# OPTIMAL INITIAL VALUE CONTROL FOR THE MULTI-TERM TIME-FRACTIONAL DIFFUSION EQUATION


***R.A. Veklych, V.V. Semenov, S.I. Lyashko***

***Taras Shevchenko National University of Kyiv, Kyiv, Ukraine***

***E-mail: vekross@gmail.com***



In this paper an initial value control problem with a quadratic cost function is considered for a system governed by a diffusion equation with a linear combination of Caputo time-fractional derivatives in an open bounded domain. We show the existence of the optimal solution by proving the existence of the weakly convergent minimization sequence satisfying the state equation. The uniqueness follows directly from the strong convexity of the cost function.



PACS: 02.30.Jr, 02.30.Yy, 51.20.+d


## INTRODUCTION

Fractional diffusion equations arise in modeling of processes with a logarithmic growth of the mean square radial displacement, and, in particular, as shown in [1], can be used to describe anomalous transport in plasma turbulence.

Some recent articles (for example, [2-4]) deal with sub-diffusion processes that can be described by a fractional equation of the following type:

$$\int_0^1 {}_0^C D_t^\alpha y(x,t)\mu(\alpha)\,\mathrm{d}\alpha = Ay(x,t),$$

where ${}_0^C D_t^\alpha y(x,t)$ denotes a Caputo fractional derivative and $\mu(\alpha)$ is positive weight function.

In this paper we consider an optimal control problem with a state equation of abovementioned type, and $\mu(\alpha)$ is a finite linear combination of the δ-functions with positive coefficients.

## 1. PRELIMINARIES

Let $f$ be a function defined on a finite interval $[0,T]$, $T<\infty$, and $\Gamma(\alpha)$ be a Gamma function. Then the left and right Riemann-Liouville fractional integrals of order $\alpha\in(0,1)$ are defined by

$$ {}_0I_t^\alpha f(t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)\,\mathrm{d}\tau,$$

$$ {}_tI_T^\alpha f(t)=\frac{1}{\Gamma(\alpha)}\int_t^T (\tau-t)^{\alpha-1} f(\tau)\,\mathrm{d}\tau.$$

The following expressions define the respective Riemann-Liouville derivatives:

$$ {}_0D_t^\alpha f(t)=\frac{1}{\Gamma(1-\alpha)}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t (t-\tau)^{-\alpha} f(\tau)\,\mathrm{d}\tau,$$

$$ {}_tD_T^\alpha f(t)=-\frac{1}{\Gamma(1-\alpha)}\frac{\mathrm{d}}{\mathrm{d}t}\int_t^T (\tau-t)^{-\alpha} f(\tau)\,\mathrm{d}\tau,$$

and Caputo derivatives:

$$ {}_0^C D_t^\alpha f(t)=\frac{1}{\Gamma(1-\alpha)}\int_0^t (t-\tau)^{-\alpha} \frac{\mathrm{d}}{\mathrm{d}t} f(\tau)\,\mathrm{d}\tau,$$

$$ {}_t^C D_T^\alpha f(t)=-\frac{1}{\Gamma(1-\alpha)}\int_t^T (\tau-t)^{-\alpha} \frac{\mathrm{d}}{\mathrm{d}t} f(\tau)\,\mathrm{d}\tau.$$

The multinomial Mittag-Leffler function is defined (see [5]) as

$$E_{(\beta_1,\ldots,\beta_m),\beta_0}(z_1,\ldots,z_m)=\sum_{k=0}^{\infty}\sum_{k_1+\ldots k_m=k}\frac{(k;k_1,\ldots,k_m)\prod_{j=1}^m z_j^{k_j}}{\Gamma\left(\beta_0+\sum_{j=1}^m \beta_j k_j\right)},$$

where $\beta\in(0,2)$, $\beta_j\in(0,1)$, $z_j\in C$, $j=1,\ldots,m$, and

$$(k;k_1,\ldots,k_m)=\frac{k!}{k_1!\cdots k_m!},$$

$$k=\sum_{j=1}^m k_j,\quad k_j\in N.$$

Also, as in [6], let

$$E_{\alpha',\beta}(z_1,\ldots,z_m)=E_{(\alpha_1,\alpha_1-\alpha_2,\ldots,\alpha_1-\alpha_m),\beta}(z_1,\ldots,z_m).$$

