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On Designing Constant-Stress Partially Accelerated Life Tests under Time-Censoring

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О планировании частично ускоренных ресурсных испытаний при постоянных нагрузках и цензурировании по времени

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Получение данных по разрушению высококачественных деталей с большой долговечностью в нормальных условиях эксплуатации является весьма трудоемким процессом, поэтому возникает необходимость использования ускоренных испытаний. Рассматриваются частично ускоренные ресурсные испытания при постоянных нагрузках и цензурировании по времени типа I для распределения Вейбулла. Планирование таких испытаний проводится таким образом, чтобы минимизировать обобщенную асимптотическую дисперсию оценочных функций максимальной вероятности параметров модели. План эксперимента позволяет определить, какую именно часть ресурсных испытаний объектов следует проводить при нормальных эксплуатационных условиях. Эффективность данного подхода проиллюстрирована на некоторых примерах численных расчетов.

Ключевые слова: частично ускоренные ресурсные испытания, постоянное напряжение, распределение Вейбулла, обобщенная асимптотическая дисперсия, план эксперимента.

Notation

ALT	–	accelerated life test
PALT	–	partially accelerated life test
n	–	total number of test items in PALTs
T	–	time of censoring
X	–	lifetime of an item at normal use condition
Y	–	lifetime of an item at accelerated use condition
λ	–	acceleration factor $(\lambda > 1)$
$(\hat{\cdot})$	–	denotes maximum likelihood estimate
η	–	Weibull scale parameter
β	–	Weibull shape parameter
x_i	–	observed lifetime of item i tested at normal use condition
y_j	–	observed lifetime of item j tested at accelerated use condition
δ_{ui}, δ_{aj}	–	indicator functions: $\delta_{ui} \equiv I(X_i \le T)$, $\delta_{aj} \equiv I(Y_j \le T)$
π	–	proportion of sample units allocated to accelerated condition
π^*	–	optimum proportion of sample units allocated to accelerated condition

n_u, n_a – numbers of items failed at normal use and accelerated use conditions, respectively

$x_{(1)} \le \ldots \le x_{(n_u)} \le T$ – ordered failure times at normal use condition

$y_{(1)} \le \ldots \le y_{(n_a)} \le T$ – ordered failure times at accelerated use condition

Introduction. It is not easy to obtain more failure data from products with high quality and long life at use condition. Hence, in order to assure rapid failure and then to shorten the testing period, all or some of test units may be subjected to stress conditions more severe than normal ones. Such accelerated life tests (ALTs) or partially accelerated life tests (PALTs) result in shorter lives than would be observed under use condition. In ALTs test items are run only at accelerated conditions, while in PALTs they are run at both use and accelerated conditions.

As Nelson [1] indicates, the stress can be applied in various ways, commonly used methods are step-stress and constant-stress. Under step-stress PALTs, a test item is first run at use condition and, if it does not fail for a specified time, then it is run at accelerated condition until failure occurs or the observation is censored. But the constant-stress PALTs run each item at either use condition, or accelerated condition only, i.e., each unit is run at a constant-stress level until the test is terminated. Accelerated test stresses involve higher than usual temperature, voltage, pressure, load, humidity, ..., etc., or some combination of them. The objective of PALTs is to collect more failure data in a limited time without necessarily using high stresses for all test units.

In practice, PALTs are easier to implement and have many advantages, which include:

(1) Time saving: PALTs can substantially shorten the duration of the test without affecting the accuracy of lifetime distribution estimates.

(2) Economical: Testing units under PALTs can reduce the costs of experiments because not all test units are run at higher stresses.

(3) Adaptable: PALTs provide a flexible test strategy, especially for new products when one presumably has little information regarding appropriate test stresses. In such situations, it may not be easy for the experimenter to determine suitable test stress levels.

Moreover, the constant-stress PALTs approach is simple and has several advantages: firstly, it is easier to maintain a constant stress level in most tests. Secondly, accelerated test models are better developed. Thirdly, data analysis for reliability estimation is well-developed and computerized [1].

