# PATTERN FORMATION IN UNSTABLE VISCOUS CONVECTIVE MEDIUM

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The convection in a thin layer of liquid (gas) with poorly heat conducting boundaries and with temperature dependent viscosity are considered. The Proctor-Sivashinsky model is examined in order to study both the pattern formation and the second-order structural phase transitions as between patterns with translational invariance as well as between structures with broken translational invariance but keeping a long-range order. The influence of the temperature dependence of viscosity on the process of pattern formation and structure transformations is discussed. Is shown that the temperature dependence of viscosity inhibits structural transition leading to formation of square cells. Initiallyquasi-stable structure of convective rolls becomes stable under this condition.

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## INTRODUCTION

Considering the various processes in continuous media, we need to take into account the dynamics of perturbations with not only different spatial and temporal scales but also different spatial orientation [1 - 12]. The last one is responsible in the common geometric sense for symmetry of the spatial structures, which possess not only short-range but also a long-range order [13 - 17].

Currently, the problem of most interest is the elucidation of the nature of spatial structures appearance, the search for physically transparent mechanisms of these processes, and then the formulation of adequate (which have clear physical background) mathematical models for description of these phenomena.

The issues of structural transformations, structural second-order phase transitions, resulting in the changes of the symmetry and some characteristic scales of spatial structures always be of great interest to researchers and developers of technologies.

The models of spatial structure formation were considered by many researches, which main ideas can be found in monographs [18 - 20]. However, of main interest, as it was pointed in [21], are the dynamical models, which could be described by differential equations in partial derivatives, the mathematical apparatus of the analysis of which is well developed. The special attention should be attended to the models that are capable to describe the imperfect quasi-periodic systems, quasi-crystals (that is characterized by a long-range order and symmetry inadmissible in a classical crystallography [22, 23]).

In particular, one of the main problems of radiative study of materials is the problem of occurrence of a complex system of defects and phase transformations caused by irradiation. The authors of [24 - 27] drew attention to the collective character of the macro-scale processes in such materials. The formation of spatio-temporal dislocation in homogeneities, dislocation channels, the moving Chernov-Luders lines, the dynamic self-wave structures (the Danilov-Zuev relaxation waves, the phase transformation front in the dislocation-vacancy ensemble etc.) may be caused namely by macroscopic processes. The self-organization of structural transformation under the action of external factors demonstrates the nonlocal properties caused most likely by the large scale instabilities. Note that in some cases the experimental and calculated data also point to the fact that local defects and disorders may be a result of imperfection in a large scale packing, occurring in particular when the system selects the characteristic scaling with broken geometric orientation on the structural elements. The Proctor-Sivashinsky model is found to be very attractive [28, 29] for studying the processes of pattern formation in systems which possess a preferred characteristic spatial scale of interaction between quasi-particles or elements of future structure. This model was developed for description of the convection in a thin layer of liquid with poorly conducting heat boundaries. Authors of [30] have found the stationary solutions with a small number of the spatial modes one of which (convective cells) was steady and the second one (convective rolls) turned out to be unstable.

The model [31] with use of the multimode description allowed to find out that at first the quasi-stable long-living state (the curved quasi-one-dimensional convective rolls) arises, and later after a lapse of time (which is considerably greater than the reverse linear growth rate of the process), the system transforms to the stable state (square convective cells). The detailed treatment of the Proctor – Sivashinsky model [32, 33] presented below have shown that this structural transition demonstrates all the characteristics of second order phase transition (the continuity of the sum of squared mode amplitudes over the spectrum $I=\sum_j a_j^2 \equiv \sum_{k_j} |a_{k_j}|^2$ or that the same, the continuity of density of this value and discontinuity of its time derivative $\partial I/\partial t$ ).

An important issue discussed in this paper is how the temperature dependence of viscosity effects on the characteristics of the pattern formation and structural transitions.

## 1. MODEL DESCRIPTION

When the Rayleigh number $Ra$ exceeds a critical value $Ra_{thr}$, i.e. $Ra = Ra_{thr}(1+\varepsilon)$, the three-dimensional convection arises in thin layer with poorly heat conducting horizontal boundaries (see, for example [2]), which can be described by the Proctor-Sivashinsky equation [28, 29].This equation determines the dynamics of temperature field in the horizontal plane (*x, y*):

$$\dot{\Phi} = \varepsilon^2\Phi + \gamma \cdot \nabla(\Phi\nabla\Phi) - (1-\nabla^2)^2\Phi + \\ + \frac{1}{3}\nabla\left(\nabla\Phi|\Phi|^2\right) + \varepsilon^2 f, \quad (1)$$

where $f$ is the random function describing the external noise, and the quantity $\varepsilon$ determines the convection threshold overriding, which is assumed to be sufficiently small ($0 < \varepsilon < 1$). The term $\gamma\nabla(\Phi\nabla\Phi)$ describes the temperature dependence of viscosity.

