ION KINETICS AND ION SOUND GENERATION UNDER THE DEVELOPMENT OF MODULATION INSTABILITY OF AN INTENSE LANGMUIR WAVE IN A PLASMA

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The development of one-dimensional parametric instabilities of intense long-wave plasma waves is considered in terms of the so-called hybrid models, when electrons are treated as a fluid and ions are regarded as particles. The analysis is performed for both cases when the average plasma field energy is lower (Zakharov's hybrid model – ZHM) or greater (Silin's hybrid model – SHM) than the plasma thermal energy. Reduced absorption of the high-frequency (HF) field leads to the retardation of the HF field burnout within plasma density cavities and to the broadening of the HF spectrum. At the same time, the ion velocity distribution tends to the normal distribution in both ZHM and SHM.

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INTRODUCTION

Considerable experimental and computational effort has been directed toward parametrically excited Langmuir turbulence over recent decades in several different arenas, including ionospheric modification, space physics, and inertial confinement fusion. The interest to the parametric instability of intense Langmuir waves, which can be easily excited in the plasma by various sources [1 - 9] was stipulated, in particular, by new possibilities in heating electrons and ions in a plasma.

The theoretical concepts proposed by V.P. Silin [10] were confirmed by the early numerical experiments on the one-dimensional simulation of the parametric decay of plasma oscillations[11]. However, the greatest experimenters' interest was provoked by the mechanism of wave-energy dissipation discovered by V.E. Zakharov. The modulation instability of intense Langmuir waves in non-isothermal plasmas also leads to collective ion perturbations, in particular, to the generation of ion-sound waves [12 - 16].

In Zakharov's model [13] that describes the instability of intense long-wave Langmuir waves in a non-isothermal plasma, just the modulation instability results in the excitation of a range of short-wave oscillations. In Silin's model, a strong Langmuir wave in a cold plasma leads to intense oscillations of the electron velocity with the amplitudes comparable to the wavelengths of the excited modes.

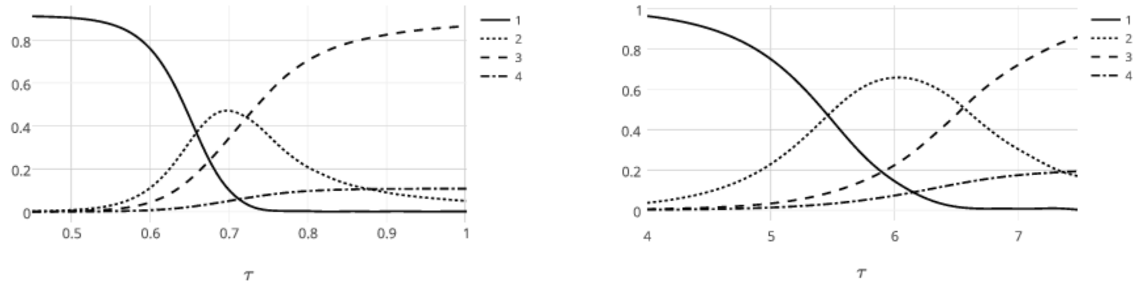
In the works [17, 18] an attempt was made to compare these models, which have similar physical nature, by the example of one-dimensional description. Of particular interest is the process of ion heating, so in this paper we use the particle description for ions because the account of inertial effects can be significant just at the nonlinear stage of the process [19]. It was observed in [19, 20] that simulation in terms of the so-called hybrid model (incorporating one of Zakharov's equations for the HF waves and using particle simulation for ions) demonstrates that fluctuations of ion density are rather significant and favor the development of parametric instability. The non-resonant interaction between ions and HF plasma oscillations, along with ion trapping by the potential wells produced by these oscillations, lead to an instability of density cavities resulting from the modulation instability and produce fast particle groups.