For an overview of constant-stress PALTs, there are few studies in the literature on designing constant-stress PALTs: Bai and Chung [2] used the maximum likelihood method to estimate the scale parameter and the acceleration factor for exponentially distributed lifetimes under type-I censoring. They also considered the problem of optimally designing constant-stress PALTs that terminates at a predetermined time. Ismail et al. [3] considered the constant-stress PALTs plans under Pareto distribution of the second kind with type-I censoring. Abdel-Ghani [4] considered only the estimation problem in constant-stress PALTs for the Weibull distribution parameters, the present investigation extends this work in which PALTs plans with two levels of stress are developed under type-I censoring.

The rest of this paper is organized as follows: In Section 1 the Weibull distribution is introduced as a failure time model and the test method is also described. Section 2 presents the maximum likelihood (ML) estimates of the model parameters. In Section 3 optimum test plans of simple constant-stress PALTs are developed. To illustrate the theoretical results, simulation studies are carried out in Section 4.

1. **The Model and Test Method**. This section introduces the assumed model for product life and also fully describes the test method.

1.1. ***The Weibull Distribution*: *a Failure Time Model***. The lifetimes of the test items are assumed to follow a two-parameter Weibull distribution. The Weibull distribution is one of the most commonly used distributions in reliability engineering because of the many shapes it attains for various values of β. It can therefore model a great variety of data and life characteristics; see Dimitri [5]. So, it is a statistical distribution frequently used in life data analysis.

The probability density function (pdf) of a two-parameter Weibull distribution is given by

$$f_T(t;\ \beta,\ \eta)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\exp\{-(t/\eta)^{\beta}\},\qquad t>0,\qquad \beta>0,\qquad \eta>0. \tag{1}$$

The Weibull reliability function takes the form:

$$R(t)=\exp\{-(t/\eta)^{\beta}\},$$

and the corresponding failure rate function is given by

$$h(t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}.$$

1.2. ***Constant-Stress PALTs***. The test procedure of the constant-stress PALTs and its assumptions are described as follows:

Test Procedure. In a constant-stress PALTs, the total sample size n of test units is subdivided into two parts such that:

1. $n\pi$ items randomly chosen among n test items sampled are allocated to accelerated condition and the remaining ones are allocated to use condition.

2. Each test item is run until the censoring time is reached or the item fails and the test condition is not changed.

Assumptions.

1. The lifetimes X_i, $i=1, \ldots, n(1-\pi)$ of items allocated to use condition, are i.i.d. r.v.'s.

2. The lifetimes Y_j, $j=1, \ldots, n\pi$ of items allocated to accelerated condition, are i.i.d. r.v.'s.

3. The lifetimes X_i and Y_j are mutually statistically-independent.

2. **Maximum Likelihood Parameter Estimation**. In a simple constant-stress PALTs, the test item is run either at use condition, or at accelerated condition only. Since the lifetimes of the test items follow the Weibull distribution, the probability density function of an item tested at use condition is given as in (1). For an item tested at accelerated condition, the pdf is given by

$$f_Y(y;\ \lambda,\ \beta,\ \eta)=\frac{\lambda\beta}{\eta}\left(\frac{\lambda y}{\eta}\right)^{\beta-1}\exp\{-(\lambda y/\eta)\},\qquad y>0,\qquad \lambda>1,\qquad \beta>0,\qquad \eta>0,$$

where $Y=\lambda^{-1}X$.

Since the test in type-I censoring terminates when a predetermined time is reached, the observed lifetimes $x_{(1)}\le\ldots\le x_{(n_u)}\le T$ and $y_{(1)}\le\ldots\le y_{(n_a)}\le T$ are ordered failure times at normal and accelerated conditions, respectively, where T is the time at which the experiment is terminated, n_u and n_a are the numbers of items failed at use and accelerated conditions, respectively.

