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# Effects of Geometrical Parameters on the Stress Field of Three-Dimensional Plates Weakened by Periodic Notches

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# Влияние геометрических параметров на поле напряжений трехмерных пластин, ослабленных периодическими надрезами

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*С использованием метода конечных элементов в трехмерной постановке исследовано напряженное состояние и определена зона максимальных напряжений в плоских пластинах с периодическими надрезами. В частности, изучен эффект варьирования шестью геометрическими параметрами (толщина пластины, отношение глубины надреза к ширине пластины, расстояние между периодическими надрезами, угол и радиус V-образного надреза) на распределение напряжений по толщине пластины и локализацию максимальных напряжений. Проанализировано более 500 геометрических конфигураций пластин с надрезами. Согласно результатам исследований в относительно тонких пластинах максимальное значение напряжения у среднего надреза достигается в серединной плоскости, что соответствует плоскому напряженному состоянию, постулируемому в рамках двухмерных конечноэлементных моделей. Однако с увеличением толщины пластины имеет место смещение области максимальных напряжений из серединной плоскости к свободной поверхности пластины. Установлено, что увеличение количества надрезов с целью снижения уровня напряжений в их вершине является эффективным лишь в том случае, если расстояние между ними сравнительно мало. Из вышеуказанных геометрических параметров наибольшее влияние на локализацию области максимальных напряжений вблизи среднего надреза оказывает расстояние между периодическими надрезами, особенно для относительно тонких пластин.*

***Ключевые слова***: периодические надрезы, конечноэлементный анализ, трехмерная постановка, растяжение.

**Introduction**. As well-known examples of periodic notches, bolt-nut joints play an important role in the safety and reliability of structural systems. In [1] the reduction of stress concentration of bolt thread geometry is achieved by an optimization technique. Reduction of stress concentration in a bolt-nut joint by using a two dimensional axisymmetric finite element model is studied in [2]. Dealing with periodic blunt notches and using the strain energy density (SED) approach, plane strain conditions are assumed to evaluate the stress concentration factors (SCF) of a number of flat plates and round bars with periodic U- and V-notches. Tension, bending, and torsion loading conditions have been considered [3]. In the presence of sharp periodic notches and by means of SED, the plane theory of elasticity has been used to study the variability of the notch stress intensity factors (NSIF) of periodic sharp notches in [4, 5]. A new model of depth reduction factor for different ratios of relative depth of the notch is proposed to match the results of the SED approach. In the case of shallow notches, the results are compared with some semianalytical solutions provided in [6]. In addition, based on the best fit of numerical data from the SED approach, some polynomials for nondimensional NSIFs, in the case of intermediate and deep notches are presented. In another work by the authors [7], some very simple expressions are derived for the direct evaluation of the SED and the NSIF of plates with infinite width as a function of the notch spacing in the case of narrow sharp notches.

Dealing with crack problems, it is crucial to know the exact stress intensity factors for different geometrical configurations because the fatigue life is strongly influenced by them, both under uniaxial and multiaxial loadings [8–11]. For a plate under in-plane loading condition, two basic assumptions for the stressed state within the framework of plane theories of elasticity, namely as plane stress (zero transverse stress) and plane strain (zero transverse strain), are commonly used [12]. On the other hand, until now there is no generally accepted criterion for identifying what thicknesses correspond to plane stress or plane strain conditions. There are numerous recent publications addressing the above issue [13–20]. In particular, in [15], an extensive review of some recent analytical, numerical and experimental results was performed to investigate the effect of plate thickness on elastic deformation, as well as quasi-brittle fracture of plate components.

In [21], an attempt was made to investigate the thickness effect on the location of maximum stress and NSIF of corresponding blunt and sharp periodic notches in three-dimensional (3D) plates weakened by periodic blunt and sharp notches. A number of 3D finite element (FE) models are constructed with this aim. In addition, different number of periodic notches, as well as different notch opening angles, are examined.

As it can be concluded from the above literature review, most of the works has been done on periodic notches are limited to two-dimensional (2D) analyses. However, the 3D stress field ahead of notch root of periodic notched components is greatly affected by the six geometrical parameters, namely: the plate thickness, the ratio of notch depth and width of the plate, the pitch of periodic notches, the notch opening angle and the notch tip radius. Therefore, in this study, a comprehensive 3D FE analysis is performed on periodic blunted notches models. The aim of this study is to investigate the influence of the above six mentioned parameters on the stress field ahead of notch root of blunted notches, as well as on the location of the maximum stress through the plate thickness.

1. **Geometry and Boundary Conditions**. Five models with different geometrical configurations (summarized in Table 1) have been considered. The geometrical parameters are introduced as: thickness of the plate ($t$), notch depth to width of the plate ($a/w$), pitch of notch ($p$), notch opening angle ($2\alpha$), notch radius ($\rho$), and number of notches ($N$). More than five hundred three-dimensional models have been analyzed by combining the following values of the above-mentioned parameters (Table 1). In all these models, the applied stress $\sigma_n = 1$ MPa is assumed.

Figure 1 shows the schematic model with boundary conditions.

2. **Numerical Analysis**. The finite element method (FEM) is employed to obtain the stress field ahead of notches and analyze the stressed state and the location of the maximum stress. All of the analyses are carried out using the ANSYS software package. Due to existence of symmetry, only one-fourth of each notched plate is modeled. The 20-node brick element is used. Fine mesh is utilized near the notch tip to improve the accuracy of results, whereas coarse mesh is used far from the notch tip, in order to reduce the numerical cost. Figure 2 shows a sample meshed model.

T a b l e 1

**Geometry of Models under Study**

| Model | $2\alpha$, deg | $\rho$, mm | $p$, mm | $t$, mm | $N$ | $a/w$ |
|---|---|---|---|---|---|---|
| 1 | 60 | 0.1 | 2.5 | 0.5, 1–10, 15, 20, 25 | 3, 7 | 0.125 |
| 2 | 60 | 0.1 | 1.5, 2.5, 3.5 4.0, 5.0 | 4–11 | 3, 7 | 0.08, 0.1, 0.125, 0.16, 0.2 |
| 3 | 60 | 0.1 | 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0 | 5–10 | 3, 7 | 0.125 |
| 4 | 10, 15, 20, 30, 45, 60, 90, 120 | 0.1 | 2.5 | 1, 5–10 | 3–5 | 0.125 |
| 5 | 60 | 0.05, 0.1, 0.2, 0.3 | 2.5 | 5–10 | 3, 7 | 0.125 |

