

# EXACT RELATIVISTIC MAXWELLIAN MAGNETIZED PLASMA DIELECTRIC TENSOR EVALUATION FOR ARBITRARY WAVE NUMBERS

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A new exact integral form of the fully relativistic permittivity tensor for plasmas in a magnetic field is given. It is suitable for numerical applications for arbitrary wave numbers since all integrals in it are one-dimensional ones. This form is interesting for applications to study propagation and absorption of electron Bernstein waves in the laboratory thermonuclear plasmas and of arbitrary electron and ion cyclotron waves in the hot astrophysical plasmas.

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## INTRODUCTION

Theoretical studying electromagnetic waves propagation and absorption in magnetized plasma in the electron cyclotron frequency range requires accurate taking into account relativistic effects, associated with increasing masses of fast enough electrons, especially in the case of wave propagation almost perpendicular to magnetic field lines and for high cyclotron harmonics numbers under consideration [1]. In both these cases in the ion cyclotron resonance frequency range similar effects can also arise for hot enough plasmas [2].

The basis for studying linear electron cyclotron waves in plasmas is an accurate evaluation of the relativistic plasma dielectric tensor. Two original equivalent exact integral forms of this tensor were given on the ground of the relativistic form of Vlasov kinetic equation [3], however, in general case of arbitrary plasma and wave parameters their applicability has been rather limited for numerical applications. Later, on the base of the exact relativistic plasma dispersion functions (PDF) and the rather compact form of this tensor, this limitation was essentially weakened [4].

Neglecting the ion dynamics, tensor elements in this form are represented as double series: on the electron cyclotron harmonic numbers and on the exact relativistic PDFs multiplied with coefficients of expansion of the functions  $A_n(\lambda) = e^{-\lambda} I_n(\lambda)$  in the small parameter  $\lambda$ . Here  $n$  is the number of the electron cyclotron harmonic;  $\lambda = (k_{\perp} \rho)^2$ ,  $k_{\perp}$  is transverse wave number,  $\rho = V_{T0} / \Omega_c$  is Larmor radius of electrons,  $V_{T0} = \sqrt{T / m_0}$  and  $\Omega_c = eB / (m_0 c)$  are their thermal velocity and fundamental cyclotron frequency,  $-e$  is the electron charge;  $I_n(\lambda)$  are modified Bessel function of the integer index. The tensor itself is presented in the form

$$\varepsilon_{ij}(\mathbf{k}, \omega, \mu) = \delta_{ij} - \mu \left( \frac{\omega_{p0}}{\omega} \right)^2 D_{ij}, \quad (1)$$

$$D_{11} = \frac{1}{\lambda} \sum_{n=-\infty}^{+\infty} n^2 \sum_{l=0}^{+\infty} (A_n)^{ln+l} Z_{ln+l+3/2},$$

$$D_{12} = D_{21} = -i \sum_{n=-\infty}^{+\infty} n \sum_{l=0}^{+\infty} \frac{d((A_n)^{ln+l})}{d\lambda} Z_{ln+l+3/2},$$

$$D_{22} = D_{11} - \frac{2}{\lambda} \sum_{n=-\infty}^{+\infty} \sum_{l=0}^{+\infty} \left( \lambda^2 \frac{dA_n}{d\lambda} \right)^{ln+l} Z_{ln+l+3/2},$$

$$D_{13} = D_{31} = -\frac{1}{\sqrt{2}\lambda} \sum_{n=-\infty}^{+\infty} n \sum_{l=0}^{+\infty} (A_n)^{ln+l} \frac{\partial Z_{ln+l+5/2}}{\partial z_n},$$

$$D_{23} = -D_{32} = -\frac{i}{\sqrt{2}\lambda} \sum_{n=-\infty}^{+\infty} \sum_{l=0}^{+\infty} \left( \lambda \frac{dA_n}{d\lambda} \right)^{ln+l} \frac{\partial Z_{ln+l+5/2}}{\partial z_n},$$

$$D_{33} = \sum_{n=-\infty}^{+\infty} \sum_{l=0}^{+\infty} \left[ (A_n)^{ln+l} Z_{ln+l+5/2} + \frac{(A_n)^{ln+l}}{2} \frac{\partial^2 Z_{ln+l+7/2}}{\partial z_n^2} \right].$$