Let us define the indicator functions: $\delta_{ui} \equiv I(X_i \le T)$ and $\delta_{aj} \equiv I(Y_j \le T)$. Then the total likelihood for $(x_1;\,\delta_{u1},\,\ldots,\,x_{n(1-\pi)};\,\delta_{un(1-\pi)},\,y_1;\,\delta_{a1},\,\ldots,\,y_{n\pi};\,\delta_{an\pi})$ is given by

$$L(\underline{x},\,\underline{y}|\lambda,\,\beta,\,\eta)=\prod_{i=1}^{n(1-\pi)} L_{ui}(x_i,\,\delta_{ui}|\beta,\,\eta)\prod_{j=1}^{n\pi} L_{aj}(y_j,\,\delta_{aj}|\lambda,\,\beta,\,\eta)=$$

$$=\prod_{i=1}^{n(1-\pi)}\left[\frac{\beta}{\eta}\left(\frac{x_i}{\eta}\right)^{\beta-1}\exp\{-(x_i/\eta)^{\beta}\}\right]^{\delta_{ui}}[\exp\{-(T/\eta)^{\beta}\}]^{\bar{\delta}_{ui}}\times$$

$$\times\prod_{i=1}^{n\pi}\left[\frac{\lambda\beta}{\eta}\left(\frac{\lambda y_j}{\eta}\right)^{\beta-1}\exp\{-(\lambda y_j/\eta)^{\beta}\}\right]^{\delta_{aj}}[\exp\{-(\lambda T/\eta)^{\beta}\}]^{\bar{\delta}_{aj}},$$

where L_{ui} and L_{aj} denote the contributions of the items i and j to the total likelihood function under use and accelerated conditions, respectively, and $\bar{\delta}_{ui}=1-\delta_{ui}$ and $\bar{\delta}_{aj}=$ $=1-\delta_{aj}$.

The ML estimate of η can be obtained by

$$\hat{\eta}=\left\{\frac{\psi}{n_u+n_a}\right\}^{1/\hat{\beta}}, \tag{2}$$

where $\psi=\sum_{i=1}^{n\bar{\pi}}\delta_{ui}x_i^{\beta}+\lambda^{\beta}+\sum_{j=1}^{n\pi}\delta_{aj}\,y_j^{\beta}+T^{\beta}(n\bar{\pi}-n_u)+(\lambda T)^{\beta}(n\pi-n_a)$ and $\bar{\pi}=1-\pi$.

Therefore, two ML non-linear equations can be expressed as follows:

$$\frac{n_a\hat{\beta}}{\hat{\lambda}}-\left[\frac{(n_u+n_a)\hat{\beta}\hat{\lambda}^{\hat{\beta}-1}}{\psi}\right]\left[\sum_{j=1}^{n\pi}\delta_{aj}\,y_j^{\hat{\beta}}+T^{\hat{\beta}}(n\pi-n_a)\right]=0, \tag{3}$$

$$\frac{n_u+n_a}{\hat{\beta}}+\sum_{i=1}^{n\bar{\pi}}\delta_{ui}\ln x_i+\sum_{j=1}^{n\pi}\delta_{aj}\ln y_j-(n_u+n_a)\ln\left(\frac{\psi}{n_u+n_a}\right)^{1/\hat{\beta}}+$$

$$+n_a\ln\hat{\lambda}+\hat{\beta}\left(\frac{n_u+n_a}{\psi}\right)^{1/\hat{\beta}}=0. \tag{4}$$

From Eq. (3), the ML estimate of λ can be easily derived from

$$\hat{\lambda}=\left\{\frac{n_a\left[\sum_{i=1}^{n\bar{\pi}}\delta_{ui}x_i^{\beta}+T^{\hat{\beta}}(n\bar{\pi}-n_u)\right]}{n_u\left[\sum_{j=1}^{n\pi}\delta_{aj}\,y_j^{\hat{\beta}}+T^{\hat{\beta}}(n\pi-n_a)\right]}\right\}^{1/\hat{\beta}}. \tag{5}$$