In papers [17, 18] the simulation of one-dimensional ion dynamics was performed in terms of the particle method [23, 24]. The number of particles used in numerical calculations was $2\cdot10^4$, which is equivalent to the number of ions, about $(2\cdot10^4)^3\sim10^{13}$, in the three-dimensional case, in agreement with the conditions of most experiments. Thus, the interaction between modeling particles and plasma oscillations in this simulation is in rather good accordance with the interaction between real particles and plasma waves, naturally with regard to the inherent limitations of the one-dimensional description. Nevertheless, there is reason to believe that the description of field energy transfer to ions within the framework of the hybrid model represents the real conditions of ion heating by intense Langmuir oscillations in plasmas. Moreover, the one-dimensional description makes it possible to select arbitrary electron-to-ion mass ratios.

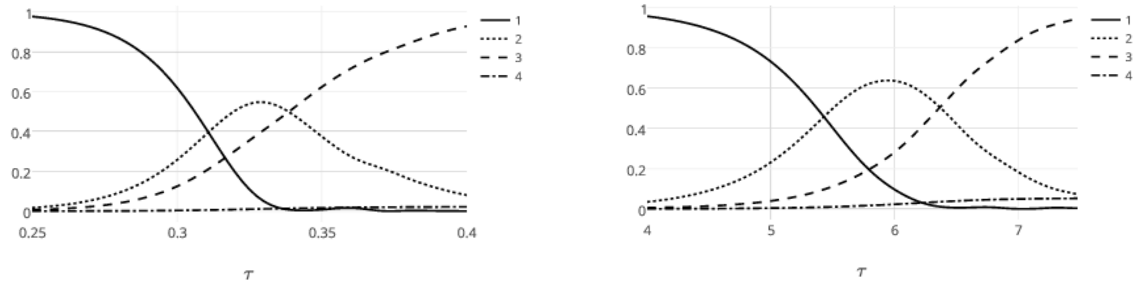
Below, we discuss the efficiency of energy transfer from Langmuir oscillations to ions and ion density perturbations under the development of the modulation instability in both cases of non-isothermic hot and cold one-dimensional plasmas within the framework of hybrid models and for different values of the electron-to-ion mass ratio. The attention is mainly concentrated on the effect of HF field burnout within density cavities accompanied by energy transfer to the ion component of the plasma.

1. STATEMENT OF THE PROBLEM AND THE INITIAL CONDITIONS

The purpose of this paper is to clarify the efficiency of the energy transfer to ions and ion density perturbations in the course of development of modulation instability for the cases of both non-isothermal hot and cold plasmas in terms of the hybrid models. The equations, governing the nonlinear dynamics of the parametric instability of an intense plasma wave, were derived in the work [21]. Both SHM and ZHM were considered for two cases of light and heavy ions. The parameters of the simulation are presented in Table 1. It is also interesting to elucidate the effect of HF spectrum damping and subsequent burnout of the Langmuir field within density cavities on the energy transfer to plasma ions.



ZHM, light ions *SHM, light ions*



ZHM, heavy ions *SHM, heavy ions*

Fig. 1. Time evolution of relative values of: energy of the main Langmuir wave (1), energy of low-scale plasma wave spectrum (2), energy transferred to electrons (3) and ions (4)

Below we employ, unless otherwise specified in the text, the following initial conditions and parameters. The number of particles simulating the dynamics of ions is $0 < s \le S = 20000$. The particles are distributed uniformly over the interval $-1/2 < \xi < 1/2$, $\xi = k_0 x / 2\pi$, initial ion velocities are set as $d\xi_s / d\tau |_{\tau=0} = v_s |_{\tau=0} = 0$, the number of spectrum modes is $-N < n < N$, $N = S/100$, M / m_e is the ion-to-electron mass ratio, $\Theta = \theta / \delta$ is the damping rate θ normalized to the linear increment of the parametric instability δ, ω_{pe} is the plasma frequency.