Above the next designations were used:  $\omega$  is the angular frequency of electromagnetic wave;  $\mathbf{k}$  is the wave vector;  $\mu = m_0 c^2 / T$ ,  $c$ ,  $m_0$ ,  $T$  are the speed of light in vacuum, the electron rest mass and temperature, respectively;  $a = \mu N_{\parallel}^2 / 2$ ,  $N_{\parallel} = k_{\parallel} c / \omega$  is longitudinal refractive index;  $z_n = (\omega - n\Omega_c) / (\sqrt{2} k_{\parallel} V_{T0})$ ;  $\omega_{p0}$  is plasma frequency of the electrons with rest mass,  $Z_{ln+l+3/2} = Z_{ln+l+3/2}(a, z_n, \mu)$  is the exact relativistic PDF with the index  $ln+l+3/2$ . Superscript  $ln+l$  of brackets in expressions  $D_{ij}$  means the order of expansion of the term in brackets in the parameter  $\lambda$ . This designation significantly reduces recording. For example, the expansion of functions  $A_n(\lambda) = e^{-\lambda} I_n(\lambda)$ , mentioned above

$$A_n(\lambda) = \sum_{l=0}^{+\infty} \frac{(-1)^l [2(ln+l)]!}{(2ln+l)!(ln+l)! 2^{ln+l}} \lambda^{ln+l} = \sum_{l=0}^{+\infty} A_{nl} \lambda^{ln+l} \quad (2)$$

can be presented on this way in a much more short form

$$A_n(\lambda) = \sum_{l=0}^{+\infty} (A_n)^{ln+l}. \quad (3)$$

In applications of relativistic tensor (1) for investigation of the fast electron cyclotron waves in laboratory thermonuclear plasmas usually takes place the condition  $\lambda \ll 1$  and, consequently, series in this parameter (formally in the index  $l$ ) converges so rapidly that its accurate calculation requires to summarize a few terms only.

However, for the study of the slow or plasma electron cyclotron waves in plasmas of laboratory or astrophysics magnetic traps the parameter  $\lambda$  can significantly increase and reach of values of order  $\lambda \sim 1$  and even ones of order  $\lambda \gg 1$ . In this case the series in the index  $l$  with an increase of the parameter  $\lambda$  begin to converge slower and slower, which can cause serious difficulties in its summation even in the case of the weakly relativistic plasmas [5]. Obviously, in the case of fully relativistic plasmas these difficulties can be proved even more significant. In these unfavorable cases it makes sense trying to find some alternative form to one (1), suitable for accurate numerical applications as the form (1) for the case  $\lambda \ll 1$ .

The main goal of the present work is the further progress in resolving the problem of exact evaluation of fully relativistic Maxwellian plasma dielectric tensor for arbitrary values of  $\lambda$  (or for arbitrary wave numbers). For each harmonic number  $n$  this scope is achieved by means of introducing the generating function for the anti-Hermitian parts of tensor and of introducing the generalized relativistic PDF with their evaluation on the basis of Kramers-Kronig formulae.

## FUNCTIONS GENERATING ANTI-HERMITIAN PARTS OF PLASMA DIELECTRIC TENSOR

In the case of unfavorable value of the parameter  $\lambda$  for each element of the fully relativistic tensor (1) and each cyclotron harmonic number  $n$  the summation of the slow convergent series mentioned above can be analytically reduced to a two-step procedure, suitable for accurate numerical applications. The first step introduces the function generating the anti-Hermitian part of this series in the parameter  $\lambda$ , and then a numerical calculation of the one-dimensional integral of this function leads directly to the anti-Hermitian part of the sum of the series. The second step calculating numerically the principal value of the integral in the sense of Cauchy of the anti-Hermitian part leads to the Hermitian part of the sum of the series. Obviously, both parts together give a value of the whole series for the corresponding tensor element.

