TRANSFORMATION RATIO AT WAKEFIELD EXCITATION BY LINEARLY SHAPED SEQUENCE OF SHORT RELATIVISTIC ELECTRON BUNCHES IN PLASMA

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Wakefield excitation by long sequence of short Gaussian bunches of relativistic electrons and electron bunch acceleration in excited wakefield is numerically simulated for the parameters of the experiments. It is shown that at change of the system parameters and shaping laws of sequence of bunches of relativistic electrons in the intervals of finite width the transformation ratio remains large.

PACS: 29.17.+w; 41.75.Lx;

INTRODUCTION

Transformation ratio is the important value in the wakefield method of charged particle acceleration (see, for example, [1 - 5]). It determines to what energy the electrons can be accelerated by sequence of electron bunches with fixed energy. The transformation ratio, determined as the ratio $T_E=E_2/E_1$ of wakefield E_2, which is excited in plasma by sequence of electron bunches to the field E_1, in which the electron bunch is decelerated, is considered. The excitation of wakefield by long sequence of short Gaussian bunches of relativistic electrons is investigated by numerical simulation, using the code lcode [6], for the parameters of the experiments [7, 8]. The distance between the bunches, equal to wavelength plus the bunch width at half-maximum, and the distance between bunches, equal to one and a half of the wavelength, are selected. It is shown that not only in these cases, but also at varying of the parameters within a certain range the transformation ratio increases with the number of bunches.

1. ANALYTICAL INVESTIGATION OF WAKEFIELD EXCITATION IN PLASMA WITH LARGE TRANSFORMATION RATIO BY SEQUENCE OF BUNCHES WITH LINEAR GROWTH OF CHARGE

First we analytically compare the wakefield, excited by resonance ($\omega_m=\omega_{pe}$, ω_m is the repetition frequency of bunches, ω_{pe} is the electron plasma frequency) sequence of rectangular (uniform) bunches with the ratio of the successive charges, equal to 1:3:5: … with $1.5\omega_m=\omega_{pe}$.

At using of shaped bunches through 1.5λ the decelerating field is equal to $E_{sl}=E_1/2$, and the accelerating field is equal to $E_{ac}=NE_1$. N is the number of bunches. I.e. after 2nd bunch $E_{ac}=2E_1$, and after 3rd bunch $E_{ac}=3E_1$.

In the case of resonant shaped bunches through λ the decelerating wakefield is approximately equal to the accelerating wakefield and accelerating wakefield after 2nd bunch is equal to $E_{ac}=4E_1$, after 3rd bunch it is equal to $E_{ac}=9E_1$, i.e. it should be $E_{ac}=N^2E_1$. Indeed

$$W_N-W_{N-1} = \eta\varepsilon_N,$$

$$\varepsilon_N = 2\pi e n_b c E_{Nc}/\omega_p,\ \ E_{Nc} = E_{N-1}+\beta\delta E_N\ \beta\approx 1/2.$$

Then after 1st bunch one can derive

$$\varepsilon_1 = \pi e n_{b1} c\delta E_1/\omega_p,\ \ W_1 = \eta\varepsilon_1,\ \ \delta E_1 = E_1,\ W_1 = E_1^2/4\pi$$

$$E_1^2/4\pi = \eta\pi e n_{b1} c E_1/\omega_p,\ \ E_1 = (2\pi)^2\eta e n_{b1} c/\omega_p.$$

After 2nd bunch one can derive

$$W_2-W_1 = \eta\varepsilon_2,\ \varepsilon_2 = 2\pi e 3 n_{b1} c E_{Nc}/\omega_p,\ E_{Nc} = E_1+\delta E_2/2.$$

$$(E_1+\delta E_2)^2/4\pi-E_1^2/4\pi =$$

$$= \delta E_2(E_1+\delta E_2/2)/2\pi = \eta 6\pi e n_{b1} c(E_1+\delta E_2/2)/\omega_p$$