The development of the instability was considered in terms of hybrid models in our previous papers [17, 18]. Here we give some results. The rate of damping of HF modes governs the rate of the field energy burnout in density caverns, from where the HF field has forced out charged particles The main part of the instability energy is initially concentrated in the HF Langmuir oscillations in parallel with the formation of the LF spectrum of density perturbations. Then the energy of the HF spectrum is transferred mainly to electrons. Thus, the shaped density cavities collapse, the trajectories of ions intercross, ion density perturbations become smoother and their characteristic scale growths with time. The relationship between ionic perturbations and the HF field is weakened and the instability is saturated. The amplitude of the main wave stabilizes after several oscillations at rather low level. The bulk energy is now contained in the perturbations of the electron component of the plasma. Some small portion of the initial energy transforms into the kinetic energy of ions. The estimate of the energy density transmitted to ions E_{kin} can be obtained from the expression

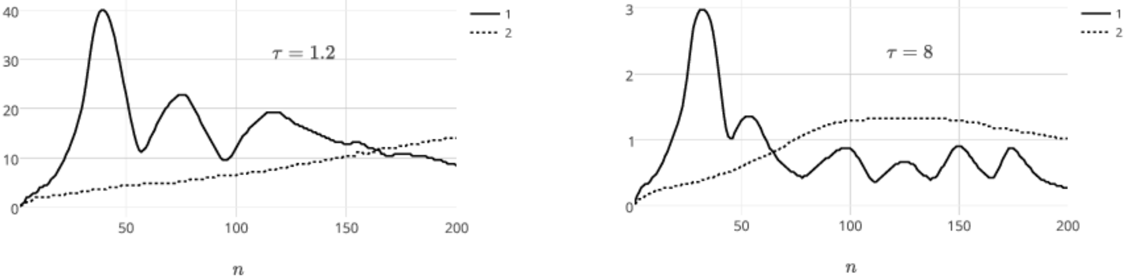
$$\frac{E_{kin}}{W_0} \approx 0.27 \cdot I \cdot \frac{M}{m} \cdot \frac{\delta^2}{\omega_{pe}^2}, \tag{1}$$

where W_0 is the initial energy density of the intense Langmuir wave, $I = \sum_s (d\xi_s / d\tau)^2$ is normalized ion kinetic energy and δ is the rate of the linear instability. The portion of energy transferred from the intense Langmuir wave to ions is determined by the ratio $W_0 / n_0 T_e$ for the case of non-isothermic plasma (ZHM) and by the ratio $(m/M)^{1/3}$ for the case of cold plasma (SHM).

Table 1

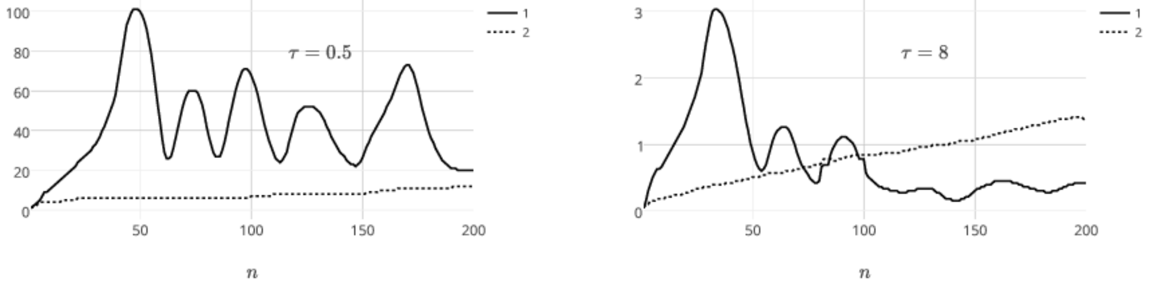
Simulation parameters for the hybrid models