Let us begin the demonstration of this two-step procedure with the element of plasma dielectric tensor  $\varepsilon_{11}$ . From the first integral form of Trubnikov it follows [6]

$$\varepsilon_{11} = 1 - \frac{\omega_{p0}^2 \mu^2}{\omega^2 2K_2(\mu)} \int_{-\infty}^{+\infty} d\bar{p}_{\parallel} \int_0^{+\infty} \bar{p}_{\perp} d\bar{p}_{\perp} \frac{e^{-\mu\gamma}}{\gamma} \sum_{n=-\infty}^{+\infty} \frac{n^2 J_n^2(v_{\perp} \bar{p}_{\perp})}{v_{\perp}^2 \gamma - N_{\parallel} \bar{p}_{\parallel} - n\Omega_c / \omega}. \quad (4)$$

Here  $K_2(\mu)$  is Macdonald function,  $\bar{p} = p/(mc)$  is normalized momentum,  $\gamma = \sqrt{1 + \bar{p}^2}$ ,  $v_{\perp}^2 = \mu\lambda$ . Using the

last definition, the expression (4) can be converted to the form

$$\varepsilon_{11} = 1 - \frac{\omega_{p0}^2 \mu}{\omega^2 2K_2(\mu)} \sum_{n=-\infty}^{+\infty} \frac{n^2}{\lambda} \int_{-\infty}^{+\infty} d\bar{p}_{\parallel} \int_0^{+\infty} \bar{p}_{\perp} d\bar{p}_{\perp} \frac{e^{-\mu\gamma}}{\gamma} \frac{J_n^2(\sqrt{\lambda\mu} \bar{p}_{\perp})}{\gamma - N_{\parallel} \bar{p}_{\parallel} - n\Omega_c / \omega}. \quad (5)$$

After a change of variable  $\bar{p}_{\perp} \rightarrow \gamma$  in (5) with transformation  $\bar{p}_{\perp}^2 = \gamma^2 - (1 + \bar{p}_{\parallel}^2)$  and its Jacobian  $d\bar{p}_{\perp} / d\gamma = \gamma / \bar{p}_{\perp}$ , we obtain the expression

$$\varepsilon_{11} = 1 - \frac{\omega_{p0}^2 \mu}{\omega^2 2K_2(\mu)} \sum_{n=-\infty}^{+\infty} \frac{n^2}{\lambda} \int_{-\infty}^{+\infty} d\bar{p}_{\parallel} \int_1^{+\infty} d\gamma e^{-\mu\gamma} \frac{J_n^2(\sqrt{\lambda\mu}[\gamma^2 - (1 + \bar{p}_{\parallel}^2)])}{\gamma - N_{\parallel} \bar{p}_{\parallel} - n\Omega_c / \omega}. \quad (6)$$

After one more change in (6)  $\gamma \rightarrow x$  with transform  $x = \gamma - \sqrt{1 + \bar{p}_{\parallel}^2}$  and Jacobian  $d\gamma / dx = 1$  and accounting relation  $x = x(x + 2\sqrt{1 + \bar{p}_{\parallel}^2}) = \gamma^2 - (1 + \bar{p}_{\parallel}^2)$ , we obtain

$$\varepsilon_{11} = 1 - \frac{\omega_{p0}^2 \mu}{\omega^2 2K_2(\mu)} \sum_{n=-\infty}^{+\infty} \frac{n^2}{\lambda} \int_{-\infty}^{+\infty} d\bar{p}_{\parallel} \times \int_0^{e^{-\mu\sqrt{1 + \bar{p}_{\parallel}^2}}} dx e^{-\mu x} \frac{J_n^2(\sqrt{\lambda\mu} x(x + 2\sqrt{1 + \bar{p}_{\parallel}^2}))}{x + \sqrt{1 + \bar{p}_{\parallel}^2} - N_{\parallel} \bar{p}_{\parallel} - n\Omega_c / \omega}. \quad (7)$$

The integral over  $x$  that appears in formula (7) with the multiplier  $\exp(-\mu\sqrt{1 + \bar{p}_{\parallel}^2})$  is one of the Cauchy type with real density, satisfying the Hölder condition of continuity and tending to 0 when  $x \rightarrow 0$  and  $x \rightarrow \infty$  at the contour. It is known that the integral of the Cauchy-type

$$F(z) = \frac{1}{2\pi i} \int_0^{+\infty} \frac{\phi(\tau) d\tau}{\tau - z}, \quad (8)$$

with the density  $\phi(\tau)$  satisfying the former conditions at the contour, is defined at the contour itself by the formulas of Sokhotskii-Plemejj

$$F^{\pm}(z) = \pm \frac{\phi(z)}{2} + \frac{1}{2\pi i} P \int_0^{+\infty} \frac{\phi(\tau) d\tau}{\tau - z}, \quad (z \geq 0). \quad (9)$$

