light ions are confined in plasma volume. Under the action of the positive radial electric field directed outside, the light ions produced at the plasma periphery, come out of the plasma volume and hit the radial collector together with heavy ions. This circumstance reduces the degree of separation of light and heavy ion components. Because of electric fields, acting in APMF, the Larmor radius of a light ion is not small in comparison with plasma radius, thus, the portion of light ions which hit a radial collector also is not small. To estimate their quantity, we consider the motion of a separate ion in crossed fields (sections 2, 3), find the condition for the ion to be confined inside the plasma cylinder (section 4), and taking into account this condition the expressions for radial ion currents and longitudinal currents will be derived, and the degree of separation of light and heavy ions in the APMF will be estimated (section 5).

## 1. GENERAL SOLUTION OF EQUATION OF MOTION IN CROSSED FIELDS

1.1. We consider the case where the basic plasma has the form of cylinder with radius $a$ and is bounded by a metal casing. The plasma is placed in crossed homogeneous magnetic field with induction $B$, directed along the axis of cylinder (axis $z$), and radial electric field with intensity $E_r$. Radial electric field of any polarity can be created by means of end electrodes, by supplying the corresponding potentials, which are transported by electrons along the magnetic field into the plasma [6]. In such a way a positive radial electric field in APMF is created [1-3].

Ions of the admixture being the subject of separation, are produced within the whole volume of basic plasma and move without collisions. The equations of motion of particle in crossed fields (Cartesian geometry) look like

$$\begin{cases} \ddot{x} - \omega_{ci}\dot{y} = (e/m_i)E_x, \\ \ddot{y} + \omega_{ci}\dot{x} = (e/m_i)E_y, \qquad \ddot{z} = 0. \end{cases} \quad (1)$$

Here $E_x = -d\Phi_0/dx,\ E_y = -d\Phi_0/dy$ are the components of a radial electric field of the strength $E_r$; $\Phi_0(r)$ is the electric potential, $e$ is the ion charge, $\omega_{ci} = eB/m_i c$ is a cyclotron frequency.

We introduce a complex quantity $u \equiv x + iy = r\exp(i\varphi)$ and take into consideration that $E_x = E_r\cos\varphi$, $E_y = E_r\sin\varphi$, and $E_x + iE_y = (E_r/r)u$. Multiplying the second equation (1) by $i$ and combining it with the first equation, the equation of particle motion in the plane perpendicular to the magnetic field is obtained in a complex form:

$$\ddot{u} + i\omega_{ci}\dot{u} - (e/m_i)(E_r/r)u = 0 \ . \quad (2)$$

The electric potential is supposed to be a square-law function of radius, $\Phi_0(r) = \Phi_0(a)(r^2/a^2)$. ($\Phi_0(a) < 0$ in the case of positive field.) Then the quantity $E_r/r$ in (2) is constant and the equation (2) becomes the second order equation with constant coefficients. With

$$\Omega_i \equiv \omega_{ci}\left[1 - 4eE_r/(\omega_{ci}^2 m_i r)\right]^{1/2} \neq 0, \quad (3)$$

the solution of the equation (2) has the form [7]

$$u = C\exp(i\omega_i t). \quad (4)$$

Substituting (4) in (2), the characteristic equation for the frequency $\omega_i$ becomes:

$$\omega_i^2 + \omega_{ci}\omega_i + (e/m_i)E_r/r = 0. \quad (5)$$

The solutions of this equation are "slow" and "fast" rotation frequencies [8]

$$\omega_i = \omega_i^{\pm} = (1/2)\left[-\omega_{ci} \pm \Omega_i\right]. \quad (6)$$

The general solution of equation of motion (2) has the form [7]:

$$u = r\exp(i\varphi) = C_+\exp\left(i\omega_i^+ t\right) + C_-\exp\left(i\omega_i^- t\right), \quad (7)$$

where $C_+$ and $C_-$ are the complex constants. By letting

$$C_+ = \mathrm{R}\exp(i\theta),\; C_- = \rho\exp(i\vartheta), \quad (8)$$

and considering $R > 0$ and $\rho > 0$, the general solution (7) becomes

$$u = r\exp(i\varphi) = \mathrm{R}\exp\left[i\left(\theta + \omega_i^+ t\right)\right] + \rho\exp\left[i\left(\vartheta + \omega_i^- t\right)\right] \quad (9)$$

Quantities $R, \theta; \rho, \vartheta$ are constants. Their values are defined by initial values of coordinate $u|_{t=0}$ and speed $\dot{u}|_{t=0}$, as well as by the strengths of electric and magnetic fields.