Then Eq. (4) after substitution of λ can be rewritten as follows:

$$\frac{n_u + n_a}{\hat{\beta}} + \sum_{i=1}^{n\bar{\pi}} \delta_{ui} \ln x_i + \sum_{j=1}^{n\pi} \delta_{aj} \ln y_j - n_u \frac{\sum_{i=1}^{n\bar{\pi}} \delta_{ui} x_i^{\hat{\beta}} \ln x_i + T^{\hat{\beta}} (n\bar{\pi} - n_u) \ln T}{\sum_{i=1}^{n\bar{\pi}} \delta_{ui} x_i^{\hat{\beta}} + T^{\hat{\beta}} (n\bar{\pi} - n_u)} -$$

$$- n_a \frac{\sum_{j=1}^{n\pi} \delta_{aj} y_j^{\hat{\beta}} \ln y_j + T^{\hat{\beta}} (n\pi - n_a) \ln T}{\sum_{j=1}^{n\pi} \delta_{aj} y_j^{\hat{\beta}} + T^{\hat{\beta}} (n\pi - n_a)} = 0. \qquad (6)$$

Obviously, it is very difficult to obtain a closed-form solution for the non-linear equation (6). Thus, an iterative procedure must be used to solve this equation numerically. The Newton–Raphson method is used to obtain the ML estimate of β. Thus, once the value of $\hat{\beta}$ is determined, the ML estimates of η and λ are easily obtained from Eqs. (2) and (5), respectively.

Concerning the asymptotic variance-covariance matrix of the maximum likelihood estimators (MLE) of the parameters, it can be obtained by numerically inverting the asymptotic Fisher information matrix F. The asymptotic (large sample) matrix F is composed of the negative second derivatives of the natural logarithm of the likelihood function evaluated at the ML estimates. Unfortunately, the exact expressions of the mathematical expectations for the second derivatives given in (7) are very difficult to obtain. Therefore, the observed Fisher information matrix can be written asymptotically by dropping the expectations as follows (see Cohen [6]):

$$F = \begin{bmatrix} -\frac{\partial^2 \ln L}{\partial \lambda^2} & -\frac{\partial^2 \ln L}{\partial \lambda \partial \eta} & -\frac{\partial^2 \ln L}{\partial \lambda \partial \beta} \\ -\frac{\partial^2 \ln L}{\partial \eta \partial \lambda} & -\frac{\partial^2 \ln L}{\partial \eta^2} & -\frac{\partial^2 \ln L}{\partial \eta \partial \beta} \\ -\frac{\partial^2 \ln L}{\partial \beta \partial \lambda} & -\frac{\partial^2 \ln L}{\partial \beta \partial \eta} & -\frac{\partial^2 \ln L}{\partial \beta^2} \end{bmatrix} \downarrow (\hat{\lambda}, \hat{\eta}, \hat{\beta}). \qquad (7)$$

Consequently, the MLEs of λ, η, and β have an asymptotic variance-covariance matrix defined by inverting the Fisher information matrix F indicated above.

3. **Optimum Test Plans**. Now, for the optimal design stage of the test, a new experiment – with test units different from those tested in the stage of parameter estimation – is conducted. The current aim is to obtain the optimal proportion of sample units π^* allocated to accelerated condition based on the outputs of the stage of parameter estimation that are at the same time considered inputs to the optimal design stage of the test. It is worth noting that the proportion of sample units π allocated to accelerated condition is pre-specified for the stage of parameter estimation. But for the optimal design stage of the test π is considered a division parameter that has to be optimally determined according to a certain optimality criterion.

This section considers the problem of designing simple constant-stress PALTs, which are terminated at a pre-specified time. The optimum test plan for products having a two-parameter Weibull lifetime distribution is developed. The optimality criterion is to find the optimal proportion of sample units π^* allocated to accelerated condition such that the generalized asymptotic variance (GAV) of the MLE of the model parameters under use condition is minimized.