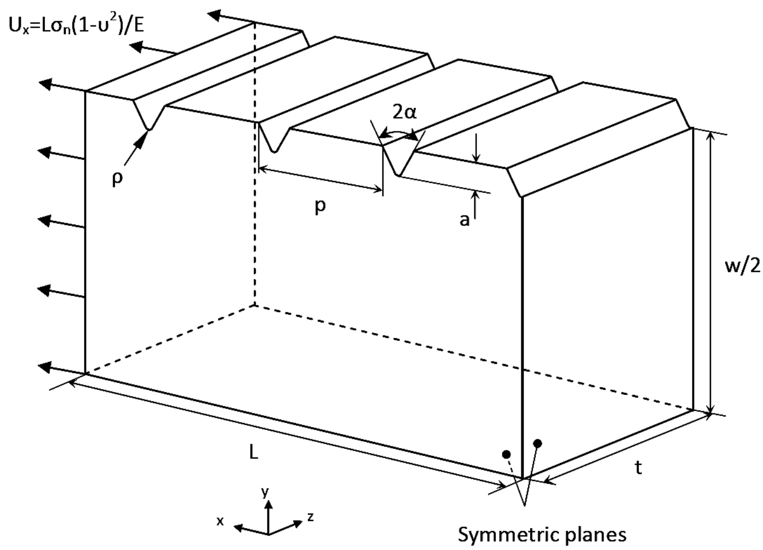



Fig. 1. The geometrical parameters of 3D periodic notched plate and boundary conditions.

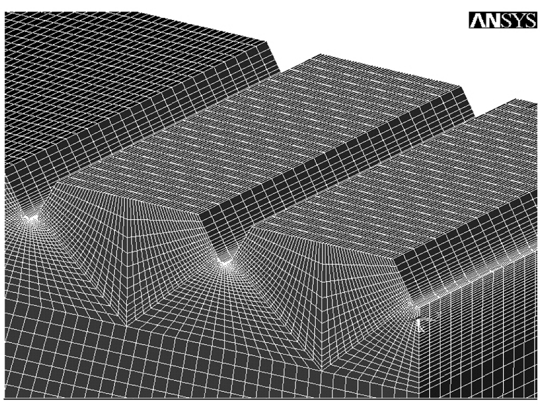



Fig. 2. A sample meshed model.

Prior to the analysis, a model verification is performed by comparing the results obtained from model 1 and corresponding results of [21]. Figure 3 depicts the normalized stress at the middle notch of the periodic notched plate as a function of normalized plate thickness for the three thickness values of 1, 5, and 10 mm. Excellent agreement is observed between the results of the present model and the ones reported in [21] with the maximum relative difference of only 0.5%, which is related to the case with a thickness of 10 mm and normalized thickness of 0.5.

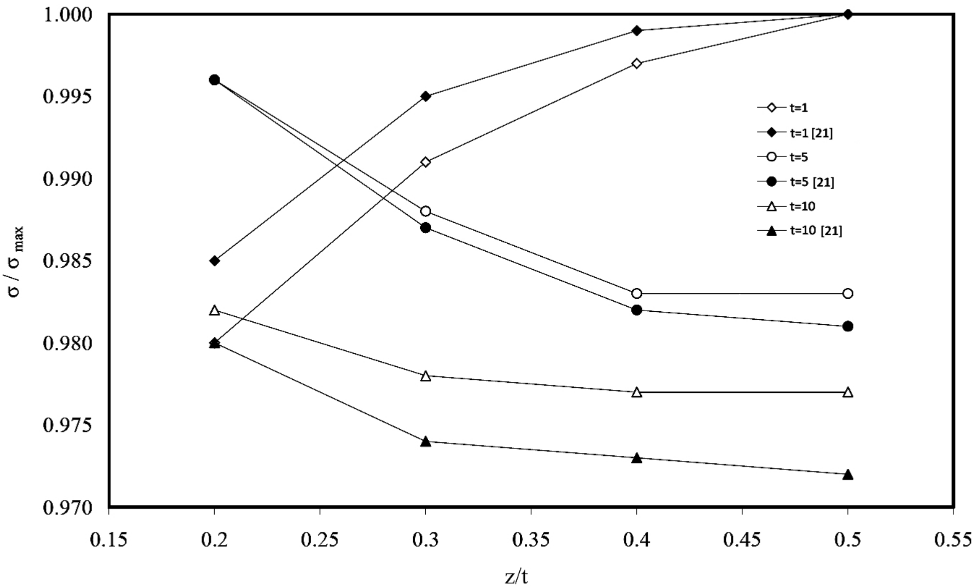



Fig. 3. Comparison between the present study results and data of [21]. (Here and in Figs. 4–25: legends have dimensions as in Table 1.)

3. **Results and Discussion**.

3.1. ***Effect of the Thickness Variation***. In order to evaluate the effect of the thickness variation on the tensile stress at the middle notch of periodic notched plates along the thickness of the plate, model 1 is used for the analysis. In this regard, stress values are calculated for 14 thicknesses (0.5, 1–10, 15, 20, and 25 mm) and two different numbers of notches ($N = 3$ and 7).

Figure 4 shows the normalized stress at the middle notch of the periodic notched plate as a function of normalized thickness of the plate for different ranges of the plate thickness.

As is seen from Fig. 4, the maximum stress is shifted from the mid-plane to the location near the free surface when the thickness of the model increases.

Figure 5 exhibits a similar trend for thinner models. According to Fig. 5, the maximum stress location starts to shift toward the free surface, in case of $t > 3$ mm.

Figure 6 depicts the results of the similar analysis for $t = 5$–10 mm. The trend is similar to the cases corresponding to thicknesses larger than 2 mm shown in Fig. 5. Moreover, it can be seen that increasing the thickness of the model leads to a slight decrease in the stress values.

Figure 7 presents the results of the analysis for $N = 7$. Again, the same patterns of the maximum stress location being shifted toward the free surface and the reduction of stress values with plate thickness can be observed. However, the difference between the stress values increases for $N = 7$.

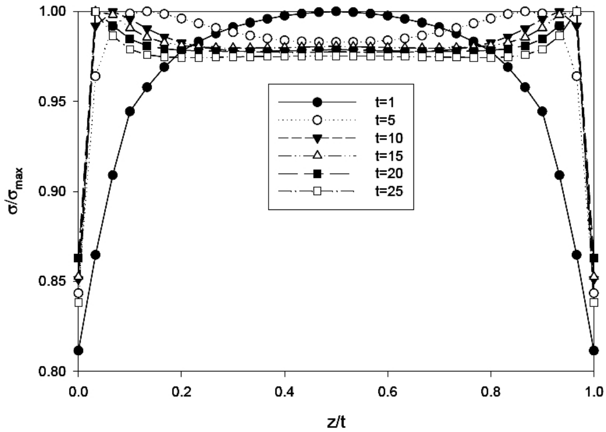



Fig. 4. Variation of tensile stress at the middle notch of periodic notched plates along the plate thickness for different thickness values ($a = 0.5$ mm, $N = 3$, $p = 2.5$ mm, $\rho = 0.1$ mm, and $2\alpha = 60°$).