The model	Light ions $M / m_e = 2 \cdot 10^3$	Heavy ions $m_e / M = 8 \cdot 10^{-6}$
SHM	$(m_e / M)(\omega_p^2 / \delta^2) = 0.43$	$(m_e / M)(\omega_p^2 / \delta^2) = 0.1$
	$\delta / \omega_0 = 0.44 \cdot (m_e / M)^{1/3} = 0.034$	$\delta / \omega_0 = 0.44 \cdot (m_e / M)^{1/3} = 0.0088$
	$\omega_0 / \delta \approx \omega_{pe} / \delta = 29.4$	$\omega_0 / \delta \approx \omega_{pe} / \delta = 113.6$
ZHM	$(m_e / M)(\omega_p^2 / \delta^2) = 2 n_0 T_e / W = 20$	$(m_e / M)(\omega_p^2 / \delta^2) = 2 n_0 T_e / W = 20$
	$\omega_0 / \delta = 2 \left(n_0 T_e / W\right)^{1/2} (M / m_e)^{1/2} = 282.6$	$\omega_0 / \delta = 2 (n_0 T_e / W)^{1/2} (M / m_e)^{1/2} = 2234.4$
	$\delta / \omega_0 = \delta / \omega_{pe} = 3.5 \cdot 10^{-3}$	$\delta / \omega_0 = \delta / \omega_{pe} = 4.5 \cdot 10^{-4}$



ZHM, light ions

SHM, light ions



ZHM, heavy ions

SHM, heavy ions

Fig. 2. Dependence of the amplitude of the LF modes M_n *(1) and the frequency* $d\Phi_n / d\tau$ *(2) on the wave-number at the stage of developed instability*

Below we consider more closely the nature of the energy redistribution with time and especially the process of energy transfer to the LF perturbations. We also discuss the specific features of the excitation of LF ion-sound waves in both non-isothermic and cold plasmas. More attention will be focused on the role of absorption of HF spectrum that is responsible for the burnout of the HF field in the density cavities. We investigate the effect of this process on the excitation of the LF spectrum and most importantly on the kind of ion velocity distribution function and on the proportion of the total energy transferred to ions.

2. THE RESULTS OF NUMERICAL SIMULATION

Fig. 1 shows the energy redistribution between the main Langmuir wave, the small-scale plasma wave spectrum and plasma electrons and ions.

The analysis of the numerical simulation results shows that the energy of intense long-wave Langmuir waves is first transferred to short-wave Langmuir oscillations. Just at this stage the cavities of plasma density, filled with HF plasma oscillations, are formed. After that, the HF field burns out due to the damping on electrons that is included in the hybrid models phenomenologically. The energy of the HF field therewith converts into the energy of plasma electrons. Under these conditions, the cavities collapse and thus excite LF waves, the ion trajectories intercross, and the energy of both collapsed caverns and LF spectrum is transferred to ions.

The root-mean-square velocity of ions, $\sigma(v) = \sqrt{\sum_s v_s^2 / S}$, at the final stage of the numerical simulation is equal to $\sigma(v) = 0.015$ for the case of light ions and $\sigma(v) = 0.006$ for heavy ions in ZHM and, respectively, to $\sigma(v) = 0.002$ for light ions and $\sigma(v) = 0.0005$ for heavy ions in SHM. The total kinetic energy of ions in assumed units $I = \sum_s (d\xi_s / d\tau)^2$ is equal to 4.689 for the case of light ions and 0.808 for heavy ions in ZHM and $\sigma(v) = 0.086$ for light ions and 0.005 for heavy ions in SHM. The variations in the values of the total energy are caused by different linear growth rates in the two models under consideration, and by different ion masses in the simulation of light and heavy ions. The final ion velocity distribution can be fitted by the normal curve with the use of the values of *rms* velocity. The particles outside the normal distribution (mainly in the so-called "tails") possess 13.8% of the total energy for light ions and 9.2% for heavy ions in ZHM model and much more in SHM: 25.6% for heavy ions and 13% for light ions, respectively. It means that in the case of instability of the intense wave in a cold plasma, a significantly greater proportion of fast particles should be expected.