Without electric field $\omega_i^+ = 0$ and $\omega_i^- = -\omega_{ci}$, and the equation (9) describes particle rotation about a Larmor center in negative direction (clockwise) with frequency $\omega_{ci}$. The quantities $R$, $\theta$ are cylindrical coordinates of the particle Larmor center, and $\rho, \vartheta$ are the quantities of Larmor radius and particle rotation phase along the Larmor circle in the moment of time $t = 0$. In the presence of radial electric field the quantities $R, \theta; \rho, \vartheta$ are the generalization of the mentioned particle coordinates for the case of crossed fields. It is interesting that the conception of a Larmor radius $\rho$ is generalized also for situation of zero magnetic field.

1.2. In a not very strong electric field

$$E_r < E_r^{cr}(m_i) \equiv m_i\omega_{ci}^2 r/(4e), \quad (10)$$

the frequencies $\Omega_i$ (3) and $\omega_i^{\pm}$ (6) are real values and particle motion (7), (9) is finite along radius. According to (9) the Larmor center rotates in azimuth direction with the "slow" rotation frequency $\omega_i^+$, and particle rotation along Larmor circle occurs with the "fast" frequency $\omega_i^-$. The electric field $E_r^{cr}$ (10) is called a “critical” electric field. We call the frequency $\Omega_i$ a "modified" cyclotron frequency. In [8], it is called a “vortex frequency”. Using (10) it is possible to present the expression for $\Omega_i$ (3) in the form

$$\Omega_i = \omega_{ci}\left(1 - E_r/E_r^{cr}(m_i)\right)^{1/2}. \quad (11)$$

The expression for $E_r^{cr}$ (10) is obtained from the condition $\Omega_i = 0$. By finding from this condition the ion mass $m_i$ or induction $B$, a critical mass can be found

$$m_{cr} = eB^2/\left(4c^2(E_r/r)\right) \quad (12)$$

or the critical value of magnetic field induction

$$B_{cr} = \left[4m_i c^2(E_r/r)/e\right]^{1/2}. \quad (13)$$

Expressions (10), (12) or (13) should be used depending on what particular quantity ($E_r$, $m_i$ or $B$) is variable in the experiment. If using (12) and (13) it is possible to present the expression for $\Omega_i$ in the form similar to (11)

$$\Omega_i = \omega_{ci}\left(1 - m_i/m_i^{cr}\right)^{1/2},\; \Omega_i = \omega_{ci}\left(1 - \left(B^{cr}\right)^2/B^2\right)^{1/2}. \quad (14)$$

2.3. As follows from (10), in the critical electric field $\Omega_i = 0$. Thus, the condition (3) is not fulfilled, and the general solution of the equation (2) has the form [7]

$$u = (C_+ + C_- t)\exp(-i\omega_{ci}t/2). \quad (15)$$

The radial motion of ion is finite only when $C_- = 0$, and it is infinite for any $C_- \neq 0$. The case $C_- = 0$ corresponds to azimuthal rotation with frequency $-(\omega_{ci}/2)$ of the ion produced at the point $u_0 = r_0\exp(i\varphi_0)$ with initial azimuthal speed $\dot{u}_0 = -i(\omega_{ci}/2)u_0$. This is the so-called Brillouin regime [9].