## 2. PROBLEM STATEMENT

Let $\Omega\subset \mathrm{R}^n$ be a bounded domain with a smooth boundary $\partial\Omega$, $U_\partial\subset H_0^1(\Omega)$ – a nonempty closed convex set of admissible controls, $y_d\in L^2(\Omega)$. We consider the following optimal control problem:

$$J(y,u)=\frac{1}{2}\|y(\cdot,T)-y_d\|^2_{L^2(\Omega)}+\frac{\beta}{2}\|u\|^2_{H^1(\Omega)}\to\inf \quad (2.1)$$

$$\sum_{j=1}^m q_j\,{}_0^C D_t^{\alpha_j} y(x,t)=Ay(x,t),\quad x\in\Omega,\quad t\in(0,T), \quad (2.2)$$

$$y(x,t)=0,\quad x\in\partial\Omega,\quad t\in(0,T), \quad (2.3)$$

$$y(x,0)=u(x),\quad x\in\Omega, \quad (2.4)$$

$$u\in U_\partial, \quad (2.5)$$

where $\alpha_j\in\mathrm{R}$, $0<\alpha_m<\cdots<\alpha_1<1$, $q_j\in\mathrm{R}$, $q_j>0$, $\beta\in\mathrm{R}$, $\beta>0$, and $A$ is a symmetrical uniformly elliptic operator:

$$Ay(x)=\sum_{i,j=1}^n\frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial y}{\partial x_j}(x)\right)+a_0(x)y(x),$$

$a_{ij}=a_{ji}\in C^1(\overline{\Omega})$, $a_0\in C(\overline{\Omega})$, $a_0\le 0$ and there exist a constant $\mu>0$, such that for all $\xi_1,\ldots,\xi_n\in\mathrm{R}$ and $x\in\overline{\Omega}$

$$\sum_{i,j=1}^n a_{ij}(x)\xi_i\xi_j\ge\mu\sum_{i=1}^n\xi_i^2 .$$

We denote the eigenvalues of the elliptic operator $-A$ as $\{\lambda_n\}_{n\in N}$, $0<\lambda_1\le\lambda_2\le\cdots$, and the respective orthonormal eigenfunctions as $\{\varphi_n\}_{n\in N}$, $\varphi_n \in H^2(\Omega)\cap H_0^1(\Omega)$.

The domain of the fractional power $(-A)^\gamma$ is defined for $\gamma \in R$ as

$$D((-A)^\gamma)=\left\{ f\in L^2(\Omega); \sum_{n=1}^{\infty}\left|\lambda_n^\gamma (f,\varphi_n)\right|^2<\infty\right\},$$

$D((-A)^\gamma)$ is a Hilbert space with a norm

$$\|f\|_{D((-A)^\gamma)}=\left(\sum_{n=1}^{\infty}\left|\lambda_n^\gamma (f,\varphi_n)\right|^2\right)^{1/2},$$

and $D((-A)^{1/2})=H_0^1(\Omega)$.

## 3. RESULTS

**Definition 3.1.** We call a function $y$ a solution to (2.2)-(2.4) if (2.2) holds in $L^2(\Omega)$, $y(\cdot,t)\in H_0^1(\Omega)$ for almost all $t\in(0,T)$, $y\in C([0,T];L^2(\Omega))$ and

$$\lim_{t\to 0}\|y(\cdot,t)-u\|_{L^2(\Omega)}=0.$$

It was shown in [6] that if $\gamma\in[0,1]$ and $u\in D((-A)^\gamma)$ then the problem (2.2)-(2.4) has a unique solution

$$y(\cdot,t)=\sum_{n=1}^{\infty}\left(1-\lambda_n t^{\alpha_1}E^{(n)}_{\alpha',1+\alpha_1}(t)\right)(a,\varphi_n)\varphi_n, \quad (3.1)$$

$$y\in C([0,T];L^2(\Omega))\cap C((0,T];H^2(\Omega)\cap H_0^1(\Omega)), \quad (3.2)$$

$$y\in L^{1/(1-\gamma)}(0,T;H^2(\Omega)\cap H_0^1(\Omega)), \quad (3.3)$$

and there exists a constant $C>0$ such that

$$\|y\|_{C([0,T];L^2(\Omega))}\le C\|u\|_{L^2(\Omega)}, \quad (3.4)$$

$$\|y(\cdot,t)\|_{H^2(\Omega)}\le C\|u\|_{D((-A)^\gamma)}t^{\alpha_1(\gamma-1)}, \quad t\in(0,T]. \quad (3.5)$$