$$\delta E_2 = 3\eta(2\pi)^2 e n_{b1} c/\omega_p = 3E_1.$$

Hence after 2nd bunch

$$E_2 = E_1+\delta E_2 = 4E_1.$$

After N-th bunch

$$W_N-W_{N-1} = \eta\varepsilon_N,\ \varepsilon_N = 2\pi e n_b c E_{Nc}/\omega_p,\ \ E_{Nc} = E_{N-1}+\delta E_N/2.$$

$$E_N^2/4\pi-E_{N-1}^2/4\pi = (E_{N-1}+\delta E_N)^2/4\pi-E_{N-1}^2/4\pi =$$

$$= \delta E_N(E_1+\delta E_N/2)/2\pi = \eta 2\pi e(2N-1) n_{b1} c(E_{N-1}+\delta E_N/2)/\omega_p,$$

$$\delta E_N = \eta(2\pi)^2 e(2N-1) n_{b1} c/\omega_p = (2N-1)E_1.$$

Then

$$E_3 = E_2+\delta E_3 = 4E_1+5E_1 = 9E_1.$$

$$E_N = E_{N-1}+\delta E_N \sim 1+3+\ldots+(2N-1)$$

$$1+3+...+(2N-1) = 1+2+...+2N-2(1+2+...+N).$$

Because

$$1+2+...+N = N(N+1)/2,$$

we have

$$1+3+\ldots+(2N-1) = 2N(2N+1)/2-2N(N+1)/2 = N^2.$$

$$E_N = N^2E_1.$$

Then after 300 bunches the accelerating fields in resonant shaped case through λ $E_N^{(res)}$ and in shaped case through 1.5λ $E_N^{(sh)}$ are different in

$$E_N^{(res)}/E_N^{(sh)}=N^2E_1/NE_1=N=300$$

times.

In the case of shaped bunches through 1.5λ the decelerating wakefield is in 300 times smaller than the accelerating wakefield and in $(300)^2\approx 10^5$ times smaller than the wakefield in resonant case.

2. NUMERICAL SIMULATION OF WAKEFIELD EXCITATION IN PLASMA WITH LARGE TRANSFORMATION RATIO BY SEQUENCE OF BUNCHES WITH LINEAR GROWTH OF CHARGE

Numerical simulation has been performed using 2d3v-code lcode [6]. Parameters: $n_{res}=10^{11}cm^{-3}$ is the resonant plasma density which corresponds to ratio $\omega_{pe}=\omega_m=2\pi\cdot 2.8\cdot 10^9$, relativistic factor of bunches equals $\gamma_b=5$, have been selected for numerical simulation. ω_m is the repetition frequency of bunches, $\omega_{pe}= (4\pi n_{res}e^2/m_e)^{1/2}$ is the electron plasma frequency. The density of bunches $n_b = 6\times 10^8\,\text{ñm}^{-3}$ is distributed in the transverse direction approximately according to Gaussian distribution,

$\sigma_r = 0.5\,\text{cm}$, $\lambda = 10.55\,\text{cm}$ is the wavelength, $\xi=V_b t-z$, V_b is the velocity of bunches. Time is normalized on ω_{pe}^{-1}, distance – on c/ω_{pe}, density – on n_{res}, current I_b – on $I_{cr}=\pi mc^3/4e$, fields – on $(4\pi n_{res} c^2 m_e)^{1/2}$.

We show that in the case of often used shaped bunches-uniform-cylinders with the ratio of charges, equal to 1:3:5:... and with the distance between bunches, equal to one and half of the wavelength $\delta\xi=1.5\lambda$, with a width of bunches, equal to $\xi_b=\lambda/2$, at of plasma wakefield excitation the problem of the formation of the accelerated bunch is solved easily. In this case, the last bunch, if its charge is small in comparison with the charge of previous bunch, becomes accelerated bunch (Figs. 1-3).