Most of the test plans are equally-spaced test stresses, i.e., the same number of test units are allocated to each stress. Such test plans are usually inefficient for estimating the mean life at design stress [7]. In this section, statistically optimum test plans are developed to determine the optimal sample proportion allocated for each stress. Therefore, to determine the optimal sample proportion π^* allocated for accelerated condition, π is chosen such that the GAV of the MLEs of the model parameters is minimized. The GAV of the MLEs of the model parameters as an optimality criterion is commonly used and defined below as the reciprocal of the determinant of the Fisher information matrix F (see Bai et al. [8].) That is

$$GAV(\hat{\lambda}, \hat{\eta}, \hat{\beta}) = \frac{1}{|F|}.$$

The minimization of the GAV over π solves the following equation:

$$\frac{\partial GAV}{\partial \pi} = 0. \tag{8}$$

In general, the solution to (8) is not in a closed form and therefore it requires usage of a numerical method, such as the Newton–Raphson method, which is applied to obtain π^* which minimizes the GAV. Accordingly, the corresponding expected optimal numbers of items failed at use and accelerated conditions can be obtained numerically, as it will be shown in the next section.

4. **Simulation Studies**. In this section, several data sets generated from Weibull distribution under type-I censored data are considered with sample sizes 25, 30, 50, 75, and 100 using 1000 replications for each sample size. The true parameter values of λ, η, and β used in this simulation study are (3, 5, 1.5) and (2, 4, 0.7). The Newton–Raphson method and programs written in the Pascal language are used for obtaining the MLEs of λ, η, and β. Tables 1 and 2 summarize the results of the ML estimates of the parameters and the estimated variances of the MLEs. Results of simulation studies provide insight into the sampling behavior of the estimators. The numerical results indicate that the ML estimates approximate the true values of the parameters as the sample size n increases. Also, as shown from the numerical results, the asymptotic variances of the estimators are decreasing as the sample size n is attaining large values.

Tables 3 and 4 depict the results of the test design. That is, the optimal sample-proportion π^* allocated to accelerated condition, the expected fraction failing at each stress, represented by n_u^* and n_a^*, and the optimal GAV of the MLEs of the model parameters are obtained numerically for each sample size. The test plans developed here are statistically optimum plans because they are more efficient than standard plans for estimating the life distribution at design stress. Standard plans usually involve equally-spaced test stresses, each with the same number of test units, and they are not the optimum test plans.

It can be observed from the numerical results, via π^*, presented in Tables 3 and 4, that the optimum test plans do not allocate the same number of test units to each stress. In practice, the optimum test plans are important for improving precision in parameter estimation and thus improving the quality of the inference. Also, these tables present the

T a b l e 1

ML Estimates and Estimated Asymptotic Variances of MLEs for the Set of Parameters (λ, η, β) at (3, 5, 1.5), Respectively, Given $\pi = 0.50$ and $T = 10$ for Different-Sized Samples under Type-I Censoring in Constant-Stress PALTs

n	Parameter	Estimate	Variance
25	λ	3.4721	2.3527
	η	5.6325	2.7104
	β	1.8362	1.2473
30	λ	3.4192	1.5521
	η	5.4571	2.2205
	β	1.6920	1.0978
50	λ	3.3255	1.0358
	η	5.2689	1.6571
	β	1.5523	1.0266
75	λ	3.2030	0.8659
	η	5.1276	0.7945
	β	1.5244	0.8571
100	λ	3.0324	0.4655
	η	5.0764	0.5106
	β	1.4923	0.3749

T a b l e 2

ML Estimates and Estimated Asymptotic Variances of MLEs for the Set of Parameters (λ, η, β) at (2, 4, 0.7), Respectively, Given $\pi = 0.50$ and $T = 10$ for Different-Sized Samples under Type-I Censoring in Constant-Stress PALTs

n	Parameter	Estimate	Variance
25	λ	2.9425	0.2711
	η	5.1162	0.2265
	β	1.0284	0.1025
30	λ	2.6782	0.2341
	η	4.6471	0.1843
	β	0.9652	0.0617
50	λ	2.2783	0.1456
	η	4.4458	0.1355
	β	0.8361	0.0268
75	λ	2.1845	0.0519
	η	4.1239	0.0813
	β	0.7632	0.0112
100	λ	2.0322	0.0343
	η	3.9852	0.0212
	β	0.7103	0.0041

optimal GAV of the MLEs of the model parameters which is obtained numerically with π^* in place of π for different sized samples. As anticipated, the optimal GAV decreases as the sample size n increases.