Here, the functions  $F^+(z)$ ,  $F^-(z)$  are the boundary values of integral (8) when the argument  $z$  tends to the contour from the right or from the left-hand side with respect to the integration direction, respectively. The letter P before integral denotes its principal value in the Cauchy sense. At the real axis out of the contour the integral (8) is not singular and, consequently,

$$F^{\pm}(z) = \pm \frac{\phi(z)}{2} + \frac{1}{2\pi i} P \int_0^{+\infty} \frac{\phi(\tau) d\tau}{\tau - z}, \quad (z < 0). \quad (10)$$

Thus, for the case  $\sqrt{1 + \bar{p}_{\parallel}^2} - N_{\parallel} \bar{p}_{\parallel} - n\Omega_c / \omega \leq 0$  the anti-Hermitian part of the integral over  $x$  in (7) with the multiplier  $e^{-\mu\sqrt{1 + \bar{p}_{\parallel}^2}}$  can be obtained by substituting the anti-Hermitian part  $-\pi i \phi(-\sqrt{1 + \bar{p}_{\parallel}^2} + N_{\parallel} \bar{p}_{\parallel} + n\Omega_c / \omega)$  of the second of the formulas (10), times  $2\pi i$ , which corresponds to the Landau rule for passing the pole,

instead of the Cauchy integral in expression (7). In this way

$$e^{-\mu\sqrt{1+\bar{p}_\parallel^2}} \int_0^{+\infty} dx e^{-\mu x} \frac{J_n^2\left(\sqrt{\lambda\mu x(x+2\sqrt{1+\bar{p}_\parallel^2})}\right)}{x+\sqrt{1+\bar{p}_\parallel^2}-N_\parallel\bar{p}_\parallel-n\Omega_c/\omega} = \text{Hermitian part} \\ -\pi i e^{-\mu(N_\parallel\bar{p}_\parallel+n\Omega_c/\omega)} J_n^2\left[\sqrt{\lambda\mu\left((N_\parallel\bar{p}_\parallel+n\Omega_c/\omega)^2-1-\bar{p}_\parallel^2\right)}\right]. \quad (11)$$

In the expression (11) it is convenient to go to the arguments introduced by Robinson in the weakly relativistic approximation:  $x = \bar{p}_\parallel\sqrt{\mu/2}$ ,  $z = \mu(1-n\Omega_c/\omega)$ ,  $a = \mu N_\parallel^2/2$ . Then (11) is converted to

$$\text{Herm. part} - \pi i e^{-2\sqrt{ax+z-\mu}} J_n^2\left\{\sqrt{-2\lambda[z-2\sqrt{ax}+x^2-(z-2\sqrt{ax})^2/(2\mu)]}\right\}. \quad (12)$$

From (12) the imaginary part of  $\mathcal{E}_{11}$  is

$$\text{Im } \mathcal{E}_{11} = \begin{cases} \left(\frac{\omega_{p0}}{\omega}\right)^2 \frac{\mu}{\lambda} \sum_{n=-\infty}^{+\infty} n^2 \frac{\pi e^{z-\mu}}{\sqrt{2\mu} K_2(\mu)} \int_x^{x^*} dx e^{-2\sqrt{ax}} J_n^2\{\Theta\}, & z < a^* \\ 0, & z \geq a^*. \end{cases} \quad (13)$$