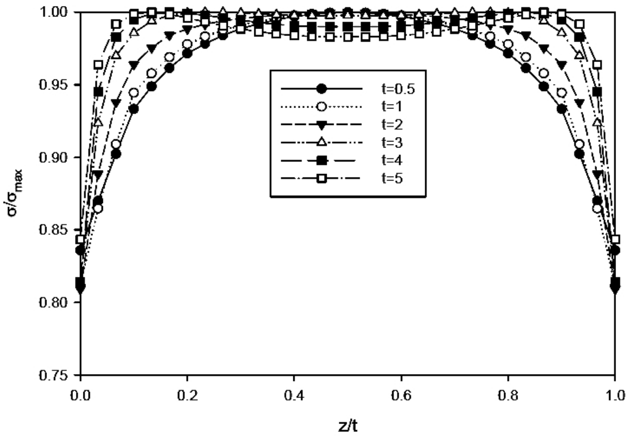



Fig. 5. Variation of tensile stress at the middle notch of periodic notched plates along the plate thickness for different thickness values ($t = 0.5$–$5.0$ mm, $a = 0.5$ mm, $N = 3$, $p = 2.5$ mm, $\rho = 0.1$ mm, and $2\alpha = 60°$).

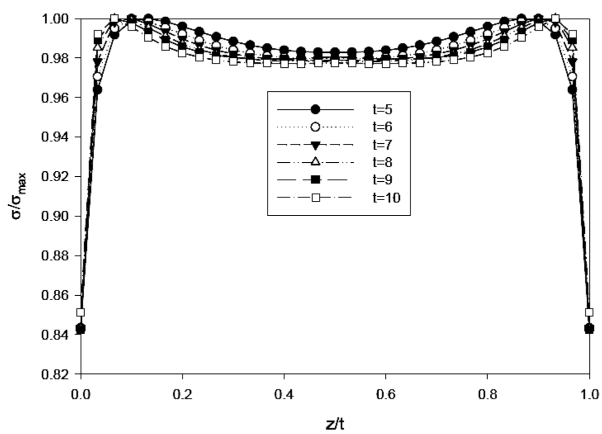



Fig. 6. Variation of tensile stress at the middle notch of periodic notched plates along the plate thickness for different thickness values ($t = 5$–$10$ mm, $a = 0.5$ mm, $N = 3$, $p = 2.5$ mm, $\rho = 0.1$ mm, and $2\alpha = 60°$).

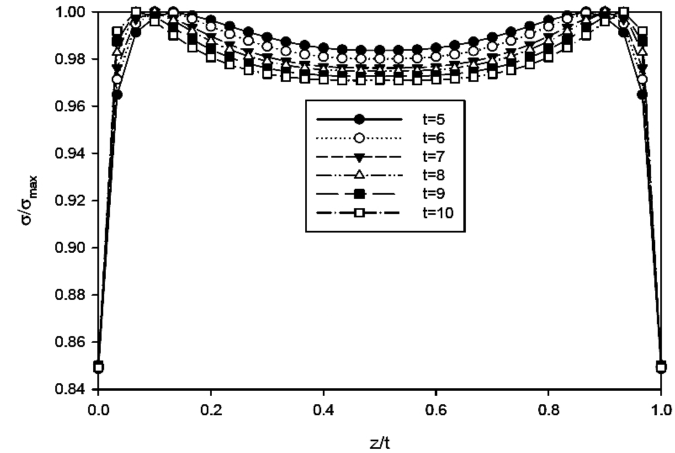



Fig. 7. Variation of tensile stress at the middle notch of periodic notched plates along the plate thickness for different thickness values ($t = 5$–$10$ mm) and $N = 7$ ($a = 0.5$ mm, $p = 2.5$ mm, $\rho = 0.1$ mm, and $2\alpha = 60°$).

The influence of thickness on the location of the normalized maximum stress will be discussed in section 3.3.

3.2. ***Effect of Notch Depth-to-Plate Width Ratio***. To investigate the effect of notch depth-to plate width ratio *a/w*, two parameters as notch pitch and number of notches are also considered. It is due to the fact that the notch pitch, as it will be discussed in section 3.3, has a strong effect on the stress field of notches, as well as on other geometrical parameters.

At first, stress variation with $a/w$ ratio is studied for model 2 with the preset number of notches $N = 3$ and thickness value $t = 5$ mm. Figures 8–10 show the results for $p = 1.5$, 3.5, and 5 mm, respectively. In these models, the notch depth ($a$) was maintained invariable, while the plate width ($w$) was varied. It might be expected that stress values increase when the width of model decreases, but based on the results depicted in Fig. 8, it can be seen that stress values start to reduce when $a/w$ increases, but stress values tend to grow at $a/w$ ratios exceeding 0.16.

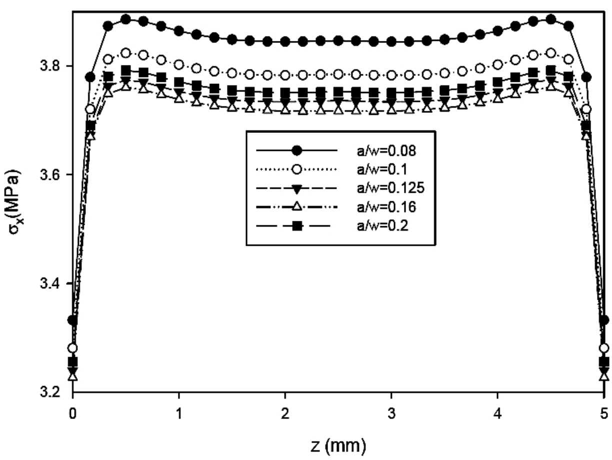



Fig. 8. Variation of tensile stress at the middle notch of periodic notched plates along the plate thickness for different $a/w$ ratios and $p = 1.5$ mm ($a = 0.5$ mm, $N = 3$, $\rho = 0.1$ mm, $2\alpha = 60°$, and $t = 5$ mm).