T a b l e 3

Results of Optimal Design of the Life Test for Different-Sized Samples under Type-I Censoring in Constant-Stress PALTs, Based on the Parameters (λ, η, β) Set at (3, 5, 1.5), Respectively

n	π^*	n_u^*	n_a^*	Optimal GAV
25	0.5738	22	36	0.2237
30	0.6177	41	84	0.0833
50	0.7182	48	173	0.0261
75	0.7617	54	262	0.0072
100	0.8215	63	312	0.0024

T a b l e 4

Results of Optimal Design of the Life Test for Different-Sized Samples under Type-I Censoring in Constant-Stress PALTs, Based on the Parameters (λ, η, β) Set at (2, 4, 0.7), Respectively

n	π^*	n_u^*	n_a^*	Optimal GAV
25	0.4273	31	19	0.3681
30	0.4972	74	37	0.1924
50	0.5481	105	46	0.1137
75	0.6152	122	62	0.0491
100	0.6507	138	71	0.0134

Conclusions. This paper discusses the problem of designing simple constant-stress PALTs plans for the Weibull distribution under type-I censoring, in which the dependence between allocations of observations and the actual values of the probability model parameters were investigated numerically. In a constant-stress test, a unit is subjected to a constant level of stress until failure occurs or the observation is censored. The ML estimates of the model parameters were numerically obtained. Optimum constant-stress PALTs plans were also developed. The minimization of the GAV of the MLEs of the model parameters was used as an optimality criterion.

In the constant-stress PALTs, the common test plans are usually standard plans which use equally-spaced test stresses, each with the same number of test units. The design problem is to determine the numbers of test units that should be allocated to each stress level to estimate the life distribution accurately based on the values of the model parameters. The test plan provides the most accurate estimate of the model parameters for a given test time and number of test units. Thus, the optimal design of the life tests can be considered as a technique to improve the quality of the inference. As shown from the results, via π^*, the optimum test plans are not standard plans because the value of π^* is different from 50% for lower or higher than unity values of shape parameter. These results coincide with finding of Yang [7] concerning the standard plans. Since the value of π was obtained to minimize the GAV of the MLEs of the model parameters, the developed test plans are statistically optimum.

It is noteworthy that when $\beta > 1$, $h(t)$ becomes suitable for representing the failure rate of units exhibiting wear-out type failures. Consequently, the majority of these units tend to fail under the accelerated conditions, which is also confirmed by the results obtained. On the other hand, when $\beta < 1$, $h(t)$ becomes suitable for representing the failure rate of units exhibiting early-type failures. Therefore, in this case, the majority of test units tend to fail under use condition, which is also corroborated by the numerical results.

Therefore, the optimal design results are quite lucrative for using the new optimal proportions, in order to run futher experiments. In fact, the test is called an optimally designed test because the test is based on the optimal values of the allocating or dividing proportions. Finally, since the developed test plans are statistically optimum plans, the usefulness of the optimal design lies in the fact that it can serve as a benchmark for comparison with other designs. As a future work, the problem of designing PALTs under the context of Bayesian approach will be considered for the same probability distribution.

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Резюме

Отримання даних щодо руйнування високоякісних деталей з великою довговічністю в нормальних умовах експлуатації є досить трудомістким процесом, тому виникає необхідність використання прискорених випробувань. Розглядаються частково прискорені ресурсні випробування за постійних навантажень і цензурування в часі типу I для розподілу Вейбулла. Планування таких випробувань проводиться таким чином, щоб мінімізувати загальну асимптотичну дисперсію оціночних функцій максимальної ймовірності параметрів моделі. План експерименту дозволяє визначити, яку частину ресурсних випробувань об’єктів необхідно проводити за нормальних експлуатаційних умов. Ефективність даного підходу проілюстровано на деяких прикладах числових розрахунків.

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