Here were used  $\Theta = \sqrt{-2\lambda[z-2\sqrt{ax}+x^2-(z-2\sqrt{ax})^2/(2\mu)]}$ ,  $a^* = \mu(1-\sqrt{1-N_\parallel^2})$  and the integration limits were obtained from the condition that the pole must appear inside the integral of expression (7), that is equivalent to  $\sqrt{1+2x^2/\mu-2\sqrt{ax}/\mu+z/\mu-1}=0$ . Consequently they are  $x^\pm = [\sqrt{a(1-z/\mu)} \pm (a-z+z^2/(2\mu))/(1-N_\parallel^2)]$ . Thus, for the cyclotron harmonic number  $n$  and for the imaginary (anti-Hermitian) part of the tensor component  $\mathcal{E}_{11}$  it was obtained the expression that is an alternative to (1) and in which the summation of a series in the index  $l$  is the reduced to the numerical calculation of the one-dimensional integral (13). After the change of variables:  $x \rightarrow t$  in accordance with relation  $t = x - \beta\sqrt{a}(1-z/\mu)$ , leading to the symmetry of the new limits of integration  $t^\pm = \pm\beta\sqrt{a-z+z^2/(2\mu)}$  about zero and one more changing:  $t \rightarrow u$  in accordance with  $t = u\beta\sqrt{a-z+z^2/(2\mu)}$ , leading to the normalization of integration limits to  $u^\pm = \pm 1$ , then from (13) it follows

$$\text{Im } \mathcal{E}_{11} = \begin{cases} \left(\frac{\omega_{p0}}{\omega}\right)^2 \frac{\mu}{\lambda} \sum_{n=-\infty}^{+\infty} n^2 \frac{\pi e^{z-\mu}}{\sqrt{2\mu} K_2(\mu)} K e^{-2\beta a(1-z/\mu)} \times \\ \int_{-1}^1 du e^{-2\sqrt{a}Ku} J_n^2\{\Theta\}, & z < a^* \\ 0, & z \geq a^*, \end{cases} \quad (14)$$

where for brevity were used the next designations  $\Theta = K\sqrt{2\lambda(1-u^2)}/\beta$  and  $K = \beta\sqrt{a-z+z^2/(2\mu)}$ .

The integral (14) in the interval  $[-1, 1]$  is not singular, and therefore for arbitrary values of the parameter  $\lambda$  can be calculated without any problems. The real (Hermitian) part of the component  $\mathcal{E}_{11}$  can also be calculated from an imaginary part (14) along with one of Kramers-Kronig formulas, linking Hermitian and anti-Hermitian parts of an integral of Cauchy type, defined on the real axis [1]

$$\text{Re } \mathcal{E}_{11}(a, z, \mu) = \delta_{11} + \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\text{Im } \mathcal{E}_{11}(a, t, \mu) dt}{t-z}. \quad (15)$$

This formula corresponds to the Landau rule for passing the pole in the expression (11). For passing to the standard coordinates in (14) and (15) it is necessary to make a change of variable:  $z = 2\sqrt{a}z_n$ . Thus, during calculation of plasma dielectric tensor element  $\mathcal{E}_{11}$  for given cyclotron harmonic number  $n$  an evaluation of series converging slowly in the index  $l$  can be reduced to one-dimensional numerical calculation of integral (14) and subsequent calculation of the principal value of an integral of Cauchy type (15).

In a similar way it can be demonstrated that calculating the remaining components of relativistic plasma dielectric tensor (1) can also be reduced to computing the same kind one-dimensional integrals in the same interval  $[-1, 1]$ . A form of this tensor which is alternative one to (1) and suitable for accurate numerical applications for arbitrary  $\lambda$  is presented below:

$$\text{Im } \mathcal{E}_{11} = \Delta \sum_{n=-\infty}^{+\infty} \frac{n^2}{\lambda} e^z K e^{-2\beta a(1-z/\mu)} \int_{-1}^1 du e^{-2\sqrt{a}Ku} J_n^2\{\Theta\}, \quad (16)$$

$$\text{Im } \mathcal{E}_{12} = i\Delta \sum_{n=-\infty}^{+\infty} \frac{n}{\lambda} e^z K e^{-2\beta a(1-z/\mu)} \int_{-1}^1 du e^{-2\sqrt{a}Ku} \Delta J_n\{\Theta\} J_n'\{\Theta\},$$

$$\text{Im } \mathcal{E}_{22} = \Delta \sum_{n=-\infty}^{+\infty} \frac{1}{\lambda} e^z K e^{-2\beta a(1-z/\mu)} \int_{-1}^1 du e^{-2\sqrt{a}Ku} (\Delta)^2 J_n\{\Theta\} J_n'^2\{\Theta\},$$