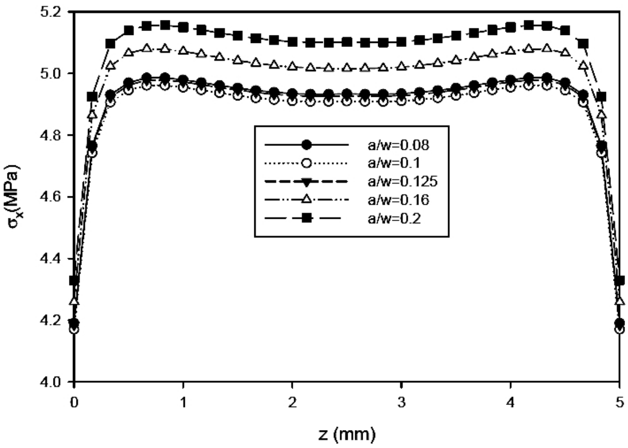



Fig. 9. Variation of tensile stress at the middle notch of periodic notched plates along the plate thickness for different $a/w$ ratios and $p = 3.5$ mm ($a = 0.5$ mm, $N = 3$, $\rho = 0.1$ mm, $2\alpha = 60°$, and $t = 5$ mm).

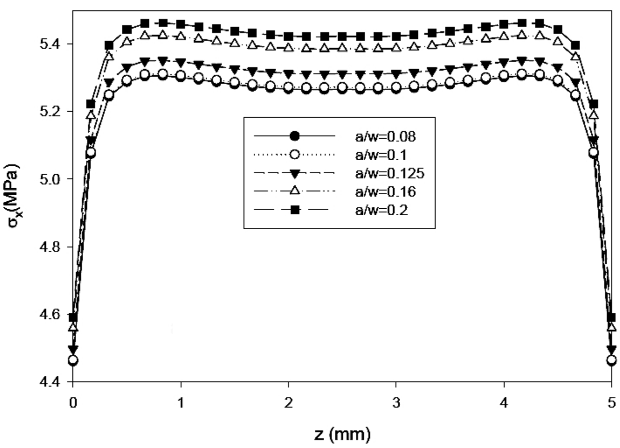



Fig. 10. Variation of tensile stress at the middle notch of periodic notched plates along the plate thickness for different $a/w$ ratios and $p = 5$ mm ($a = 0.5$ mm, $N = 3$, $\rho = 0.1$ mm, $2\alpha = 60°$, and $t = 5$ mm).

The same trend can be observed in Fig. 9, with the only difference that the increase in stress values starts at larger widths (for $a/w > 0.1$), which obviously indicates the effect of width variation on the stress field.

It can be concluded from what Figs. 8–10 that further increasing of the model plate pitch leads to a trend in stress variation, which was expected from the beginning. In other words, as it is presented in Fig. 10, reducing the model width leads to an increment of stress values.

Figure 11 depicts the tensile stress at the middle point of the plate thickness for different pitch $p$ values versus $a/w$ ratios.

According to Fig. 11 and based on what it can be observed in Figs. 8–10, when the periodic notches are far enough from each other, i.e., a large pitch value, stress values increase due to the reduced distance between the notch tip and the bottom edge. In other words, when the pitch of periodic notches is large enough so that the notches have a feeble effect on each other, one can expect the increase of stress values by decreasing the width of the model plate.

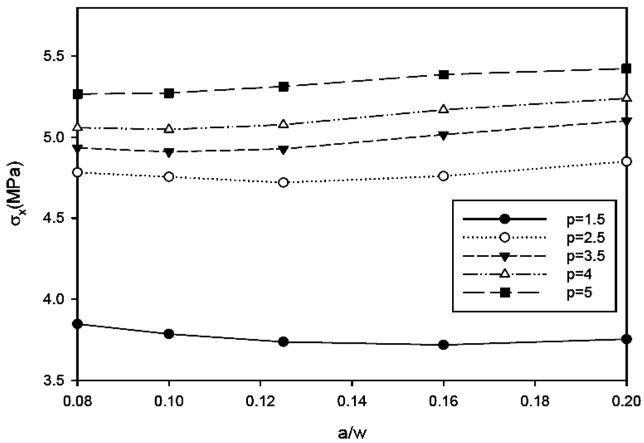



Fig. 11. Tensile stress at the middle point of the plate thicknes versus $a/w$ ratios for different pitch values $p$ ($a = 0.5$ mm, $N = 3$, $\rho = 0.1$ mm, $2\alpha = 60°$, and $t = 6$ mm).

The effect of $a/w$ ratio on the stress at the middle notch as a function of number of notches ($N$) and pitch ($p$) is also studied. The results are shown in Fig. 12. Generally, it is expected that the stresses would decrease through the thickness, when the number of notches increases. But as it can be seen from Fig. 12, this pattern is valid only in cases with a relatively small pitch. For example, for models with $p = 1.5$ mm, there is a relatively large difference between the stress values corresponding to models with $N = 3$ and 7. But for $p = 5$ mm (except for the first point, which is related to the ratio of $a/w = 0.08$), increasing the number of notches has no effect on stress values, and the respective stress lines almost coincide. Thus, when the periodic notches are placed at a small distance from each other, the effect of each notch on the stress field of others becomes quite strong, so that one can reduce the stress values by increasing the number of notches.

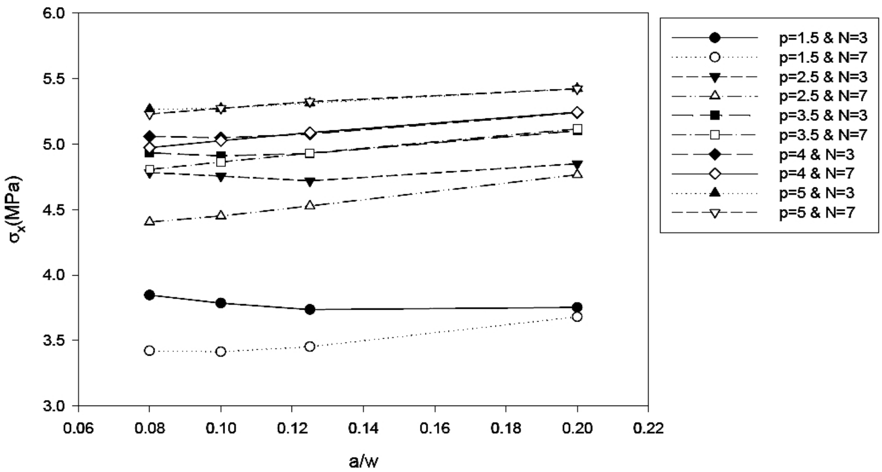



Fig. 12. The effect of $a/w$ ratio on stress at the middle notch as a function of number of notches and pitch ($a = 0.5$ mm, $\rho = 0.1$ mm, $2\alpha = 60°$, and $t = 5$ mm).

Another important point is the role of $a/w$ ratio. As it is depicted in Fig. 12, increasing $a/w$ ratio reduces the effect of adding notches to the model. This is true for the cases with small pitch as it is stated earlier and also is evident from Fig. 12.