Moreover, $\partial_t y\in C((0,T];L^2(\Omega))$, and there exists a constant $C>0$ such that

$$\|\partial_t y(\cdot,t)\|_{L^2(\Omega)}\le C\|u\|_{D((-A)^\gamma)}t^{\alpha_1\gamma-1}, \quad t\in(0,T]. \quad (3.6)$$

**Theorem 2.1.** Optimal control problem (2.1)-(2.5) has a unique solution.

*Proof:* From (3.6) we have that $\partial_t y\in L^p(0,T;L^2(\Omega))$ for $p\in(1,1/(1-\alpha\gamma))$.

Let $X_p$ denote a space

$$X_p=\{y\in L^2(0,T;H^2(\Omega)\cap H_0^1(\Omega)):\partial_t y\in L^p(0,T;L^2(\Omega))\}$$

with a following norm:

$$\|y\|_{X_p}=\|y\|_{L^2(0,T;H^2(\Omega))}+\|\partial_t y\|_{L^p(0,T;L^2(\Omega))}.$$

Let $\Psi$ denote a set of admissible pairs for the problem (2.1)-(2.5), that is, $\Psi$ is a set of $(y,u)\in X_p\times U_\partial$ such that (2.2)-(2.4) hold and $J(y,u)<\infty$.

Then there exists such a minimizing sequence $\{(y_n,u_n)\}_{n\in N}\in\Psi$ that

$$\lim_{k\to\infty}J(y_n,u_n)=\inf_{(y,u)\in\Psi}J(y,u)>-\infty.$$

Hence, there exists a constant $C_1>0$, that does not depend on $n$, and $\sup_{n\in N}J(y_n,u_n)\le C_1$. From (3.5) and (3.6) we obtain

$$\|y\|_{X_p}+\|u\|_{H^1(\Omega)}\le C_2\|u\|_{H^1(\Omega)}\le C_3J(y,u)+C_4,$$

so $\sup_{n\in N}\left(\|y_n\|_{X_p}+\|u_n\|_{H^1(\Omega)}\right)\le C_5$. Thus, we can choose weakly convergent subsequences $\{y_n\}_{n\in N}$ and $\{u_n\}_{n\in N}$ (still indexed by $n$ for simplicity):

$$y_n\xrightarrow{w}\hat y \text{ in } X_p, \quad (3.7)$$

$$u_n\xrightarrow{w}\hat u \text{ in } H_0^1(\Omega). \quad (3.8)$$

The set $U_\partial$ is convex and closed, hence $\hat u\in U_\partial$.

Now we show that $y$ is a solution to a system (2.2)-(2.4) for $u=\hat u$. It follows from Definition 3.1 that for $y_n$ and for any $v\in C_0^\infty(\Omega)$

$$\int_\Omega\left(\sum_{j=1}^{n}q_j\,{}_0^CD_t^{\alpha_j}y(x,t)-Ay(x,t)\right)v(x)\,dt=0. \quad (3.9)$$

Multiplying this equality by $\psi\in C_0^\infty(0,T)$, we obtain

$$\int_0^T\int_\Omega\left(\sum_{j=1}^{n}q_j\,{}_0^CD_t^{\alpha_j}y(x,t)-Ay(x,t)\right)v(x)\psi(t)\,dx\,dt=0. \quad (3.10)$$

Since $\|y\|_{L^2(0,T;L^2(\Omega))}\le\|y\|_{X_p}$ $\forall y\in X_p$, then from weak convergence in the space $\left(X_p,\|\cdot\|_{X_p}\right)$ we can obtain also

$$y_n\xrightarrow{w}\hat y \text{ in } \left(X_p,\|\cdot\|_{L^2(0,T;L^2(\Omega))}\right).$$