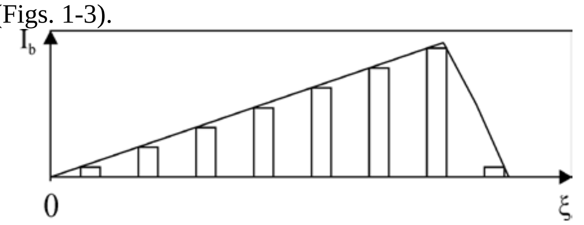

Fig. 1. Current of bunches with $\xi_b=\lambda/2$, $\delta\xi=1.5\lambda$, q_1:q_2:q_3: … =1:3:5: … at linear growth and sharp decrease at back front

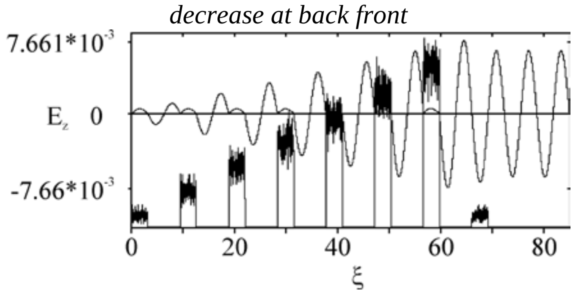

Fig. 2. Longitudinal distribution of density n_b of sequence of bunches and of longitudinal wakefield E_z in the case of sequence of bunches with the ratio of the successive charges, equal to 1:3:5:…, at $\xi_b=\lambda/2$, $I_b=0.2\cdot10^{-3}$

The wakefield excitation by sequence of N bunches with the ratio of the charges of the successive bunches, equal to 1:3:5: ..., the distance between bunches, equal to the sum of the excited wavelength λ and bunch width at half maximum ξ_b is considered. One can see (Fig. 4), that bunches on part of the sequence, on which the charges of bunches increases, get into small decelerating wakefield. Thus the large transformation ratio achieves, which can be determined as the ratio of the maximum accelerating wakefield E_{ma} to the maximum decelerating wakefield E_{md} in the area of bunch $TR=E_{ma}/E_{md}$. Also one can see that on the back front of the sequence, where the charges of bunches decrease, the bunches automatically get into large accelerating wakefield.

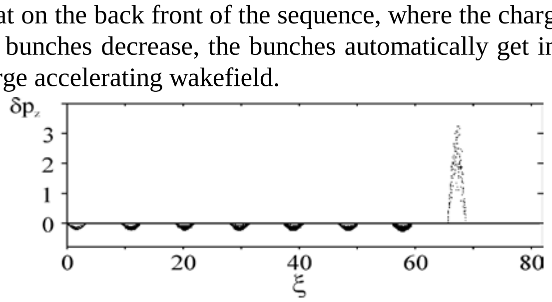

Fig. 3. Perturbation of longitudinal momenta δp_z of bunches

When bunches are formed due to trapping of electrons with linearly increasing charge density by electric field on each successive identical time interval and due to their bunching and when on half periods of time electrons are lost, then the ratio of charges of consecutive bunches is equal to 1:5:9: ... If the distance between the bunches is equal to the sum of the excited wavelength and of the width of the bunch at half-maximum, then (Fig. 5) the amplitude of the beatings of decelerating wakefield, in which bunches get, is more. However, the transformation ratio remains large. Again at the back front of the sequence, in which the charges of the bunches decrease, bunches get into large accelerating field.

In the case of a sequence of 45 bunches with the ratio of charges of the successive bunches, equal 1:5:9: ..., excited wakefield has the shape, shown in Fig. 6. In the case of a sequence of 100 bunches with the ratio of charges of the successive bunches, equal 1:3:5: ..., excited wakefield has the shape, shown in Fig. 7.

If the ratio of the charges of successive bunches is equal to 1:2:3: ... [2] and the distance between bunches is equal to the sum of excited wavelength and of width of the bunch at half-maximum, and the charge of bunches distributed in the longitudinal direction according to the Gaussian distribution, the excited wakefield has the shape, shown in Fig. 8. One can see that at the back front of the sequence, in which the charges of the bunches decrease, bunches get automatically into large accelerating wakefield.