$$\text{Im } \mathcal{E}_{13} = \Delta \sum_{n=-\infty}^{+\infty} \frac{n}{\sqrt{\lambda}} e^z K^2 e^{-2\beta a(1-z/\mu)} \int_{-1}^1 u du e^{-2\sqrt{a}Ku} J_n^2\{\Theta\},$$

$$\text{Im } \mathcal{E}_{23} = i\Delta \sum_{n=-\infty}^{+\infty} \frac{1}{\sqrt{\lambda}} e^z K^2 e^{-2\beta a(1-z/\mu)} \int_{-1}^1 u du e^{-2\sqrt{a}Ku} \Delta J_n\{\Theta\} J_n'\{\Theta\},$$

$$\text{Im } \mathcal{E}_{33} = \Delta \sum_{n=-\infty}^{+\infty} 2e^z K^3 e^{-2\beta a(1-z/\mu)} \int_{-1}^1 u^2 du e^{-2\sqrt{a}Ku} J_n^2\{\Theta\},$$

$$\Delta = \left(\frac{\omega_{p0}}{\omega}\right)^2 \frac{\pi e^{-\mu}}{K_2(\mu)} \sqrt{\frac{\mu}{2}}, \quad \Theta = K\sqrt{2\lambda(1-u^2)}/\beta,$$

$$\text{Re } \mathcal{E}_{ij}(a, z, \mu) = \delta_{ij} + \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\text{Im } \mathcal{E}_{ij}(a, t, \mu) dt}{t-z},$$

where  $\delta_{ij}$  is designation of Kronecker symbol. At last transition to the standard coordinates usual in the non-relativistic case can be made in (16) through the change  $z = 2\sqrt{a}z_n$ .

## CONCLUSIONS

1. It was shown that a fully relativistic dielectric tensor of plasma in magnetic field can be presented in the alternative to (1) the one dimensional integral form (16),

suitable for numerical applications for an arbitrary value of the parameter  $\lambda$ .

2. The form of the relativistic tensor (16) is rather of interest from the point of view applications to studying propagation and absorption of Bernstein electron cyclotron waves in the laboratory thermonuclear plasmas and arbitrary electron cyclotron waves in the hot astrophysical plasmas.

3. On the same way can be obtained the form of the exact fully relativistic dielectric tensor of magnetized plasma for ion plasma components suitable for applications with arbitrary wave numbers. This tensor can be used for the studying propagation and absorption of the arbitrary ion cyclotron waves in extremely hot astrophysical plasmas, for example in such ones as in the conditions of supernova explosions of stars.

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### ТОЧНОЕ ВЫЧИСЛЕНИЕ РЕЛЯТИВИСТСКОГО ТЕНЗОРА ДИЭЛЕКТРИЧЕСКОЙ ПРОНИЦАЕМОСТИ ПЛАЗМЫ В МАГНИТНОМ ПОЛЕ ДЛЯ ПРОИЗВОЛЬНЫХ ВОЛНОВЫХ ЧИСЕЛ

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Даётся новая интегральная форма полностью релятивистского тензора диэлектрической проницаемости плазмы в магнитном поле. Она годится для численных приложений при произвольных волновых числах, поскольку все интегралы в ней являются одномерными. Эта форма представляет интерес с точки зрения приложений для изучения бернштейновских электронных циклотронных волн в лабораторной термоядерной плазме и произвольных электронных и ионных циклотронных волн в горячей астрофизической плазме.

### ТОЧНЕ ОБЧИСЛЕННЯ РЕЛЯТИВІСТСЬКОГО ТЕНЗОРА ДІЕЛЕКТРИЧНОЇ ПРОНИКНОСТІ ПЛАЗМИ В МАГНІТНОМУ ПОЛІ ДЛЯ ДОВІЛЬНИХ ХВИЛЬОВИХ ЧИСЕЛ

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Дається нова інтегральна форма повністю релятивістського тензора діелектричної проникності плазми в магнітному полі. Вона годиться для численних додатків при довільних хвильових числах, оскільки всі інтеграли в ній є одновимірними. Ця форма являє інтерес з точки зору додатків для вивчення бернштейнівських електронних циклотронних хвиль в лабораторній термоядерній плазмі і довільних електронних та іонних циклотронних хвиль в гарячій астрофізичній плазмі.