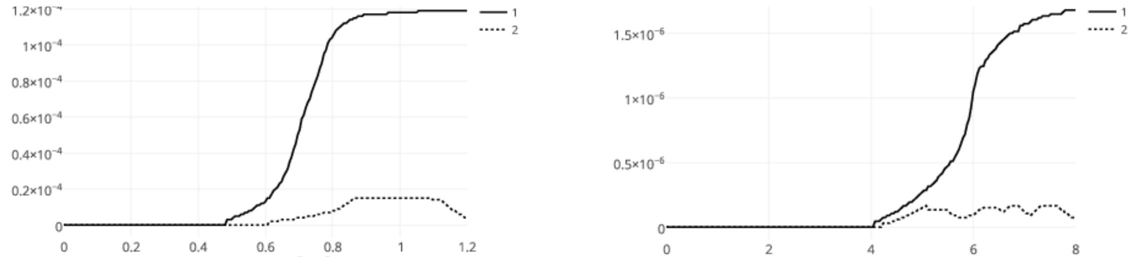
We are interested not only in the ion kinetic energy distribution, but also in the collective excitation of ion oscillation (Fig. 2), hence we define the frequency of the mode with the wave vector nk_0 associated with these oscillations, i.e.

$$\frac{d\Phi_n}{d\tau} = -\frac{d}{d\tau}\left(\frac{M_{nr}}{\sqrt{M_{nr}^2 + M_{ni}^2}}\right) \Bigg/ \left(\frac{M_{ni}}{\sqrt{M_{nr}^2 + M_{ni}^2}}\right), \qquad (2)$$

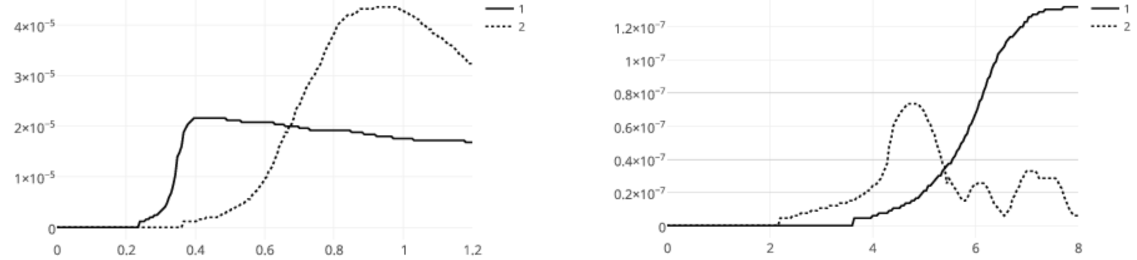
where the phases of LF modes can be found from the expression

$$M_n = M_{nr} + iM_{ni} = \sqrt{M_{nr}^2 + M_{ni}^2} \cdot \exp(i\Phi_n).$$

It should be noticed that the intensity of the LF spectrum in the case of a non-isothermal plasma (ZHM) is quite high in a wide range of wave numbers, that corresponds to the spectrum of ion sound after the destruction of density cavities detected in numerical experiments [15]. In a cold plasma (SHM), in contrast, the long-wave oscillations dominate in the spectrum.



ZHM, light ions *SHM, light ions*



ZHM, heavy ions *SHM, heavy ions*

Fig. 3. Evolution of 1) the ion kinetic energy and 2) LF field energy, multiplied by factor 70, with time

For both models, the ion kinetic energy in the assumed units can be written as

$$\frac{1}{2}\int_{-1/2}^{1/2} d\xi_{s0}\left(\frac{d\xi_s}{d\tau}\right)^2 , \qquad (3)$$

and the energies of collective excitations for ZHM and SHM, respectively, reduce to

$$\frac{1}{8\pi^2}\frac{m}{M}\frac{1}{n_M^2}\frac{\delta}{\omega_{pe}}\sum_n |M_n|^2,$$
$$\frac{1}{8\pi^2}\frac{m}{M}\sum_n\frac{1}{n^2}\left[1-J_0^2(a_n)+\frac{2}{3}J_2{}^2(a_n)\right]|M_n|^2 . \qquad (4)$$

Note that in Zakharov's model these oscillations are referred to as ion-sound waves.