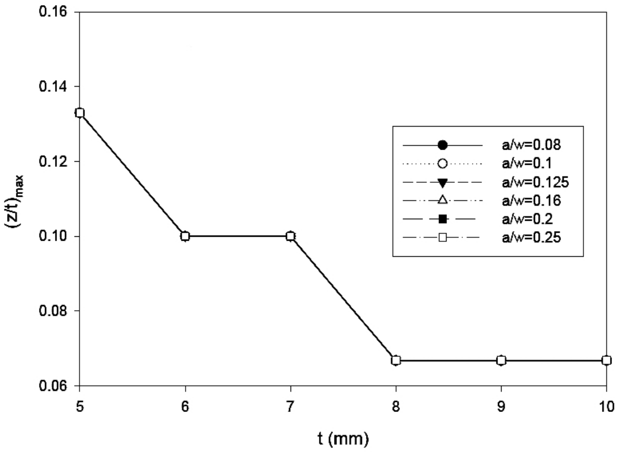
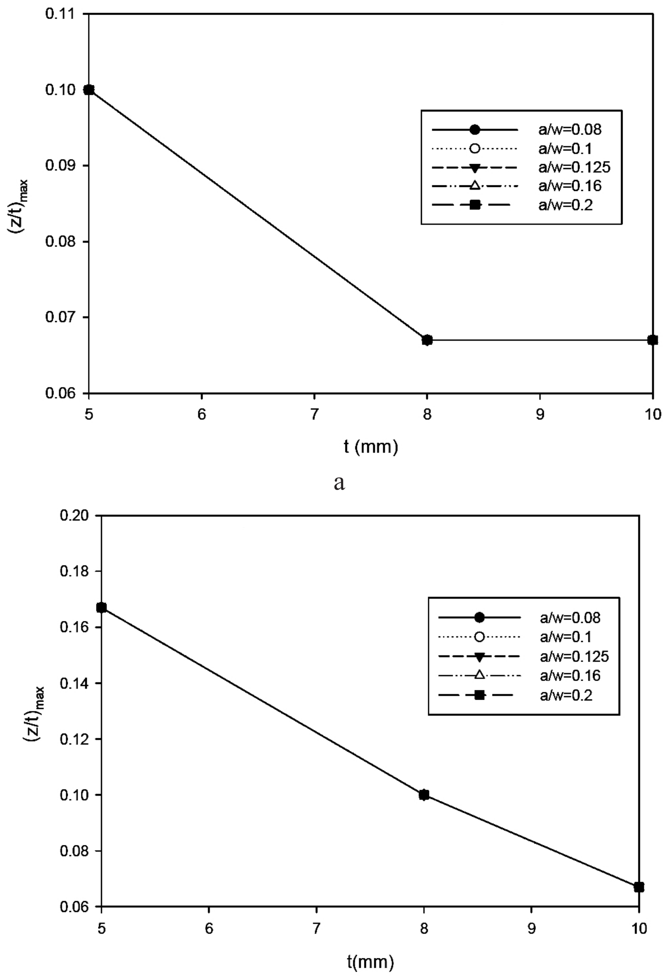



Fig. 13. The effect of $a/w$ ratio on the location of the maximum stress $(z/t)_{max}$ for $p = 2.5$ mm ($a = 0.5$ mm, $N = 3$, $\rho = 0.1$ mm, and $2\alpha = 60°$).




a

b

Fig. 14. The effect of $a/w$ ratio on the location of the maximum stress $(z/t)_{max}$ for (a) $p = 1.5$ mm and (b) $p = 4$ mm ($a = 0.5$ mm, $N = 3$, $\rho = 0.1$ mm, and $2\alpha = 60°$).

The effect of $a/w$ ratio on the location of the maximum stress is also studied. Figure 13 presents the $z/t$ values regarding the location of the maximum tensile stress for models with $p = 2.5$ mm. As is seen in Fig. 13, $a/w$ variation exerts no effect on the location of the maximum stress values. Figure 14 illustrates the similar analysis for models with pitch values of 1.5 and 4 mm, and thickness values of 5, 8, and 10 mm. It is clear from Fig. 14 that even under conditions of pitch variation, $a/w$ ratio still does not affect the maximum stress location in the plate within the thickness range of 5–10 mm.

3.3. ***Effect of Pitch Variation***. Figure 15 shows the tensile stress variation along the plate thickness in the middle notch of the periodic notched plate for different pitch values. As it is depicted in Fig. 15, increasing the notch pitch leads to an increment of the stress values. Furthermore, the rate of this increment tends to be reduced for higher pitch values.

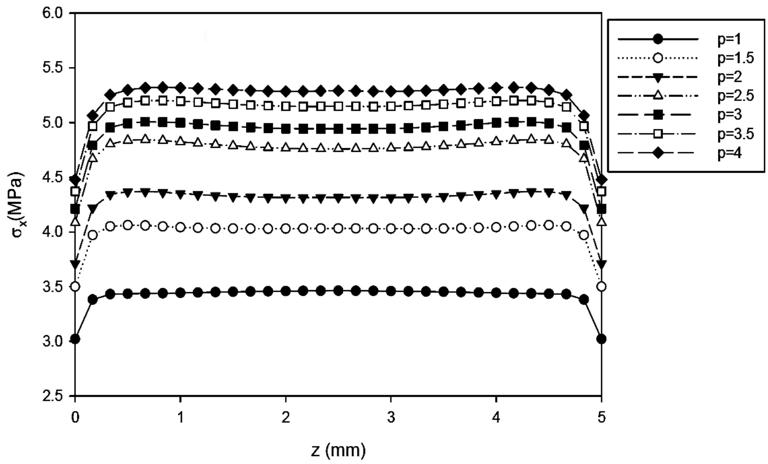



Fig. 15. Variation of tensile stress at the middle notch of periodic notched plates along the plate thickness for different notch pitches ($a = 0.5$ mm, $\rho = 0.1$ mm, $t = 5$ mm, and $2\alpha = 60°$).

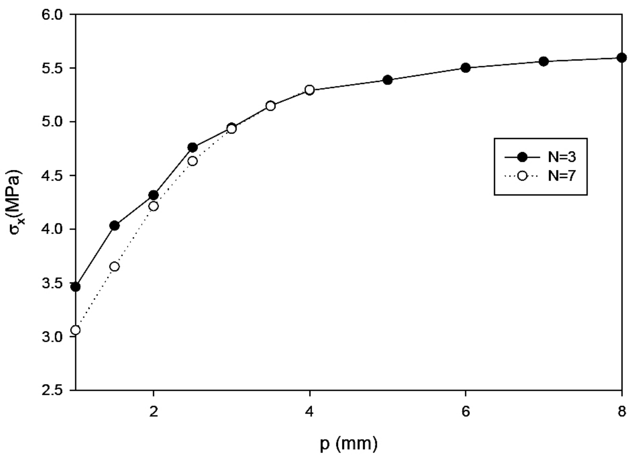



Fig. 16. Tensile stress at the middle point of the plate thickness as a function of pitch for different number of notches ($a = 0.5$ mm, $\rho = 0.1$ mm, $t = 5$ mm, and $2\alpha = 60°$).

