UDC 539.4

Lateral Torsional Buckling Response of Steel Beam with Different Boundary Conditions and Loading

L. Dahmani and A. Boudjemia

Mouloud Mammeri University, Tizi-Ouzou, Algeria

УДК 539.4

Торсионная потеря устойчивости стального стержня при различных граничных условиях и продольных изгибных нагрузках

Л. Дахмани, А. Буджемиа

Университет им. Мулуда Маммери, Тизи-Узу, Алжир

Евростандарт EN 1993-1-1 описывает общий метод определения предельной нагрузки для стальных стержней при продольном изгибе с кручением. В методе учитываются кривые, описывающие потерю устойчивости при продольном изгибе. Предельная нагрузка при продольном изгибе с кручением может быть рассчитана методом конечных элементов на основе геометрического и нелинейного анализа материалов стержня с дефектами. Проведено сопоставление значений предельной нагрузки в соответствии с нормами Евростандарта EN 1993-1-1 для продольного изгиба поперечно закрепленных стержней кручения с таковыми, полученными путем моделирования методом конечных элементов на основе параметрического исследования.

Ключевые слова: стальной стержень, продольный изгиб с кручением, Евростандарт EN 1993-1-1, метод конечных элементов, программная система ANSYS.

Introduction. Buckling and lateral stability are among the key parameters in the design of steel structures [1–4]. Flexural members subjected to bending about their major axis may develop buckling in the compression flange combined with lateral bending, leading to what is known as lateral torsional buckling [3, 4]. For doubly symmetric laterally unbraced slender beams, lateral torsional buckling can govern their ultimate limit state.

Lateral Torsional Buckling. A short beam with a compact cross section can reach its full plastic moment capacity without any lateral instability. However, if the beam is slender and the compression flange is not adequately braced in the lateral direction, a different phenomenon occurs. As the beam is loaded in bending about its strong axis, it deforms in the direction of loading, but after buckling it demonstrates an angular deformation (Fig. 1).

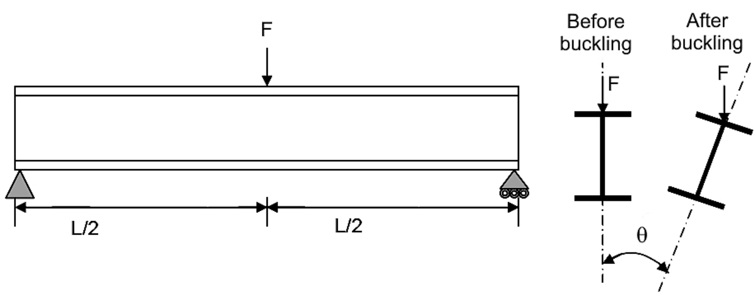


Fig. 1. Lateral torsional buckling at angle θ.

The lateral torsional buckling capacity depends upon a number of material and geometric properties, support conditions, and location of the applied load relative to the shear centre and bending moment distribution along the length of the member. The critical elastic lateral torsional buckling capacity for the uniform moment gradient is given by [5]:

$$M_{cr} = \frac{\pi^2 EI_z}{L^2} \left[\frac{I_w}{I_z} + \frac{L^2 GI_t}{\pi^2 EI_z} \right]^{0.5}, \tag{1}$$

where M_{cr} is the elastic lateral torsional buckling strength, E is the modulus of elasticity, G is the shear modulus, I_z is the moment of inertia about weak axis, I_t is the Saint-Venant torsional constant, and I_w is the warping constant of the section.

Generally, consideration of the nonuniform bending moment diagram is taken into account by means of the equivalent uniform moment factor C_1 [5]. The elastic critical moment of a simply supported beam with a uniform moment is multiplied by this factor to obtain the elastic critical moment for any bending moment diagram,

$$C_1 = 1.88 - 1.40\psi + 0.52\psi^2 \le 2.7,$$

where ψ is the ratio of the smaller factored moment to the larger one at the end points of lateral support, $\psi = M_a / M_b$ for $M_a \prec M_b$ $(-1 \le \psi \le 1)$. This ratio is positive for the double curvature and negative for the single curvature. The moments are applied at the end points of lateral support.

