APPLICATION OF NUCLEAR METHODS

APPROXIMATION OF THE DEUTERON WAVE FUNCTIONS AND POLARIZING CHARACTERISTICS FOR Reid93 POTENTIAL

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Numerical coefficients of analytical forms for deuteron wave function in configuration representation for Reid93 potential are designed. The obtained wave functions do not contain superfluous knots. The designed parameters of a deuteron well agree with the experimental and theoretical data. The polarization performances T_{20} and A_{yy} designed on wave functions are proportionate with earlier published.

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INTRODUCTION

The deuteron is most the elementary nucleus, that will consist from two strongly interreacting particles (a proton and a neutron). Simplicity of a structure of a deuteron makes by its convenient laboratory for the study of nucleon-nucleon forces. Currently the deuteron is well investigated observationally and theoretically.

The experimental defined values of static performances of deuteron well agree with theoretical calculations [1]. Despite of it, there are some theoretical inconsistencies. For example, one (for CD-Bonn potential) or both (for Moscow potential) components of the deuteron wave function have knots [2, 3] about an origin of coordinates. Such behavior of a wave function contradicts the common mathematical theorem of number of knots of eigenfunctions of boundary value problems [4]. Presence of the knots in wave functions of the basic and unique state of a deuteron testifies to inconsistencies and inaccuracies in embodying numerical algorithms in a solution of detailed problems. Influence of a select of numerical algorithms on solutions is reduced in Ref. [5 - 7].

Such potentials a nucleon-nucleon interaction as Bonn [2], Moscow [3], potentials of Nijmegen group (NijmI, NijmII, Nijm93 [8]), Argonne v18 [9] or Paris [10] potential have uneasy enough structure and bulky record. The original potential Reid68 [20] was parameterized on the basis of phase analysis in Nijmegen group and has received title Reid93. The parametrization has been carried out for 50 parameters potential, and χ^2/N_{data}=1.03 [8].

Besides deuteron wave function can be submitted as the table: through corresponding files of values of radial wave functions. Sometimes at numerical calculations to operate with such files of numbers difficultly enough. And the text of programs for numerical calculations is overloaded. Expedient reception of more simple analytical forms of representation of deuteron wave functions therefore is.

1. ANALYTICAL FORM OF THE DEUTERON WAVE FUNCTIONS

Known numerical values of a radial wave function of a deuteron in coordinate representation can be approximated with the help of convenient expansions [11] in the analytical shape:

$$\begin{cases} u_a(r) = \sum_{i=1}^{N_a} A_i \exp(-a_i r^2), \\ w_a(r) = r^2 \sum_{i=1}^{N_a} B_i \exp(-b_i r^2), \end{cases} \quad (1)$$

or as asymmetric double sigmoidal [7]:

$$R_l = C_0 + C_1 \frac{1}{1+\exp\left(-\frac{r - C_2 + C_3/2}{C_4}\right)} \times \left(1 - \frac{1}{1+\exp\left(-\frac{r - C_2 - C_3/2}{C_5}\right)}\right). \quad (2)$$

In (2) parameter C_0 not always is the positive number, therefore there is a superfluous knot.

Despite of unwieldy both long-lived calculations and minimizations χ^2 (to size smaller for 10^{-4}), it was necessary to approximate numerical values of wave functions of a deuteron, which arrays of numbers made 839×2 values to an interval r=0…25 fm for potential Reid93 [8]. The value of coefficients A_i, a_i, B_i, b_i for formulas (1) is reduced in Tables 1 and 2 (N_a=17). In Ref. [11] value N_a=13.

Except for (1), there is one more analytical form of the deuteron wave function [2, 10, 12]:

$$\begin{cases} u_b(r) = \sum_{j=1}^{N_b} C_j \exp(-m_j r), \\ w_b(r) = \sum_{j=1}^{N_b} D_j \exp(-m_j r)\left[1 + \frac{3}{m_j r} + \frac{3}{(m_j r)^2}\right], \end{cases} \quad (3)$$

where $m_j = \beta + (j-1)m_0$, $\beta = \sqrt{ME_d}$, m_0=0.9 fm^{-1}. M – nucleon mass, E_d – binding energy of deuteron.