If the distance between the bunches, the charge of which is distributed in the longitudinal direction according to Gauss distribution, is equal to one and half of the wavelength, the length of the bunch at the base equals to the wavelength and the charge ratio of successive bunches is equal to 1:3:5: ... the excited wakefield has the shape, shown in Fig. 9. One can see that decelerating wakefield is small and approximately the same for the majority of the bunch electrons. The bunches are in fairly homogeneous close focusing wakefields (see Fig. 9) due to the fact that the bunches get into the dips of the plasma electron density (Fig. 10).

If the distance between the bunches, the charge of which is distributed in the longitudinal direction according to Gauss distribution, is equal to two and half of the wavelength, the length of the bunch at the base equals to the wavelength and the charge ratio of successive bunches is equal to 1:3:5: ... the excited wakefield has the shape, shown in Fig. 11. One can see that decelerating wakefield is also small and approximately the same for the majority of the bunch electrons.

In the case of linear shaping of charges of bunch sequence along the sequence as well as along each bunch with use of bunch-precursor the large transformation ratio is achieved (Fig. 12)

$$TR\approx2\pi N\xi_b/\lambda.$$

One can see that the wakefield amplitude and the transformation ratio increase with increasing of number of bunches, exciting wakefield.

Now we consider the range of change parameters in which the transformation ratio remains large. Numerical simulation shows that TR and the accelerating wakefield are large in the case of parameter changes $2\pi V_0/\omega_m=\lambda+\xi_b$ in range $2\pi V_0/\omega_m=\lambda+\xi_b\pm\xi_b/2$. However, in these cases the decelerating wakefield and their spatial distributions are different (see Figs. 4, 13, 14).

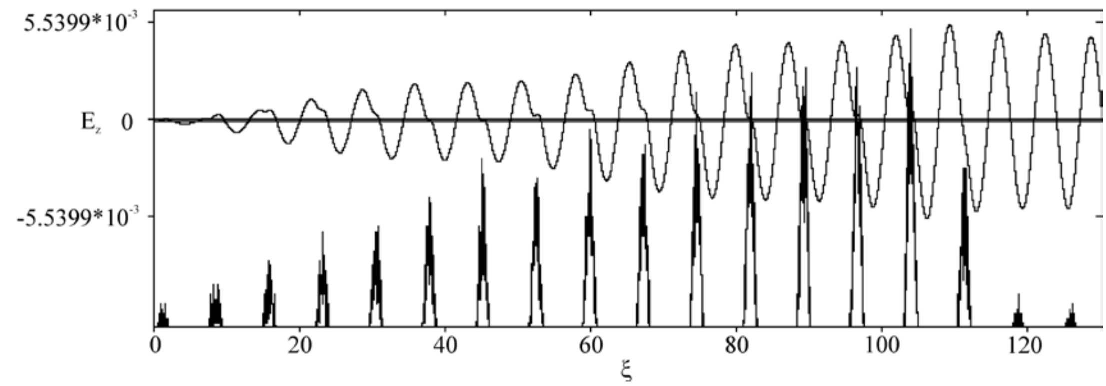

Fig. 4. Longitudinal distribution of density n_b of sequence of bunches and of longitudinal wakefield E_z in the case of sequence of bunches with the ratio of charges of the successive bunches, equal to 1:3:5: … with the distance between the Gaussian bunches, equal to the sum of excited wavelength and of width of the bunch at half-maximum at $\xi_b=\lambda/6$, $I_b=0.7\cdot10^{-3}$