The operator $\partial_t:X_p\to L^p(0,T;L^2(\Omega))$ is bounded and linear, so it preserves weak convergence, thus

$$\partial_t y_n\xrightarrow{w}\partial_t\hat y.$$

The fractional integration operator ${}_0I_t^{1-\alpha_j}$ is linear, and from [7] we have that for $\alpha_j\in(0,1)$ and $p\in(0,1/(1-\alpha_j))$ it acts boundedly from $L^p$ into $L^q$, where $q=1/(1-(1-\alpha_j)p)$. Therefore, in $L^q(0,T;L^2(\Omega))$

$${}_0^CD_t^{\alpha_j}y_n={}_0I_t^{1-\alpha_j}\partial_t y_n\xrightarrow{w}{}_0I_t^{1-\alpha_j}\partial_t\hat y={}_0^CD_t^{\alpha_j}\hat y. \quad (3.11)$$

Let $q^*=p/((2-\alpha_j)p-1)$, that is, $1/q+1/q^*=1$. Then $\psi\in L^{q^*}(0,T)=\left(L^q(0,T)\right)^*$ and from (3.11) we obtain for $n\to\infty$:

$$\int_0^T\left(\sum_{j=1}^{m}q_j\,{}_0^CD_t^{\alpha_j}y_n(x,t)\right)\psi(t)\,dt\to$$
$$\to\int_0^T\left(\sum_{j=1}^{m}q_j\,{}_0^CD_t^{\alpha_j}\hat y(x,t)\right)\psi(t)\,dt. \quad (3.12)$$

Using the Green formula and taking into account the boundary condition (2.3), we have

$$\int_\Omega Ay_n(x,t)v(x)\,\mathrm{d}x=\int_\Omega y_n(x,t)A^*v(x)\,\mathrm{d}x+$$
$$+\int_{\partial\Omega}\left(v(x)\frac{\partial y_n}{\partial N_A}(x,t)-y_n(x,t)\frac{\partial v}{\partial N_{A^*}}(x)\right)\mathrm{d}s=$$
$$=\int_\Omega y_n(x,t)A^*v(x)\,\mathrm{d}x,$$

where $A^*$ is an adjoint of $A$, that is

$$A^*v(x)=\sum_{i,j=1}^n\frac{\partial}{\partial x_j}\left(a_{ij}(x)\frac{\partial v}{\partial x_i}(x)\right)+a_0(x)v(x),$$

$N=(N_1,\dots,N_n)$ is an outward unit normal to $\partial\Omega$, and

$$\frac{\partial y}{\partial N_A}(x,t)=\sum_{i,j=1}^n\left(a_{ij}(x)N_i(x)\frac{\partial y}{\partial x_j}(x,t)\right),$$
$$\frac{\partial v}{\partial N_{A^*}}(x)=\sum_{i,j=1}^n\left(a_{ij}(x)N_j(x)\frac{\partial v}{\partial x_i}(x)\right).$$

Proceeding to the limit and using the Green formula once again, we obtain:

$$\int_\Omega Ay_n(x,t)v(x)\,\mathrm{d}x\xrightarrow[n\to\infty]{}\int_\Omega \hat{y}(x,t)A^*v(x)\,\mathrm{d}x= \qquad (3.13)$$
$$=\int_\Omega A\hat{y}(x,t)v(x)\,\mathrm{d}x.$$

Therefore, from (3.12) and (3.13)

$$\int_0^T\int_\Omega\left(\sum_{j=1}^m q_j\,{}_0^C D_t^{\alpha_j}y_n(x,t)-Ay_n(x,t)\right)v(x)\psi(t)\,\mathrm{d}x\,\mathrm{d}t\to$$
$$\to\int_0^T\int_\Omega\left(\sum_{j=1}^m q_j\,{}_0^C D_t^{\alpha_j}\hat{y}(x,t)-\hat{y}(x,t)\right)v(x)\psi(t)\,\mathrm{d}x\,\mathrm{d}t,$$

that is $\hat{y}(x,t)$ satisfies (2.2).

Now we need to show that $\hat{y}(x,t)$ also satisfies the initial condition (2.4).