Figure 16 shows the tensile stress of the middle point of the plate ($z = t/2$) as a function of model pitch for $N = 3$ and 7. It is clear from this figure that the effect of increasing the number of notches becomes negligible when the pitch of the model increases. For example, in this specific model with $p = 3$ mm, stress values for $N = 3$ and 7 are almost identical. In addition, for the model with $N = 3$, influence of larger pitches is considered, i.e., pitch values of 5, 6, 7, and 8 mm. As it is depicted in Fig. 16, when the pitch attains a relatively high value, the stress values of the middle notch become insensitive to further increase in the pitch value.

The effect of pitch variation on the maximum stress location is also evaluated. Figure 17 shows the $z/t$ values regarding the location of maximum tensile stress for different thickness and pitch values (these data are tabulated in Table 2). As it is depicted in Fig. 17, pitch variation has the most important effect on the location of maximum stress values. According to this figure, for a given pitch, increasing the plate thickness leads to a shift in the location of maximum stress value toward the free surface. In addition, for a given thickness, the location of the maximum stress becomes shifted from the mid-plane for the small pitch values to the near surface for the case of large pitch values. In fact, for thinner plates the shift in the maximum stress location is more evident, in comparison with relatively thicker plates ($t = 10$ and 11 mm).

Table 2

**Effect of Pitch Variation on the Maximum Stress Location for Different Thicknesses**

| The location of the maximum tensile stress $(z/t)_{max}$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| Thickness (mm) | Pitch of notches (mm) | | | | | | |
| | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| 5 | 0.5 | 0.133 | 0.133 | 0.1667 | 0.200 | 0.2000 | 0.2000 |
| 6 | 0.5 | 0.100 | 0.133 | 0.1330 | 0.133 | 0.1667 | 0.1667 |
| 7 | 0.5 | 0.100 | 0.100 | 0.1000 | 0.100 | 0.1000 | 0.1330 |
| 8 | 0.5 | 0.067 | 0.100 | 0.1000 | 0.100 | 0.1000 | 0.1000 |
| 9 | 0.5 | 0.067 | 0.067 | 0.0670 | 0.100 | 0.1000 | 0.1000 |
| 10 | 0.2 | 0.067 | 0.067 | 0.0670 | 0.067 | 0.0670 | 0.0670 |
| 11 | 0.2 | 0.067 | 0.067 | 0.0670 | 0.067 | 0.0670 | 0.0670 |

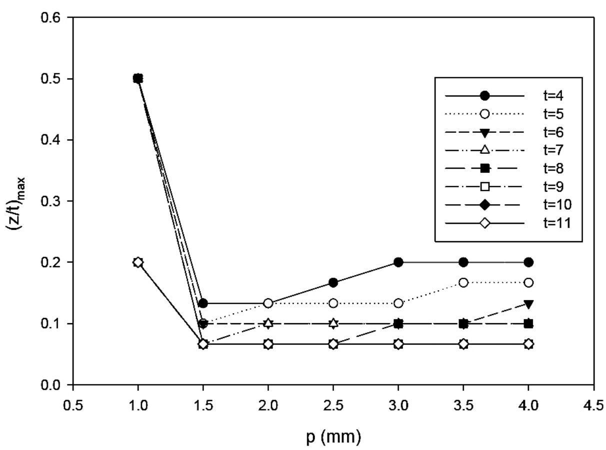



Fig. 17. The effect of $p$ variation on the location of the maximum stress $(z/t)_{max}$ for different thicknesses ($a = 0.5$ mm, $\rho = 0.1$ mm, $N = 3$, and $2\alpha = 60°$).

3.4. ***Effect of the Notch Opening Angle Variation***. In order to evaluate the effect of the notch opening angle variation on tensile stress at the middle notch of periodic notched plates along the thickness of the plate, model 4 is analyzed.

Figure 18 shows the variation of tensile stress for different notch opening angles($t = 1$ mm and $N = 3$).

As it can be seen from Fig. 18, increasing the notch opening angle leads to reduction of the tensile stress in the middle notch of a periodic notched plate. But this trend is more perceptible for notch angles exceding 90°, while the increase in the stress value is low for the notch opening angles of 60° and smaller.

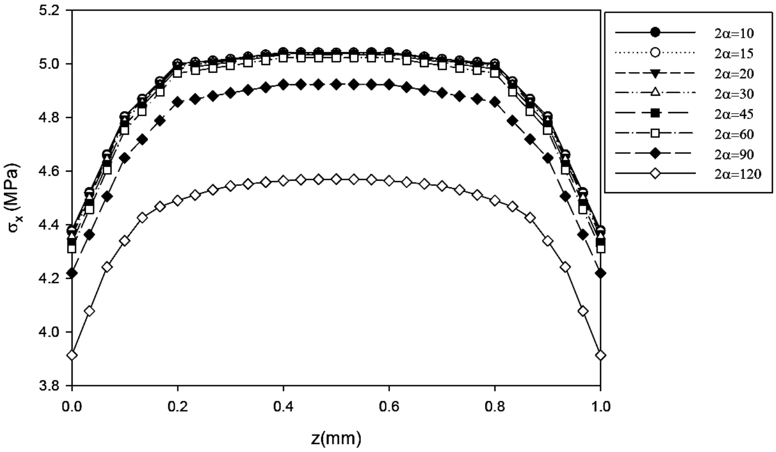



Fig. 18. Variation of tensile stress for different notch opening angles ($t = 1$ mm, $N = 3$, $a = 0.5$ mm, $\rho = 0.1$ mm, and $p = 2.5$ mm).

Figure 19 shows the results of the similar analysis for $N = 7$. According to Fig. 19, the same trend of Fig. 18 can be observed. In addition, increasing the number of notches leads to reduced stress values, which reduction in stress values is higher for notch angles of 90 and 120° and is insignificant for angles of 10–60°.