## 2. TRAJECTORIES OF IONS PRODUCED IN A STATE OF REST

2.1. The quantities $R, \theta; \rho, \vartheta$ in (9) define the form of ion trajectories. We suppose that ions under study are produced in crossed fields by ionization of neutrals (atoms, molecules) by electron impact. Because of big electron-ion mass difference, at small energies of ionizing electrons (typical for gas discharges), with good accuracy one may consider that ions are produced in a state of rest in any noticeable electric fields. The kinetic energy acquired in electric field does considerably exceed the initial ion kinetic energy, which is close to thermal energy of neutral. Let’s find the values $R, \theta; \rho, \vartheta$ for the ion produced in crossed fields in the state of rest at the point with coordinates $r_0, \varphi_0$:

$$u|_{t=0} = r_0\exp(i\varphi_0),\; \dot{u}|_{t=0} = 0. \quad (16)$$

We designate them as: $R_0, \theta_0; \rho_0, \vartheta_0$. As will be shown, just these quantities ultimately define the efficiency of the method of separation of light and heavy ions, realized in APMF [1-3]. If the value of positive radial electric field is less than the critical value ($0 < E_r < E_r^{cr}$), it can be found by substituting (9) in (16):

$$\begin{aligned} \mathrm{R}_0 &= -\left(\omega_i^-/\Omega_i\right)r_0 = \left(\omega_{ci}/\Omega_i + 1\right)(r_0/2),\; \theta_0 = \varphi_0, \\ \rho_0 &= -\left(\omega_i^+/\Omega_i\right)r_0 = \left(\omega_{ci}/\Omega_i - 1\right)(r_0/2),\; \vartheta_0 = \varphi_0 + \pi. \end{aligned} \quad (17)$$

Here $\Omega_i < \omega_{ci}$, $\omega_i^+ < 0$, $\omega_i^- < 0$. The ion trajectory has the form of epicycloid: the point on a generating circle of radius $\rho_0$ is rolling skidless on the outer side of the circle of radius $r_0$ in a negative azimuthal direction. From (17) it follows that $R_0 > r_0$, and $\rho_0$ can be less than $r_0$ or exceed it.

2.2. Let’s consider the case when the electric field is equal to the critical value (10) ($E_r = E_r^{cr}$). Representing in solution (15) the constants $C_+$ and $C_-$ in the form $C_+ = \mathrm{R}_0\exp(i\theta_0)$ and $C_- = (\omega_{ci}/2)\rho_0\exp(i\vartheta_0)$, we obtain from the initial conditions (16) the relationships

$$R_0 = \rho_0 = r_0, \quad \theta_0 = \varphi_0, \quad \vartheta_0 = \varphi_0 + \pi/2. \quad (18)$$

Thus, the solution of the equation of ion motion (2) takes the form:

$$u(t) = r_0 \exp(i\varphi_0)(1 + i\omega_{ci}t/2)\exp(-i\omega_{ci}t/2). \quad (19)$$

As follows from this expression, in the critical radial electric field the motion of the ion produced at rest, is radially infinite (when $r_0 \neq 0$). For such ions the Brillouin regime [9] is unrealized.

## 3. CONDITION OF ION CONFINEMENT

The condition of particle confinement in the plasma volume is formulated trivially: the maximum distance of particle from the axis should be less than the radius of metal casing $a$.In coordinates $R, \rho$ it takes the form [10]

$$R_0 + \rho_0 < a. \quad (20)$$

Using the expressions (17) we obtain the condition of ion confinement in positive radial electric field with the value less than the critical value (10), $0 < E_r < E_r^{cr}$:

$$r_0 < b_i \equiv (\Omega_i/\omega_{ci})a. \quad (21)$$

It is naturally to name the quantity $b_i$ as the radius of confinement of an $i$ – kind ion. Those ions born inside the cylinder of radius $b_i$ ($r_0 < b_i$) do not fall outside the limits of the plasma cylinder when moving along their trajectories. The ions produced on larger radii ($r_0 > b_i$), under the action of electric field fall outside the plasma limits and will hit the chamber wall or the collector.

The quantity of confinement radius $b_i$ (21) depends on the strengths of electric and magnetic fields, and on the ion mass. If the strength of radial electric field $E_r$ approaches the critical value (10) for the $i$ – kind ion, the confinement radius $b_i$ (21) decreases to zero. The radius $b_i$ depends on an ion mass $m_i$, therefore in the mixture of different kind ions the radii $b_i$ will be different: for ions with heavier masses $b_i$ is less than for lighter ions.