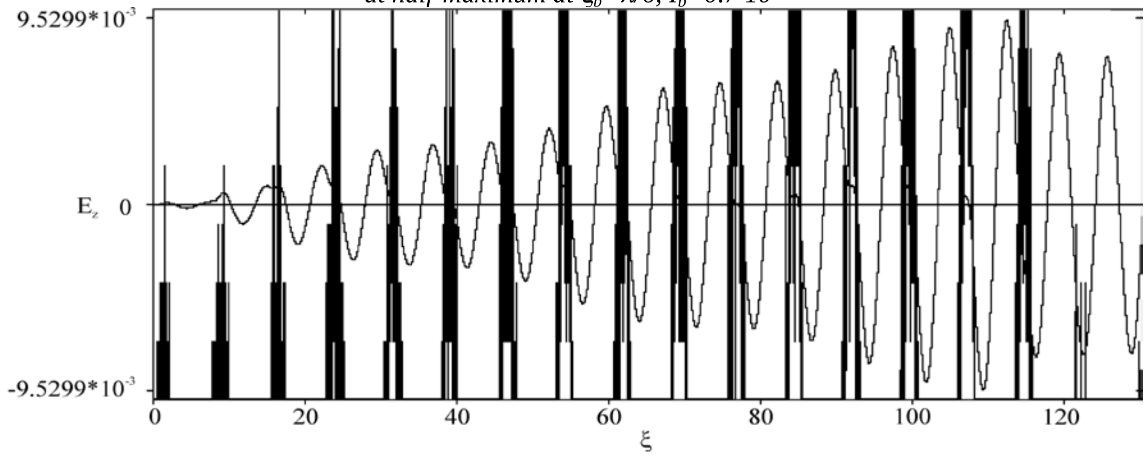

Fig. 5. Longitudinal distribution of density n_b of sequence of bunches and of longitudinal wakefield E_z in the case of sequence of 17 bunches with the ratio of charges of the successive bunches, equal to 1:5:9: … with the distance between the the Gaussian bunches, equal to the sum of excited wavelength and of width of the bunch at half-maximum at $\xi_b=\lambda/6$, $I_b=0.7\cdot10^{-3}$

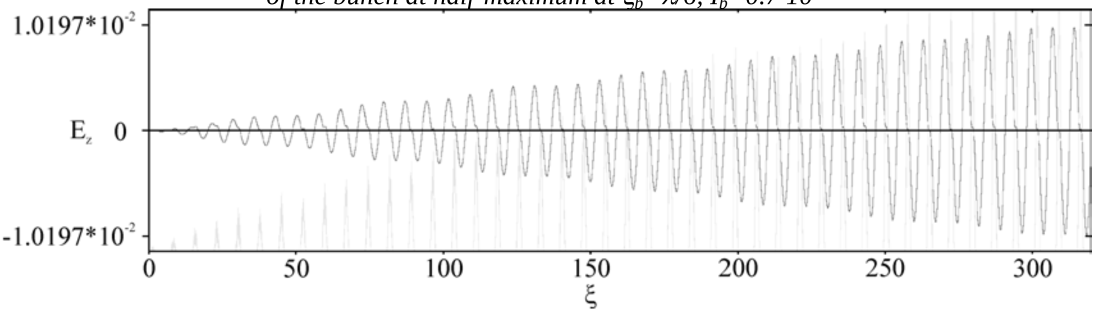

Fig. 6. Longitudinal distribution of density n_b of sequence of bunches and of longitudinal wakefield E_z in the case of sequence of 45 bunches with the ratio of charges of the successive bunches, equal to 1:5:9: … with the distance between the Gaussian bunches, equal to the sum of excited wavelength and of width of the bunch at half-maximum at $\xi_b=\lambda/6$, $I_b=0.23\cdot10^{-3}$