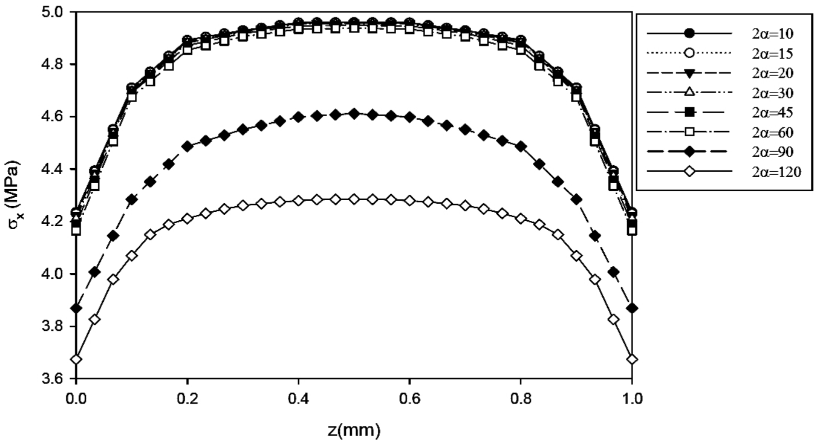



Fig. 19. Variation of tensile stress for different notch opening angles ($t = 1$ mm, $N = 7$, $a = 0.5$ mm, $\rho = 0.1$ mm, and $p = 2.5$ mm).

In addition, the effect of the notch opening angle variation on the stress values has been studied for $t = 5$ mm. Figure 20 shows the results of this analysis for $N = 3$. Again, the trend of stress reduction due to increase of the notch opening angle could be observed and, similar to the analysis for $t = 1$ mm, the increase in stress value is low for notch angles of 60° and smaller.

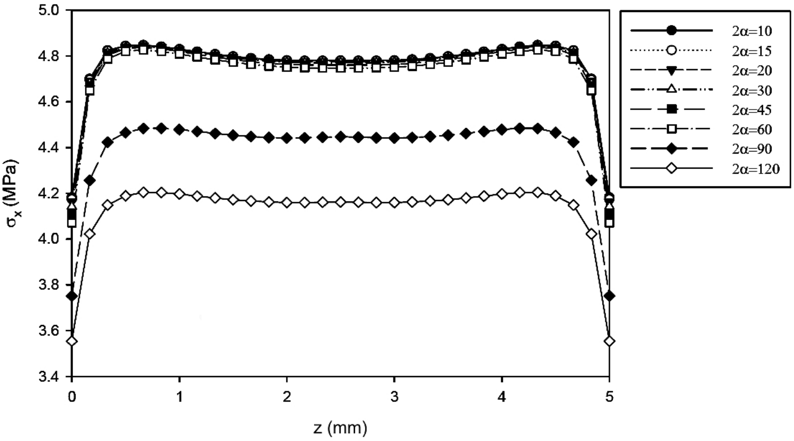



Fig. 20. Variation of tensile stress for different notch opening angles ($t = 5$ mm, $N = 3$, $a = 0.5$ mm, $\rho = 0.1$ mm, and $p = 2.5$ mm).

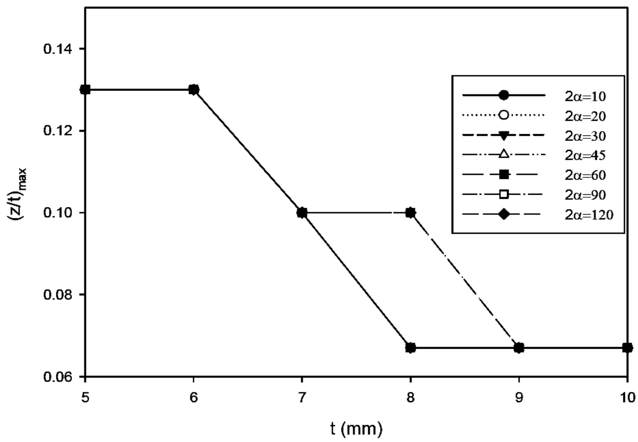



Fig. 21. Effect of the notch opening angle variation on the location of maximum tensile stress $(z/t)_{\max}$ ($a = 0.5$ mm, $N = 3$, $\rho = 0.1$ mm, and $p = 2.5$ mm).

Figure 21 shows the results of the analysis regarding the effect of notch opening angle variation on the location of the maximum tensile stress. As it can be seen from this figure, the notch opening angle variation has no strong effect on the maximum stress location within the thickness range of 5–10 mm. There are two exceptions, which are related to the case of $t = 8$ mm with the opening angles of 90 and 120°.

3.5. ***Effect of Notch Radius Variation***. Effect of notch radius variation on the through the thickness tensile stress in the middle notch of the plate is studied using model 5, in which four notch tip radii of 0.05, 0.1, 0.2, and 0.3 mm are considered. Figure 22 shows the results of the analysis for $N = 3$ and $t = 5$ mm.

According to Fig. 22, reducing the notch tip radius leads to a considerable increase in stress values. Figure 23 presents the results of the same analysis for $N = 7$, in which a similar trend of Fig. 22 can be observed.

The effect of the notch radius variation on the location of the maximum stress is considered as well. Figure 24 shows the $z/t$ values regarding the maximum tensile stress location for different notch radii and thicknesses. According to this figure, for a given plate thickness, the maximum stress location is not affected by the notch radius variation, except for two radii, in case of $t = 8$ mm. In addition, the same analysis is done for model 1 with

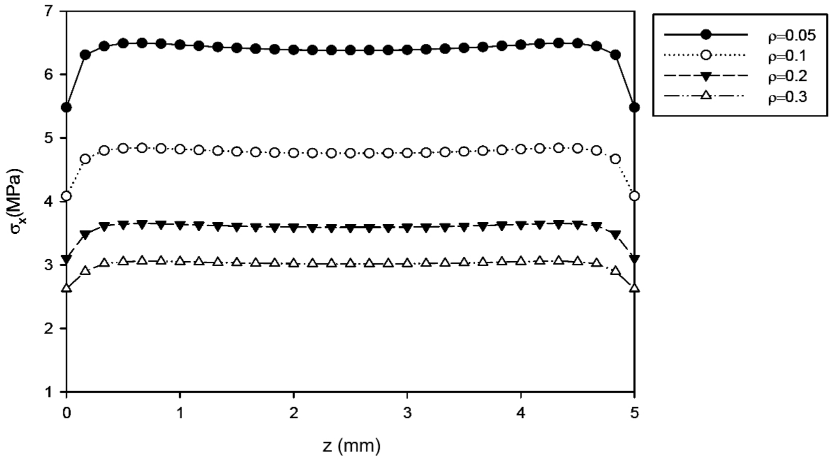



Fig. 22. Effect of notch radius variation on tensile stress in the plate middle notch for $N = 3$ ($a = 0.5$ mm, $p = 2.5$ mm, $t = 5$ mm, and $2\alpha = 60°$).