For an arbitrary function $\psi\in C_0^\infty(0,T)$ we can use fractional (see [7]) and ordinary integration by parts:

$$\int_0^T\left(\sum_{j=1}^m q_j\,{}_0^C D_t^{\alpha_j}y_n(x,t)\right)\psi(t)\,\mathrm{d}t=$$
$$=\int_0^T\left(\sum_{j=1}^m q_j\,{}_0I_t^{1-\alpha_j}\partial_t y_n(x,t)\right)\psi(t)\,\mathrm{d}t=$$
$$=\int_0^T\left(\sum_{j=1}^m q_j\,{}_tI_T^{1-\alpha_j}\psi(t)\right)\partial_t y_n(x,t)\,\mathrm{d}t=$$
$$=\left(\sum_{j=1}^m q_j\,{}_tI_T^{1-\alpha_j}\psi(t)\right)y_n(x,t)\Bigg|_{t=0}^{t=T}-$$
$$-\int_0^T\left(\sum_{j=1}^m q_j\frac{\mathrm{d}}{\mathrm{d}t}\,{}_tI_T^{1-\alpha_j}\psi(t)\right)y_n(x,t)\,\mathrm{d}t.$$

Then, using $y_n(x,t)$ in (3.10) and taking into account that $y_n(x,0)=u_n(x)$ and ${}_tI_T^{1-\alpha_j}\psi(t)\big|_{t=T}=0,$

$$\int_0^T\int_\Omega\left(\sum_{j=1}^m q_j\,{}_0^C D_t^{\alpha_j}y_n(x,t)-Ay_n(x,t)\right)v(x)\psi(t)\,\mathrm{d}x\,\mathrm{d}t=$$
$$=\int_\Omega\left(\sum_{j=1}^m q_j\,{}_tI_T^{1-\alpha_j}\psi(t)\Big|_{t=0}\right)u_n(x)v(x)\,\mathrm{d}x+ \qquad (3.14)$$
$$+\int_0^T\int_\Omega\left(\left(\sum_{j=1}^m q_j\,{}_tD_T^{\alpha_j}\psi(t)\right)y_n(x,t)v(x)-\right.$$
$$\left.-y_n(x,t)\psi(t)A^*v(x)\right)\mathrm{d}x\,\mathrm{d}t=0.$$

Similarly for $\hat{y}(x,t)$

$$\int_0^T\int_\Omega\left(\sum_{j=1}^m q_j\,{}_0^C D_t^{\alpha_j}\hat{y}(x,t)-A\hat{y}(x,t)\right)v(x)\psi(t)\,\mathrm{d}x\,\mathrm{d}t=$$
$$=\int_\Omega\left(\sum_{j=1}^m q_j\,{}_tI_T^{1-\alpha_j}\psi(t)\Big|_{t=0}\right)\hat{y}(x,0)v(x)\,\mathrm{d}x+ \qquad (3.15)$$
$$+\int_0^T\int_\Omega\left(\left(\sum_{j=1}^m q_j\,{}_tD_T^{\alpha_j}\psi(t)\right)\hat{y}(x,t)v(x)-\right.$$
$$\left.-\hat{y}(x,t)\psi(t)A^*v(x)\right)\mathrm{d}x\,\mathrm{d}t=0.$$

Thus, subtracting (3.14) from (3.15) and proceeding to the limit, we have

$$\int_\Omega\left(\sum_{j=1}^m q_j\,{}_tI_T^{1-\alpha_j}\psi(t)(\hat{y}(x,t)-\hat{u})\Big|_{t=0}\right)v(x)\,\mathrm{d}x=0,$$

and, as $v(x)$ can be arbitrary,

$$\lim_{t\to0}\|\hat{y}(\cdot,t)-\hat{u}\|_{L^2(\Omega)}=0.$$

Finally, we show that $y_n(x,T)\to\hat{y}(x,T)$ in $L^2(\Omega)$. Indeed, from (3.1)

$$\|y_n(x,T)-\hat{y}(x,T)\|^2_{L^2(\Omega)}=$$
$$=\sum_{n=1}^\infty\left|1-\lambda_n t^{\alpha_1}E^{(n)}_{\alpha',1+\alpha_1}(t)\right|^2|(u_n-\hat{u},\varphi_n)|^2\le$$
$$\le\sum_{n=1}^\infty C_1(u_n-\hat{u},\varphi_n)^2\le$$
$$\le C_2\|u_n-\hat{u}\|^2_{L^2(\Omega)}.$$