The asymptotics of deuteron wave functions (1) for $r \to \infty$

$$u(r) \to A_S \exp(-\beta r),$$

$$w(r) \to A_D \exp(-\beta r)\left[1 + \frac{3}{\beta r} + \frac{3}{(\beta r)^2}\right],$$

where A_S and A_D are the asymptotic S- and D-state normalizations.

The accuracy of the parametrization (1) is characterized

$$I_S = \left(\int_0^\infty [u(r) - u_a(r)]^2 dr\right)^{1/2} = 2.5 \times 10^{-4},$$

$$I_D = \left(\int_0^\infty \left[w(r) - w_a(r) \right]^2 dr \right)^{1/2} = 1.7 \times 10^{-4} .$$

Table 1

Coefficients A_i, a_i

i	A_i	a_i
1	-0.0293645998298	2.0291635490395
2	0.0823581508029	0.0142620649234
3	0.0807980381556	0.0236567335601
4	0.2776372774260	4.5335812432041
5	-0.4348821007714	2.3259949260170
6	0.0844998823432	0.0411919136076
7	0.0787954538986	0.1089476353731
8	0.2654767966317	2.2248822859584
9	-0.4348821007713	2.3259949260119
10	0.0440768783933	0.0045691318899
11	0.0831819945356	0.0104362641201
12	0.2654767966318	2.2248822859584
13	-0.4348821007712	2.3259949260146
14	0.0878101336022	0.0412039254988
15	0.0735806665400	0.1088944012465
16	0.2654767966318	2.2248822859584
17	-0.3532779919126	1.0378165184475

Table 2

Coefficients B_i, b_i

i	B_i	b_i
1	-0.2053724318749	6.0941159580164
2	-0.0847216453447	0.2592222819651
3	-0.0847216468360	0.2592222816757
4	-0.2615777513621	0.7377945559650
5	1.2812879375816	0.7460129600248
6	-0.1067533670936	0.2565139300968
7	-0.0847216468291	0.2592222816770
8	-0.2615777513623	0.7377945559650
9	0.3972095973113	0.2544132039622
10	-0.1067533722081	0.2565139308288
11	-0.0847215171541	0.2592223068180
12	-0.2615777513623	0.7377945559650
13	0.39720960090286	0.2544129992542
14	-0.1067533722050	0.2565139308284
15	-0.0847216364107	0.2592222836969
16	-0.2615777513623	0.7377945559650
17	0.00443162550424	0.0493377965549

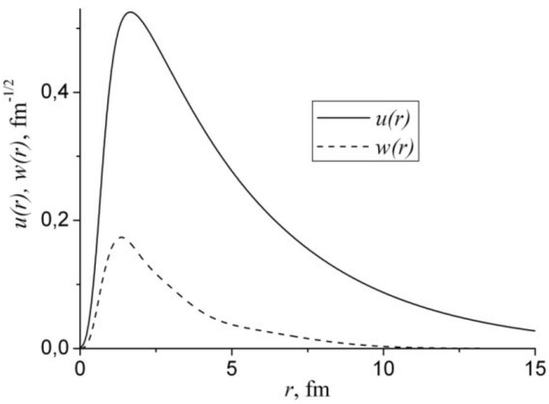

Fig. 1. Deuteron wave functions

The designed the deuteron wave functions (1) in configuration space do not contain superfluous knots. On Fig. 1 radial wave functions *u(r)* and *w(r)* for *L*=0 and *L*=2 are specified. They well correlate with the data in Ref. [13].