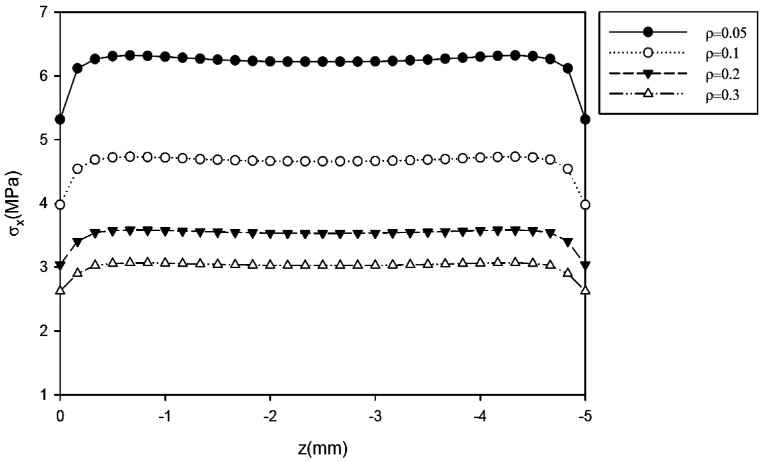



Fig. 23. Effect of notch radius variation on tensile stress in the plate middle notch for $N = 7$ ($a = 0.5$ mm, $p = 2.5$ mm, $t = 5$ mm, and $2\alpha = 60°$).

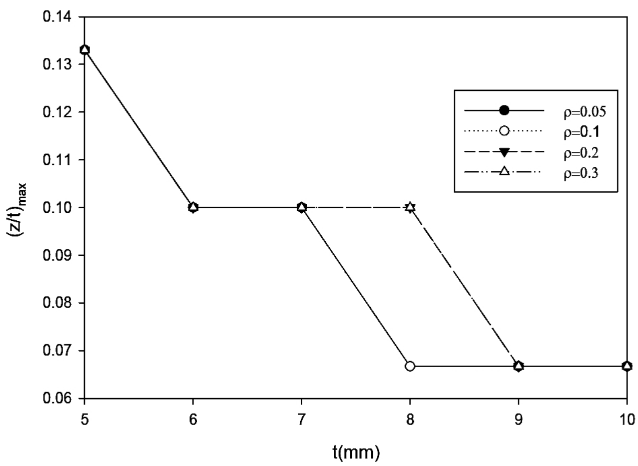



Fig. 24. Effect of notch radius variation on the location of the maximum tensile stress $(z/t)_{\max}$ for different thicknesses ($a = 0.5$ mm, $p = 2.5$ mm, $N = 3$, and $2\alpha = 60°$).

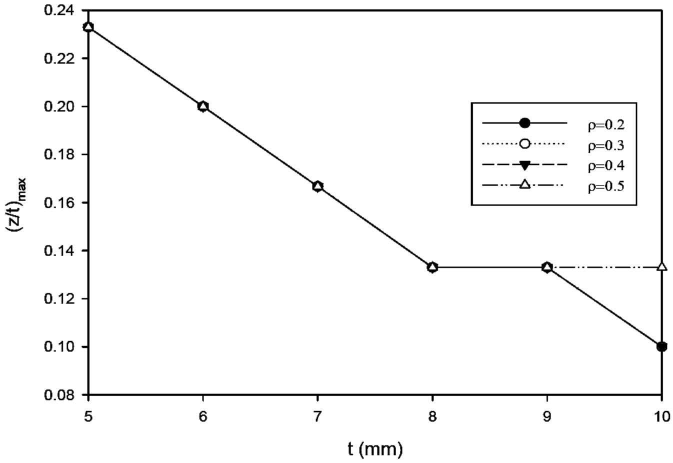



Fig. 25. Effect of notch radius variation on the location of maximum tensile stress $(z/t)_{\max}$ for different thicknesses ($a = 1.0$ mm, $p = 4.0$ mm, $N = 3$, and $2\alpha = 60°$).

$a = 1$ mm and $p = 4$ mm, which results are shown in Fig. 25. Again, it can be observed that for a given thickness the maximum stress location is not affected by variation of the notch radius. However, there is an exception, which is the notch radius of 0.5 mm for the case of $t = 10$ mm.

**Conclusions**. The main findings of this study are summarized as follows:

1. For relatively thin plates (thinner than 2 mm for this model), the maximum stress in the middle notch appears to be in the middle of the plate, which confirms the plane stress assumption of 2D FE models. However, increasing the plate thickness leads to a shift in the location of the maximum stress towards the free surface. In addition, increasing the number of notches has no influence on this pattern.

2. Reducing the plate width leads to an increment of the maximum stress in the middle notch.

3. Increasing number of notches with the purpose of decreasing stress values in the notch tip is possible only when the pitch of the notches is small.

4. Increasing the notch opening angle leads to reduction in the value of tensile stress in the middle notch of a periodic notched plate for the notch opening angles over 90°. Moreover, this pattern is invariant to the number of notches.

5.Reducing the notch tip radius leads to a considerable increase in stress values, and this trend can be observed for different number of notches.

6. Among the above-mentioned geometrical parameters, pitch variation has the strongest influence on the maximum tensile stress location in the middle notch of periodic notched plates. For a given thickness, the maximum stress location varies from the mid-plane for the small pitch values to the near surface for the case of large pitch values. In fact, for thinner plates the shift in the maximum stress location is more pronounced, in comparison with relatively thicker plates ($t = 10$ and 11 mm).

7. Variation of other geometrical parameters, such as notch depth-to-plat width ratio, notch opening angle, and notch tip radius, does not affect the maximum stress location, except for a few cases as shown in Figs. 21, 24, and 25.


**Р е з ю м е**

За допомогою методу скінченних елементів у тривимірній постановці досліджено напружений стан і визначено зону максимальних напружень у плоских пластинах із періодичними надрізами. Зокрема, вивчено ефект варіювання шістьма геометричними параметрами (товщина пластини, відношення глибини надрізу до ширини пластини, відстань між періодичними надрізами, кут і радіус V-подібного надрізу) на розподіл напружень по товщині пластини і локалізацію максимальних напружень. Проаналізовано більш як 500 геометричних конфігурацій пластин із надрізами. Згідно з результатами досліджень у відносно тонких пластинах максимальне значення напруги біля середнього надрізу має місце в серединній площині, що відповідає плоскому напруженому стану, постульованому в рамках двовимірних скінченно-елементних моделей. Однак зі збільшенням товщини пластини має місце зміщення області максимальних напружень із серединної площини до вільної поверхні пластини. Установлено, що збільшення кількості надрізів із метою зниження рівня напружень у їх вершині є ефективним тільки в тому випадку, якщо відстань між ними порівняно мала. Із вищевказаних геометричних параметрів найбільший вплив на локалізацію області максимальних напружень поблизу середнього надрізу має відстань між періодичними надрізами, особливо для відносно тонких пластин.

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