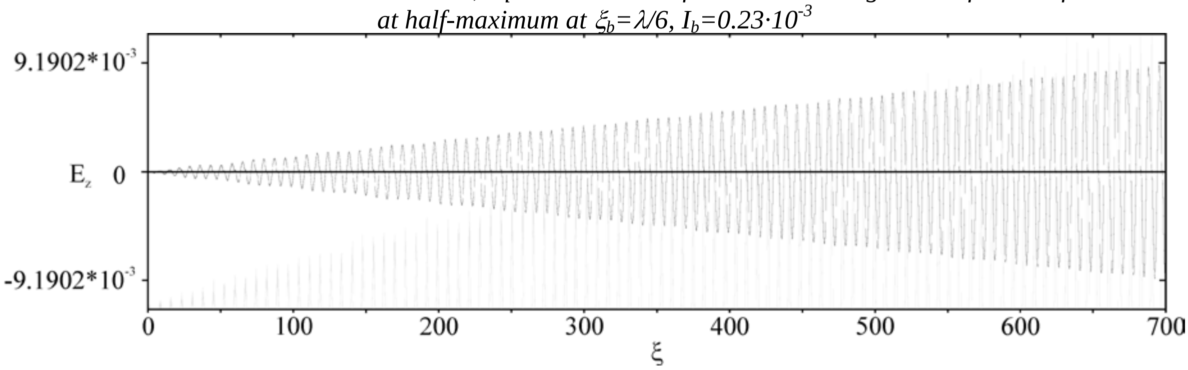

Fig. 7. Longitudinal distribution of density n_b of sequence of the Gaussian bunches and of longitudinal wakefield E_z in the case of sequence of 100 bunches with the ratio of charges of the successive bunches, equal to 1:3:5: … at $\xi_b=\lambda/6$, $I_b=0.18\cdot10^{-3}$

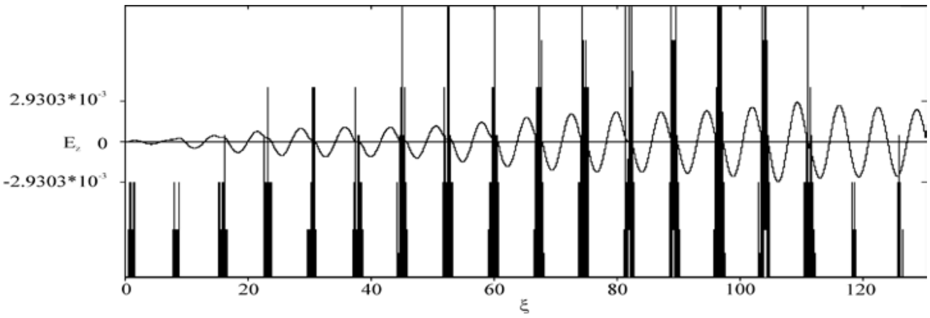

Fig. 8. Longitudinal distribution of density n_b of sequence of bunches and of longitudinal wakefield E_z in the case of sequence of bunches with the ratio of charges of the successive bunches, equal to 1:2:3: … with the distance between the bunches, equal to the sum of excited wavelength and of width of the bunch at half-maximum at $\xi_b=\lambda/6$, $I_b=0.7\cdot10^{-3}$

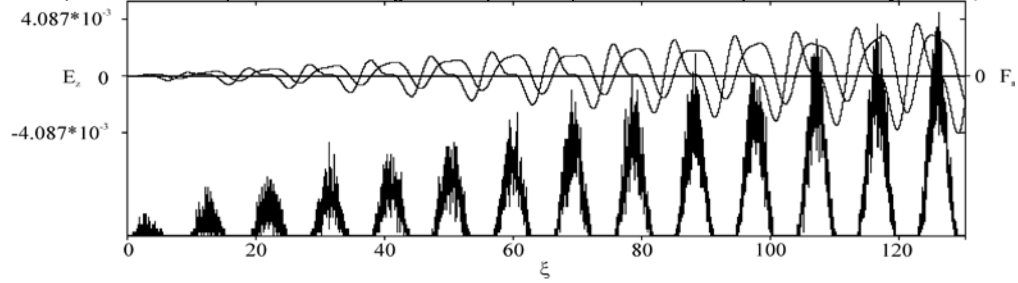

Fig. 9. Longitudinal distribution of density n_b of sequence of bunches, of longitudinal wakefield E_z and of radial wake force F_r in the case of sequence of the Gaussian bunches with the ratio of charges of the successive bunches, equal to 1:3:5: … with the distance between the bunches, equal to one and half of the wavelength, the length of the bunch at the base equals to the wavelength at $\xi_b=\lambda/2$, $I_b=0.7\cdot10^{-3}$