2. PROPERTIES OF THE DEUTERON

If the deuteron wave functions are known, then it is possible to calculate deuteron properties [2, 6]:

- the root-mean-square or matter radius:

$$r_d = \frac{1}{2} \left\{ \int_0^\infty r^2 \left[u^2(r) + w^2(r) \right] dr \right\}^{1/2} ;$$

- the quadrupole moment:

$$Q_d = \frac{1}{20} \int_0^\infty r^2 w(r) \left[\sqrt{8} u(r) - w(r) \right] dr ;$$

- the magnetic moment:

$$\mu_d = \mu_s - \frac{3}{2} (\mu_s - \frac{1}{2}) P_D ;$$

- the *D*- state probability:

$$P_D = \int_0^\infty w^2(r) dr ;$$

- the "*D/S*- state ratio":

$$\eta = A_D / A_S .$$

The designed properties of a deuteron it is reduced in Table 3. They well agree with the experimental and theoretical datas [1, 8].

Table 3

Deuteron properties

Property	(1)	[8]
P_D, %	5.70677	5.699
r_d , fm	1.9681	1.969
Q_d , fm^2	0.252849	0.2703
μ_d	0.847289	0.8853
η	0.027601	0.0251

3. POLARIZATION CHARACTERISTICS OF THE DEUTERON

On the designed deuteron wave functions o(1) expedient there are calculations of polarization characteristics [14]. Measuring of polarization characteristics of a response of a fragmentation of deuteron *A(d,p)X* at the intermediate and high energies remains to one of the basic tools for examination of structure of a deuteron.

In impulse representation it is possible to calculate the following polarization characteristics:

1) component of a tensor of sensitivity polarization of deuterons T_{20} [15]:

$$T_{20} = \frac{1}{\sqrt{2}} \frac{2\sqrt{2} u(p) w(p) - w(p)^2}{u(p)^2 + w(p)^2} ; \qquad (4)$$

2) polarization transmission K_0:

$$K_0 = \frac{u(p)^2 - w(p)^2 - u(p) w(p) / \sqrt{2}}{u(p)^2 + w(p)^2} ; \qquad (5)$$

3) tensor analyzing power A_{yy} [14]:

$$A_{yy} = \frac{T_{00}^2 - T_{11}^2 + 4P^2 T_{10}^2}{T_{00}^2 + 2T_{11}^2 + 4P^2 T_{10}^2} ; \qquad (6)$$

4) tensor-tensor transmission of polarization K_{yy} [14]

$$K_{yy} = \frac{5T_{11}^2 + T_{00}^2 - 8P^2T_{10}^2}{T_{00}^2 + 2T_{11}^2 + 4P^2T_{10}^2}, \qquad (7)$$

where $P=0.4p$ – the entered parameter; $u(p)$ and $w(p)$ – deuteron wave functions in impulse representation that calculate through $u(r)$ and $w(r)$ with the help of transformation Hankel

$$u(p) = \int_0^\infty u(r) j_0(rp)dr; \quad w(p) = \int_0^\infty w(r) j_2(rp)dr.$$

The amplitudes $T_{ij}(p/2)$ determined with the help of wave functions $u(r)$ and $w(r)$ as follows [6, 14]:

$$T_{00} = S_0(p/2) + \sqrt{2}S_2(p/2);$$

$$T_{11} = S_0(p/2) - \frac{1}{\sqrt{2}}S_2(p/2);$$

$$T_{10} = \frac{i}{\sqrt{2}}\int_0^\infty \left(u^2(r) - \frac{w^2(r)}{2}\right) j_0(rp/2)dr +$$

$$+\frac{i}{2}\int_0^\infty w(r)\left(u(r) + \frac{w(r)}{\sqrt{2}}\right) j_2(rp/2)dr.$$

Here S_0 and S_2 – spherical and quadrupole shape factors of a deuteron:

$$S_0(p/2) = \int_0^\infty (u^2(r) + w^2(r)) j_0(rp/2)dr;$$

$$S_2(p/2) = \int_0^\infty 2w(r)\left(u(r) - \frac{1}{2\sqrt{2}}w(r)\right) j_2(rp/2)dr.$$

The designed polarization performances T_{20}, K_0, A_{yy}, and K_{yy} are reduced on Fig. 2.