Since $H_0^1(\Omega)$ is compactly embedded into $L^2(\Omega)$, from (3.8)

$$\|u_n-\hat{u}\|^2_{L^2(\Omega)}\xrightarrow[n\to\infty]{}0,$$

and $y_n(x,T)$ is strongly converges to $\hat{y}(x,T)$ in $L^2(\Omega)$. Thus,

$$\|\hat{y}(x,T)-y_d(x)\|^2_{L^2(\Omega)}=\lim_{n\to\infty}\|y_n(x,T)-y_d(x)\|^2_{L^2(\Omega)}. \qquad (3.16)$$

The mapping $u\to\|u\|_{H^1(\Omega)}$ is weakly lower semicontinuous, so

$$\|\hat{u}\|_{H^1(\Omega)}\le\underline{\lim}_{n\to\infty}\|u_n\|_{H^1(\Omega)}. \qquad (3.17)$$

Therefore, combining (3.16) and (3.17)

$$J(\hat{y},\hat{u})\le\underline{\lim}_{n\to\infty}J(y_n,u_n)=\inf_{(y,u)\in\Psi}J(y,u),$$

and $(\hat{y},\hat{u})$ is optimal solution to the problem (2.1)-(2.5).

To prove the uniqueness of the solution, let us assume that $(\hat{y}_1, \hat{u}_1)$ and $(\hat{y}_2, \hat{u}_2)$ are distinct optimal solutions.

Let

$$\hat{y}_0 = \frac{\hat{y}_1 + \hat{y}_2}{2}, \ \hat{u}_0 = \frac{\hat{u}_1 + \hat{u}_2}{2}.$$

As the state system (2.2)-(2.4) is linear and $U_\partial$ is convex, we have that $(\hat{y}_0, \hat{u}_0) \in \Psi$, and from strong convexity of the cost function in (2.1)

$$J(\hat{y}_0, \hat{u}_0) < \frac{1}{2}\left(J(\hat{y}_1, \hat{u}_1) + J(\hat{y}_2, \hat{u}_2)\right) = \inf_{(y,u)\in\Psi} J(y,u).$$

Therefore, we came to contradiction, and the optimal solution is unique.

## CONCLUSIONS

In this paper existence and uniqueness of the solution to optimal control problem for a diffusion equation with multiple time-fractional Caputo derivatives has been proved.

In the future, a characterization for an optimal control problem can be obtained similarly to the results for parabolic equations with a time derivative of integer order, presented in [8].

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*Article received 15.09.2016*

## ОПТИМАЛЬНОЕ НАЧАЛЬНОЕ УПРАВЛЕНИЕ ДЛЯ МНОГОЧЛЕННОГО ДРОБНОГО ПО ВРЕМЕНИ УРАВНЕНИЯ ДИФФУЗИИ

***Р.А. Веклич , В.В. Семёнов , С.И. Ляшко***

Рассмотрена задача оптимального управления начальными условиями с квадратичной функцией стоимости для системы, описываемой уравнением диффузии с линейной комбинацией дробных производных Капуто по времени в открытой ограниченной области. Мы показываем существование оптимального решения, доказав существование слабосходящейся минимизирующей последовательности, удовлетворяющей уравнение состояния. Единственность непосредственно следует из строгой выпуклости функции стоимости.

## ОПТИМАЛЬНЕ ПОЧАТКОВЕ КЕРУВАННЯ ДЛЯ БАГАТОЧЛЕННОГО ДРОБОВОГО ЗА ЧАСОМ РІВНЯННЯ ДИФУЗІЇ

***Р.А. Веклич , В.В. Семенов , С.І. Ляшко***

Розглянута задача оптимального керування початковими умовами з квадратичною функцією вартості для системи, що описується рівнянням дифузії з лінійною комбінацією дробових похідних Капуто за часом у відкритій обмеженій області. Ми показуємо існування оптимального розв'язку, довівши існування слабкозбіжної мінімізуючої послідовності, що задовольняє рівняння стану. Єдиність безпосередньо випливає зі строгої опуклості функції вартості.