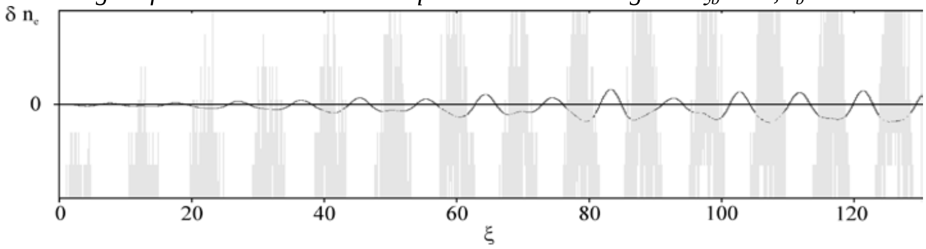

Fig. 10. Longitudinal distribution of density n_b of sequence of bunches and of density of plasma electrons n_e in the case of sequence of bunches with the ratio of charges of the successive bunches, equal to 1:3:5: … with the distance between the bunches, equal to one and half of the wavelength, the length of the bunch at the base equals to the wavelength

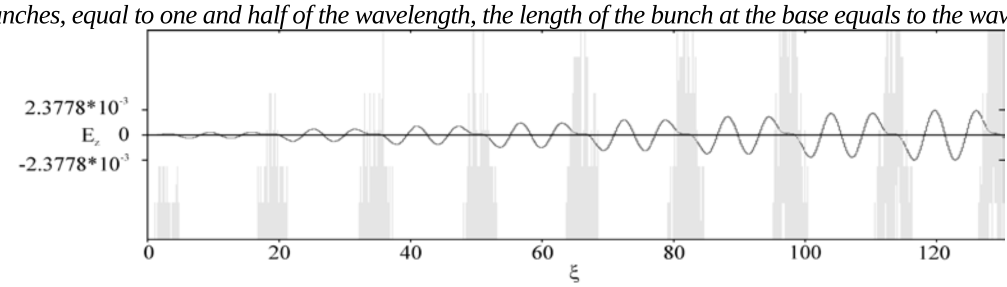

Fig. 11. Longitudinal distribution of density n_b of sequence of the Gaussian bunches and of longitudinal wakefield E_z in the case of sequence of bunches with the ratio of charges of the successive bunches, equal to 1:3:5: … with the distance between the bunches, equal to two and half of the wavelength, the length of the bunch at the base equals to the wavelength at $\xi_b=\lambda/2$, $I_b=0.7\cdot10^{-3}$

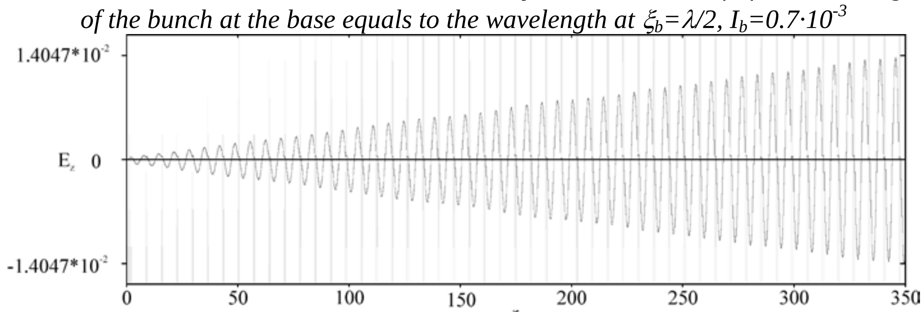

Fig. 12. Longitudinal distribution of density n_b of sequence of short bunches and of longitudinal wakefield E_z in the case of linear shaping of charges of bunch sequence along the sequence as well as along each bunch with use of bunch-precursor at $\xi_b=\lambda/20$, $I_b=1.0\cdot10^{-3}$