Code Requirements. For both the general and specific methods in Eurocode 3 [5] to determine the ultimate lateral torsional buckling (LTB) load of beams in bending, the design buckling resistance moment should be taken as

$$M_{b.Rd} = \chi_{LT} \frac{W_y f_y}{\gamma_{M1}}, \tag{2}$$

in which W_y is the appropriate section modulus: $W_y = W_{pl.y}$ for class 1 or 2 sections. The reduction factor χ_{LT} is a function of the imperfection factor α_{LT} and the relative slenderness is given by

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}. \tag{3}$$

This relative slenderness will be used in subsequent equations to determine the reduction factor. It should be noted that the elastic critical bending moment for LTB is not specified by Eurocode 3 [5], but its determination is left to a designer.

General Method. This method is presented in clause 6.3.2.2 of Eurocode 3 [5] as the "general case," hereafter referred to as the general method (GM). According to the GM, the reduction factor χ_{LT} for LTB of beams is similar to that for column buckling [6, 7]:

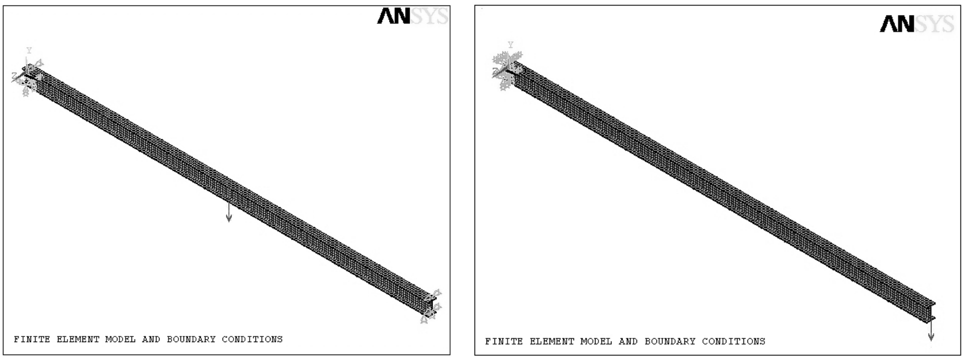
$$\chi_{LT} = \frac{1}{\varphi_{LT} + \sqrt{[\varphi_{LT}^2 - \bar{\lambda}_{LT}^2]}} \le 1, \tag{4}$$

$$\varphi_{LT} = 0.5[1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2]. \tag{5}$$

When $\bar{\lambda}_{LT} \leq 0.4$ or the design bending moment $M_{Ed} \leq 0.16 M_{cr}$, then $\chi_{LT} = 1$. The imperfection factor α_{LT} is selected according to the required buckling curve for the design of the beam. The appropriate buckling curve is given in Eurocode 3 [5].

Finite Element Method. ANSYS [8], a commercial finite element software, was used for the analysis. An eigenvalue analysis was used to get the deflected shape (mode shape or eigenvector) and the associated load factor (eigenvalue). The resulting eigenvalues are actually the load factors to be multiplied by the applied loading, in order to obtain the critical buckling load.

The element used in ANSYS [8], BEAM 188, is a quadratic three-dimensional beam element suitable for analyzing slender to moderately stocky beams. It possesses warping degrees of freedom, in addition to the conventional six degrees of freedom (Fig.2). The results of the buckling analysis are shown in Fig. 3, where the buckled shape and the load factor (μ) are indicated.



a b

Fig. 2. Finite element model and boundary conditions for (a) simply supported and (b) cantilever beams.