Fig. 3 demonstrates the time evolution of the ion kinetic energy and LF field energy. It should be noted that the energy of the LF field is far smaller than ion energies in all cases. Reducing the field energy with time is caused by the energy transfer to ions as well as by the destruction of plasma density cavities [15].

The rate of the HF field burnout within density cavities is determined by the value $\Theta = \theta/\delta$. It is of interest how the simulation results depend on this parameter. Obviously, the decrease of this parameter not only inhibits the burnout of the HF field in the cavities, but also broadens the spectrum of HF modes, i.e. it increases the contribution of small-scale components that leads to the deepening of plasma density cavities and to the growth of the kinetic energy of ions ejected from the cavities.

Note that for both models the ion velocity distribution function approaches the Maxwellian curve with decreasing damping rate of HF modes, as may be seen from Fig. 4.

Table 2 demonstrates the extent of deviation of the ion velocity distribution function, obtained by numerical modeling, from the fitted Maxwellian curve for the cases shown in Fig. 2.

Fig. 5 shows that in the case of a non-isothermal plasma the maximum energy of ion-sound oscillations remains practically unchanged as the damping rate of HF field decreases, whereas the formation of the LF spectrum occurs with higher rate. In a cold plasma, on the contrary, the intensity of LF oscillations grows with the decrease of the damping rate of the HF field. After that, the LF spectrum is suppressed and its energy is transferred to ions. As might be expected, the energy, transmitted to ions, increases with the decrease of the damping rate of HF oscillations almost in the same proportion in both non-isothermal and cold plasmas (Fig. 6).

Table 2

Deviation of the ion velocity distribution function, obtained by numerical simulation, from the fitted Maxwellian curve

Damping rate	ZHM	SHM
Θ = 0.05	19.9%	13%
Θ = 0.015	9.9%	13.4%
Θ = 0.001	6.9%	8.8%

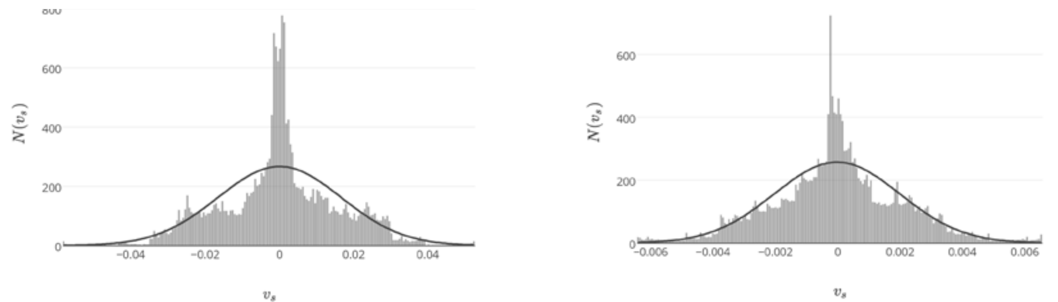
It should be noted in conclusion that the ion-density perturbations with spatial scale smaller than the Debye radius $r_{Di} = v_{Ti}/\omega_{pi}$ do not contribute to the formation of LF electric fields by virtue of the screening effect. The Debye radius can be estimated from the expression [25]

$$r_{Di}k_0/2\pi = R_{Di} \propto \left\langle \frac{v_i k_0}{2\pi\gamma_L}\right\rangle \left(\frac{\delta}{\omega_{pe}}\right)\left(\frac{M}{m_e}\right)^{1/2} = $$
$$= < v_s > \left(\frac{\delta}{\omega_{pe}}\right)\left(\frac{M}{m_e}\right)^{1/2} . \qquad (5)$$