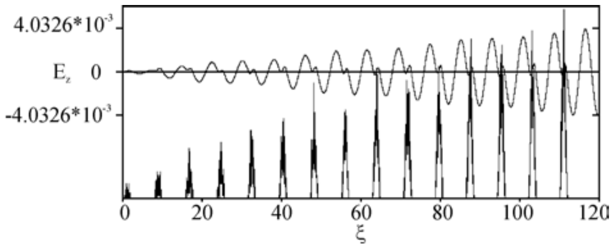

Fig. 13. Longitudinal distribution of density n_b of sequence of the Gaussian bunches and of longitudinal wakefield E_z in the case of sequence of bunches with the ratio of charges of the successive bunches, equal to 1:3:5: … with the distance between the bunches, equal $\lambda+\xi_b+\xi_b/2$ at $\xi_b=\lambda/6$, $I_b=0.7\cdot10^{-3}$

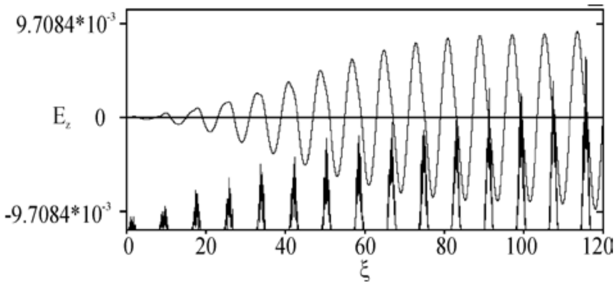

Fig. 14. Longitudinal distribution of density n_b of sequence of the Gaussian bunches and of longitudinal wakefield E_z in the case of sequence of bunches with the ratio of charges of the successive bunches, equal to 1:3:5: … with the distance between the bunches, equal $\lambda+\xi_b-\xi_b/2$ at $\xi_b=\lambda/6$, $I_b=0.7\cdot10^{-3}$

CONCLUSIONS

The transformation ratio has been investigated by the numerical simulation at wakefield excitation in plasma by sequence of Gaussian short relativistic electron bunches with linearly increasing charges. It has been shown that the transformation ratio increases with the increasing of number of bunches when the distance between bunches is equal to the sum of excited wavelength and of width of the bunch at half-maximum and when the distance between the bunches is equal to the one and half of the wavelength.

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Article received 20.10.2015

КОЭФФИЦИЕНТ ТРАНСФОРМАЦИИ ПРИ ВОЗБУЖДЕНИИ КИЛЬВАТЕРНОГО ПОЛЯ ЛИНЕЙНО ПРОФИЛИРОВАННОЙ ПОСЛЕДОВАТЕЛЬНОСТЬЮ КОРОТКИХ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОННЫХ СГУСТКОВ В ПЛАЗМЕ

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Численным моделированием исследуется для параметров экспериментов возбуждение кильватерного поля длинной последовательностью коротких гауссовских сгустков релятивистских электронов и ускорение электронных сгустков с малым зарядом в возбужденном поле. Показано, что при изменении параметров системы и законов профилирования последовательности сгустков релятивистских электронов в интервалах конечной ширины коэффициент трансформации остается большим.

КОЕФІЦІЄНТ ТРАНСФОРМАЦІЇ ПРИ ЗБУДЖЕННІ КІЛЬВАТЕРНОГО ПОЛЯ ЛІНІЙНО ПРОФІЛЬОВАНОЮ ПОСЛІДОВНІСТЮ КОРОТКИХ РЕЛЯТИВІСТСЬКИХ ЕЛЕКТРОННИХ ЗГУСТКІВ У ПЛАЗМІ

І.П. Левчук, В.І. Маслов, І.М. Оніщенко

Числовим моделюванням досліджується для параметрів експериментів збудження кільватерного поля довгою послідовністю коротких гауссівських згустків релятивістських електронів і прискорення електронних згустків з малим зарядом у збудженому полі. Показано, що при зміні параметрів системи і законів профілювання послідовності згустків релятивістських електронів в інтервалах кінцевої ширини коефіцієнт трансформації залишається значним.