Note, that conditions (20) and (21) can be hold only for light ions, $m_i < m_{cr}$.

## 4. SEPARATION OF LIGHT AND HEAVY IONS IN APMF

5.1. The authors of [1-3] represent the process of ion separation in APMF as follows. In the basic plasma of a cylindrical form placed in the crossed fields the ionization process of vapour of a nuclear waste introduced from the outside does occur. Thus, within the basic plasma a secondary plasma is produced consisting of many kinds of ions.

Depending on mass, ions are divided into two groups: heavy ions ($H$) with masses $m_{i'} > m_{cr}$, and light ions ($L$) with masses $m_i < m_{cr}$. The heavy ions are not confined radially. They are thrown out from the plasma, hit a radial collector and generate a radial ion current $I_r$. The light ions are confined radially. Making radial oscillations, they drift in a longitudinal direction, get on the face collectors and generate a longitudinal ion current $I_z$.

This process can be characterized mathematically, as it is done below.

Let $Q$ is the total power of ion production (total quantity of ions of secondary plasma produced per 1 sec within the whole plasma volume), $Q^L$ is the total power of light ion production ($L$), $Q^H$ is total power of heavy ion production ($H$). Power $Q^L$ and $Q^H$ are caused by many kinds of ions, so

$$Q^L = \sum_i Q_i^L, \quad Q^H = \sum_{i'} Q_{i'}^H, \quad Q = Q^L + Q^H = I. \quad (22)$$

Here $Q_i^L$ and $Q_{i'}^H$ are the total power of light ion production ($L$) of $i$ – kind and of heavy ion production ($H$) of $i'$ – kind, $I$ is a total current of secondary ions from plasma volume.

According to [1-3], the complete separation of light and heavy ions has to be realized in APMF:

$$Q^L = I_z, \quad Q^H = I_r. \quad (23)$$

5.2. However, as shown in section 4, the light ions produced at the plasma periphery, at radii $a > r > b_i$, also leave the plasma cylinder radially. In the APMF these ions will hit a radial collector together with heavy ions. Thus, the total radial current increases in comparison with (23) at the expense of the light ions produced at the periphery of plasma:

$$I_r' = \sum_{i'} Q_{i'}^H + \sum_i Q_i^L \left(1 - \frac{\Omega_i^2}{\omega_{ci}^2}\right) = Q^H + \sum_i Q_i^L \frac{E_r}{E_r^{cr}(m_i)}. \quad (24)$$

Respectively, the total current of the light ions leaving the plasma to the end face collectors decreases in comparison with (23) and becomes:

$$I_z' = \sum_i Q_i^L \frac{\Omega_i^2}{\omega_{ci}^2} = \sum_i Q_i^L \left(1 - \frac{E_r}{E_r^{cr}(m_i)}\right) = Q^L - \sum_i Q_i^L \frac{E_r}{E_r^{cr}(m_i)}. \quad (25)$$

For simplicity the production power of $i$ – kind ion $Q_i^{L,H}$ in (24), (25) is supposed to be homogeneous on radius. It's obvious, that $I = I_z + I_r = I_z' + I_r'$.

As follows from eqs (24), (25) the ion currents depend linearly on the strength of radial electric field. Just such dependence on radial electric field was observed in the experiments in APMF [3] for the amount of the material deposited on collectors (radial and face).

As formula (24) predicts, it is impossible to separate completely the light (not radioactive) ion component of a waste from the heavy (radioactive) ion component in a single-pass through the APMF device, as the authors of [1-3] declared. In reality, the light component contributes in the radial current $I_r'$, and its contribution is big in comparison with the heavy component contribution. This is because in a nuclear waste the value $Q^L$ exceeds the value $Q^H$ by an order of magnitude, and the field $E_r$, generally speaking, is not low, but of the order of $E_r^{cr}(m_i)$ ($E_r \lesssim E_r^{cr}(m_i)$).

The composition of the light component of waste in a radial current $I_r'$ (24) differs from the initial value (the first equality (22)): it is impoverished by lighter ions. In the longitudinal current $I_z'$ (25) - on the contrary: the composition of light ions is depleted by less light ions.