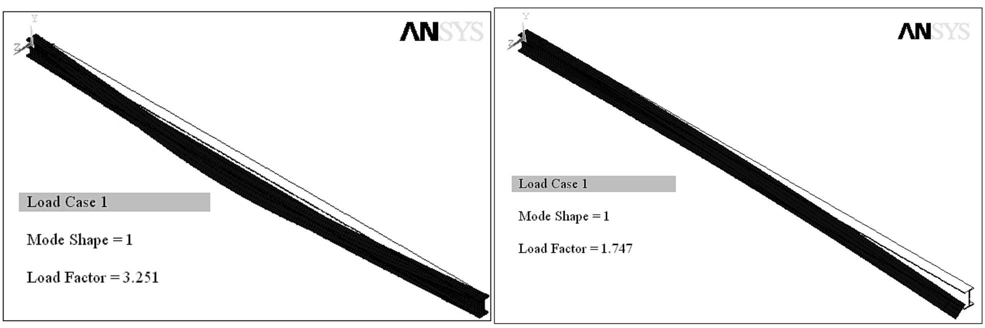


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Fig. 3. Buckled shape and load factor for (a) simply supported and (b) cantilever beams.

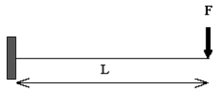
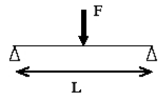
The above figure depicts the behavior of the lateral torsional buckling, where lateral displacement combined with twisting can be observed.

Validation. In order to validate the finite element model developed for this investigation, an eigenvalue buckling analysis was carried out for the model shown in Fig. 2, and the predicted load factors (Table 1) were compared with the theoretical values of the lateral torsional buckling capacity.

The difference between the results calculated using formula is $\Delta = \dfrac{|\mu_{ANSYS} - \mu_{theor}|}{\mu_{ANSYS}}$.

Table 1

Predicted Load Factors

Section	Boundary conditions loading	Loading location	Load factors		
			μ_{theor}	μ_{ANSYS}	Δ, %
Double symetrical I section $h = 300$ mm $b_{f1} = b_{f2} = 15$ mm $t_{f1} = t_{f2} = 10$ mm $t_w = 10$ mm	 $L = 10$ m, $F = 10$ kN At beam fixing: v, θ, v', θ' (fixed)	Upper flange	1.735	1.747	0.57
		Shear center	2.333	2.343	0.43
		Lower flange	2.670	2.676	0.21
Double symetrical I section $h = 300$ mm $b_{f1} = b_{f2} = 15$ mm $t_{f1} = t_{f2} = 10$ mm $t_w = 10$ mm	 $L = 10$ m, $F = 10$ kN At beam fixing: v, θ (free), v', θ' (fixed)	Upper flange	3.245	3.251	0.18
		Shear center	4.245	4.250	0.11
		Lower flange	4.643	4.650	0.15

The buckling capacity predicted using the beam element BEAM 188 of ANSYS [8] is within 0.6% of the theoretical value.

Conclusions. This paper compares the ultimate lateral torsional buckling loads of unrestrained beams in bending based on the design rules in Eurocode 3 for the ultimate loads obtained via finite element simulations. For the calculations performed in the parameter study, worrisome results have been obtained on the validity of the general methods for lateral torsional buckling of rolled sections. It can be concluded that the general method can lead to the underestimations of even less than 0.6% of the ultimate lateral torsional buckling load of unrestrained beams obtained via the finite element simulations. The general method gives good results for lateral torsional buckling of steel beams without restraints between the supports.

For these situations, there is quite good agreement between the values given by the Eurocode 3 design code and the numerical results of the finite element methods.

Р е з ю м е

Євростандарт EN 1993-1-1 описує загальний метод визначення граничного навантаження для стальних стрижнів при поздовжньому згині з крутінням. У методі враховуються криві, що описують втрату стійкості при поздовжньому згині. Граничне навантаження при поздовжньому згині з крутінням може бути розраховано методом скінченних елементів на основі геометричного і нелінійного аналізу матеріалів стрижня з дефектами. Проведено зіставлення значень граничного навантаження у відповідності з нормами Евростандарту EN 1993-1-1 для поздовжнього згину поперечно закріплених стрижнів крутіння з такими, що отримані шляхом моделювання методом скінченних елементів на основі параметричного дослідження.

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Received 16. 07. 2013