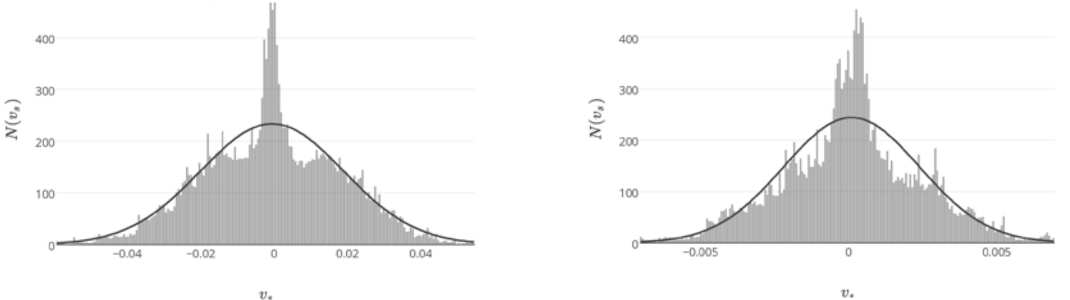
At the stage of developed instability this value is of the order of $R_{Di} \le 10^{-3}$ and the number of spatial spectral modes of ion density does not exceed $1/R_{Di}$ that is in agreement with the previous analysis.

CONCLUSIONS

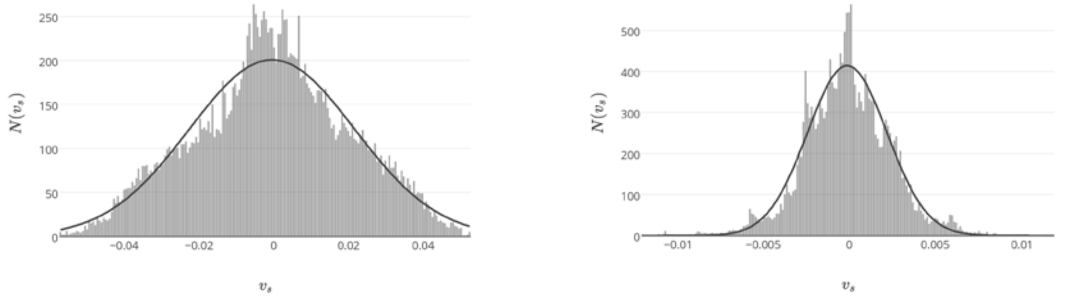
In the case of a non-isothermic plasma (ZHM), the amplitudes of modes of the LF spectrum (ion-sound waves) are of the same order in a wide range of wave numbers. In a cold plasma (SHM), the long-wave oscillations dominate in the LF spectrum. The energy of the LF field is found to be much lower than the total kinetic energy of ions for all the cases discussed above. Reducing the energy of the LF field with time happens due to the energy transfer to ions.