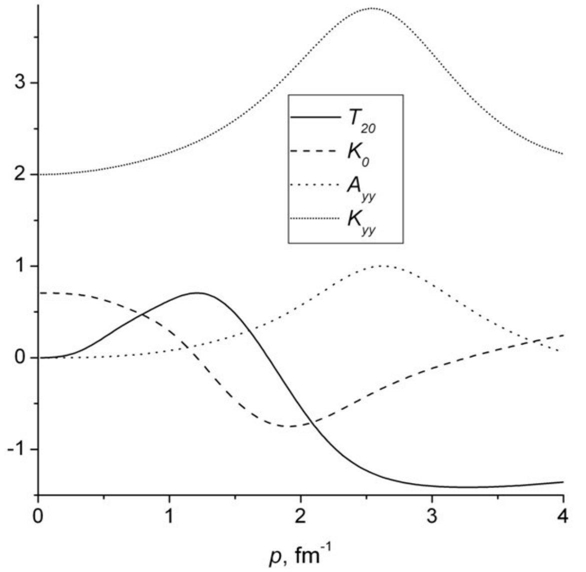

Fig. 2. Polarization characteristics T_{20}, K_0, A_{yy}, and K_{yy}

The magnitude T_{20} designed on the formula (4) for potential Reid93 well correlates with outcomes Ref. [16], and magnitude A_{yy} is proportionate to outcomes [17] for Bonn potential. Calculations of polarization performances K_0 and K_{yy} for potential Reid93 it is realized for the first time.

It is necessary to note, that magnitudes of polarization performances T_{20} and A_{yy} for potential Reid93 in the given operation almost coincide with values, as at their definition in Ref. [6], where deuteron wave functions also do not contain surplus knots in coordinate and space representations. Deviations make 1…2%.

The obtained outcomes for tensor analyzing power A_{yy} well agree with outcomes within the framework of model of an momentum approximation of plane waves [14] and experimental data of [18] responses of inelastic scattering on the carbon, obtained at initial impulse of a deuteron 9 GeV/c and an angle of detection of secondary deuterons 85 mr in the field of excitation of a resonance with the mass 2190 MeV/c^2.

CONCLUSIONS

Numerical coefficients of analytical forms for deuteron wave function in configuration representation for realistic phenomenological potential Reid93 are designed. The obtained wave functions do not contain superfluous knots. On these wave functions it is designed deuteron properties.

Using deuteron wave functions in coordinate and space representations, are designed a component of a tensor of sensitivity polarization of deuterons T_{20}, polarization transmission K_0 and tensor analyzing power A_{yy}. The obtained outcomes are compared to the published experimental and theoretical outcomes.

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АППРОКСИМАЦИЯ ВОЛНОВОЙ ФУНКЦИИ И ПОЛЯРИЗАЦИОННЫЕ ХАРАКТЕРИСТИКИ ДЕЙТРОНА ДЛЯ ПОТЕНЦИАЛА Reid93

В.И. Жаба

Численно рассчитаны коэффициенты аналитической формы для волновой функции дейтрона в координатном представлении для потенциала Reid93. Полученные волновые функции не содержат лишних узлов. Рассчитанные параметры дейтрона хорошо согласуются с экспериментальными и теоретическими данными. Рассчитанные по волновым функциям поляризационные характеристики T_{20} и A_{yy} соразмерны с раньше опубликованными.

АПРОКСИМАЦІЯ ХВИЛЬОВОЇ ФУНКЦІЇ ТА ПОЛЯРИЗАЦІЙНІ ХАРАКТЕРИСТИКИ ДЕЙТРОНА ДЛЯ ПОТЕНЦІАЛУ Reid93

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Чисельно розраховані коефіцієнти аналітичної форми для хвильової функції дейтрона в координатному представленні для потенціалу Reid93. Отримані хвильові функції не містять надлишкових вузлів. Розраховані параметри дейтрона добре узгоджуються з експериментальними і теоретичними даними. Розраховані за хвильовими функціями поляризаційні характеристики T_{20} і A_{yy} співрозмірні з раніше опублікованими.