In the considered schema of separation in the APMF device the radial current $I_r'$ (24) should be considered as a product, and a longitudinal current $I_z'$ (25) – as a waste.

If the current settled on a radial collector (24) is passed through APMF once again, the outlet currents will be equal:

$$I_r'' = \sum_{i'} Q_{i'}^H + \sum_i Q_i^L \left(1 - \frac{\Omega_{i1}^2}{\omega_{ci1}^2}\right)\left(1 - \frac{\Omega_{i2}^2}{\omega_{ci2}^2}\right) = \sum_{i'} Q_{i'}^H + \sum_i Q_i^L \left(\frac{E_{r1}}{E_r^{cr1}(m_i)}\right)\left(\frac{E_{r2}}{E_r^{cr2}(m_i)}\right), \quad (26)$$

(26)

$$I_z'' = \sum_i Q_i^L \left(1 - \frac{\Omega_{i1}^2}{\omega_{ci1}^2}\right)\frac{\Omega_{i2}^2}{\omega_{ci2}^2} = \sum_i Q_i^L \frac{E_{r1}}{E_r^{cr1}(m_i)}\left(1 - \frac{E_{r2}}{E_r^{cr2}(m_i)}\right). \quad (27)$$

In these relations the indexes 1, 2 correspond to quantities after the first or after the second pass through APMF. If we estimate $E_{r1,2} / E_r^{cr1,2} \sim 1/2$, the current of light ions in (26) does still exceed the current of the heavy ions. Thus, after the second pass the situation is still far from the complete separation of light and heavy components of nuclear waste.

## CONCLUSIONS

1. The confinement condition of light ions (21) in a volume of plasma placed in crossed fields with positive radial electric field is found.
2. The currents of ions to radial and to end face collectors in the APMF are evaluated taking into account a confinement condition (21).
3. The conclusion is made that the complete separation of light and heavy components of a nuclear waste is impossible in a single pass through the APMF.

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## ДВИЖЕНИЕ ИОНОВ В СКРЕЩЕННЫХ ПОЛЯХ И МЕХАНИЗМ РАЗДЕЛЕНИЯ В "ARCHIMEDES PLASMA MASS FILTER"

***Ю.Н. Елисеев***

Определены траектории ионов, которые образовались в результате ионизации нейтралей (атомов, молекул) электронным ударом в плазме в скрещённых полях при положительном радиальном электрическом поле. Показано, что в таком поле лёгкие ионы, образовавшиеся на периферии плазмы, выходят за её пределы. В "Archimedes Plasma Mass Filter" это должно приводить к попаданию на радиальный коллектор не только тяжёлых, но и лёгких ионов, что существенно снизит степень разделения лёгкой и тяжёлой компонентов отходов ядерного производства. Рассчитаны потоки лёгких и тяжёлых ионов по радиусу (на радиальный коллектор) и вдоль оси устройства (на торцевой коллектор). Сделан вывод о невозможности полного разделения лёгкого и тяжёлого компонентов отходов в "Archimedes Plasma Mass Filter" за один проход.

## РУХ ІОНІВ У СХРЕЩЕНИХ ПОЛЯХ І МЕХАНІЗМ ПОДІЛУ В "ARCHIMEDES PLASMA MASS FILTER"

***Ю.М. Єлісеєв***

Визначені траєкторії іонів, які утворилися в результаті іонізації нейтралей (атомів, молекул) електронним ударом у плазмі в схрещених полях при позитивному радіальному електричному полі. Показане, що в такому полі легкі іони, що утворилися на периферії плазми, виходять за її межі. У "Archimedes Plasma Mass Filter" це повинне приводити до влучення на радіальний колектор не тільки важких, але й легких іонів, що суттєво знизить ступінь поділу легкої й важкої компонентів відходів ядерного виробництва. Розраховані потоки легких і важких іонів по радіусу (на радіальний колектор) і уздовж осі пристрою (на торцевий колектор). Зроблений висновок про неможливість повного поділу легкої та важкої компонентів відходів в "Archimedes Plasma Mass Filter" за один прохід.