ZHM, Θ = 0.05 *SHM*, Θ = 0.05

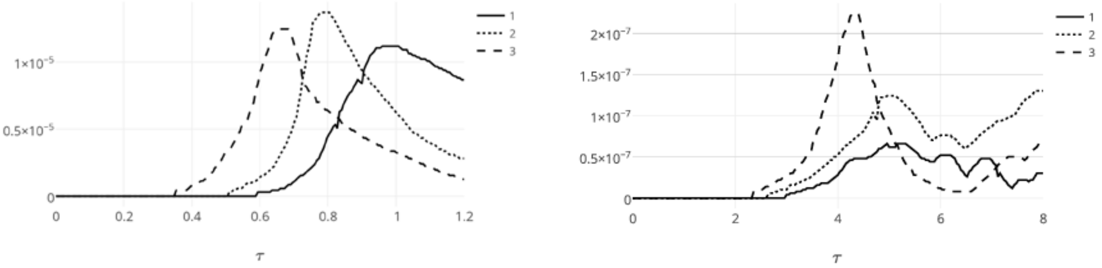


ZHM, Θ = 0.015 *SHM*, Θ = 0.015



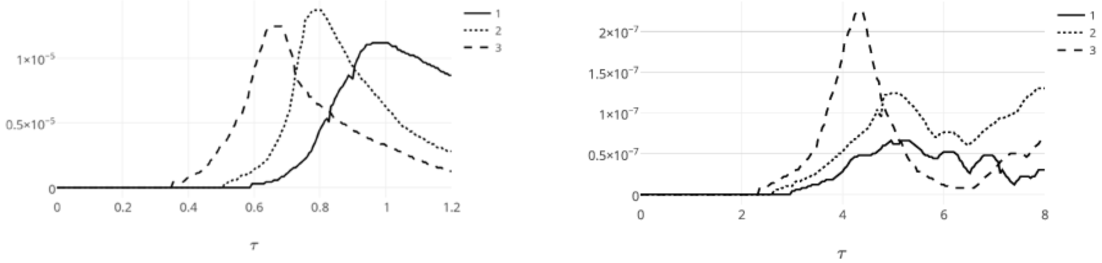
ZHM, Θ = 0.001 *SHM*, Θ = 0.001

Fig. 4. The ion velocity distribution for the case of light ions



ZHM *SHM*

Fig. 5. Time evolution of the LF spectrum energy for the case of light ions,
1 – Θ = 0.05 *, 2 –* Θ = 0.015 *, 3 –* Θ = 0.001



ZHM *SHM*

Fig. 6. Time evolution of the ion kinetic energy for the case of light ions:
1 – Θ = 0.05 *; 2 –* Θ = 0.015 *; 3 –* Θ = 0.001

The decrease of the damping rate of the HF field corresponds to the slowing of the HF field burnout in the cavities and leads to the broadening of the HF spectrum that causes the deepening of the cavities and increase of the kinetic energy of ions ejected from them. It should be noted that as the absorption rate of the HF field decreases, the ion velocity distribution function approaches the Maxwellian distribution in both models under consideration. In a cold plasma, the intensity of the long-wave LF oscillations is high and it increases with the decrease of the absorption of HF modes. It is important to note that the total energy transferred to ions increases as the absorption of the HF spectrum reduces.

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ДИНАМИКА ИОНОВ И ГЕНЕРАЦИЯ ИОННОГО ЗВУКА ПРИ РАЗВИТИИ МОДУЛЯЦИОННОЙ НЕУСТОЙЧИВОСТИ ИНТЕНСИВНЫХ ЛЕНГМЮРОВСКИХ ВОЛН В ПЛАЗМЕ

А.Г. Загородний, А.В. Киричок, В.М. Куклин, А.В. Приймак

Анализируется развитие 1D-параметрических неустойчивостей интенсивных длинноволновых ленгмюровских волн в рамках так называемых гибридных моделей, когда электроны описываются как жидкость, а ионы описываются частицами. Рассматриваются случаи, когда средняя плотность энергии поля меньше (гибридная модель Захарова) и больше (гибридная модель Силина) плотности тепловой энергии плазмы. Уменьшение уровня поглощения высокочастотного поля соответствует замедлению выгорания ВЧ-поля в образовавшихся кавернах плотности плазмы и уширению спектра ВЧ-мод. При этом функция распределения ионов по скоростям в моделях Захарова и Силина по форме приближается к нормальному распределению.

ДИНАМІКА ІОНІВ І ГЕНЕРАЦІЯ ІОННОГО ЗВУКУ ПРИ РОЗВИТКУ МОДУЛЯЦІЙНОЇ НЕСТІЙКОСТІ ІНТЕНСИВНИХ ЛЕНГМЮРІВСЬКИХ ХВИЛЬ В ПЛАЗМІ

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Аналізується розвиток 1D-параметричних нестійкостей інтенсивних довгохвильових ленгмюрівських хвиль у термінах так званих гібридних моделей, коли електрони описуються як рідина, а іони описуються частинками. Розглядаються випадки, коли середня енергія поля менше (гібридна модель Захарова) і більше (гібридна модель Силіна) теплової енергії плазми. Зменшення поглинання високочастотного поля відповідає уповільненню вигорання ВЧ-поля в утворених кавернах щільності плазми і уширенню спектра ВЧ-мод. При цьому функція розподілу іонів за швидкостями в моделях Захарова та Силіна за формою наближається до нормального розподілу.