In this case we shall find the solution in the form

$$\Phi = \varepsilon \sum_j a_j \exp(i\vec{k}_j \vec{r}), \quad (2)$$

with $|\vec{k}_j| = 1$. Renormalizing the time units $\propto \varepsilon^2$, we obtain the evolution equation for slow amplitudes $a_j$ [30]:

$$\dot{a}_j = a_j - 2\gamma \cdot a_{j+j_0} a_{j+2j_0} - \\ - \sum_{m=1}^{N} V_{mj} |a_m|^2 a_j + f, \quad (3)$$

where interaction coefficients are determined as follows

$$V_{jj} = 1, \quad (4)$$

$$V_{ij} = (2/3)\left(1 - 2\left(\vec{k}_i \vec{k}_j\right)^2\right) = (2/3)\left(1 + 2\cos^2\vartheta\right). \quad (5)$$

Here $\vartheta$ is the angle between vectors $\vec{k}_i$ and $\vec{k}_j$. Let $\vartheta_{j_0} = 2\pi/3$, $\vartheta_{j+j_0} = \vartheta_j + 2\pi/3$ and $\vartheta_{j+2j_0} = \vartheta_j + 4\pi/3$.

The instability interval in $k$-space represents a ring with average radius equal to unit and the width is order of relative above-threshold parameter $\varepsilon$, i.e. much less than unity. During the development of the instability, the effective growth rate of modes that lies outside of the very small neighborhood near the unit circle decreases due to the growth of the nonlinear terms and can change sign that leads to a narrowing of the spectrum to the unit circle in the $k$-space. Since the purpose of further research will be the study of stability of spatial structures with characteristic size of order $2\pi/k \propto 2\pi$ and the important characteristic for visualization of simulation results will be evidence of these structures, so we restrict ourselves by considering some idealized model of the phenomenon, assuming that the oscillation spectrum is already located on the unit circle in the $k$-space.

## 2. SIMULATION RESULTS

Development of perturbations in the system, as shown by the numerical analysis of Eq. (5.1) will be as follows [12, 31]. Starting from initial fluctuations, the modes over a wide range of $\vartheta$ begin grow. The value of the quadratic form of the spectrum $I = \sum_j a_j^2$ can be estimated by equating the r.h.s of Eq. (3) to zero and to obtain as result a value close to 0.75.

***Convection with temperature independent viscosity.*** It was shown in [31 - 33] that in the absence of temperature dependence of viscosity and when the number of modes is sufficiently large the system delayed the development while remaining in a dynamic equilibrium. For further development - "crystallization", one of the modes must get a portion of the energy which excesses some threshold value. That is, in these case, it is necessary a certain level of noise (fluctuations). This can be achieved either at finite noise level $f \neq 0$ or by decreasing the accuracy of calculations that is the same as noted in [32]. Similar cases, when the noise can trigger or accelerate instability are reviewed in the book [34].

If one of the modes gets the proper amount of energy, then the process of formation of a simplest convective structure – rolls begins. Note that in the nature, the thin clouds also can form the roll structure. The value of $I$ in this case tends to unity ($I \to 1$). However, this state is not stable and then we can see the next structural transition: convective rolls are modulated along the axis of fluid rotation, and the typical size of this modulation phases down. In this transition state, the system stays for a sufficiently long time (which slightly increases within some limits with increase in the number of modes), and the value $I \approx 1.07$ remains constant during this time. After a rather long time, ten times more than the inverse linear growth rate of the initial instability only the one mode "survives" from newly formed "side" spectrum, which amplitude is comparable with the amplitude of the primary leading mode. In the end, the stable convective structure –square cells is generated, and the quadratic form $I$ reaches the value of $I = 1.2$.

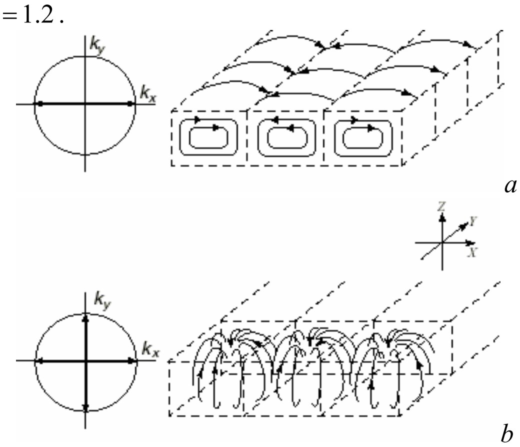


*Fig. 1. Convective structures: rolls (a) and square cells (b)*

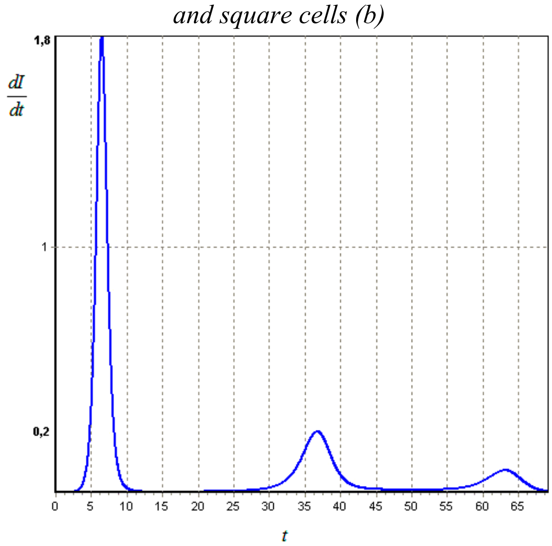


*Fig. 2. The evolution of the derivative $dI/dt$ (in relative measuring units) of the integral quadratic form $I = \sum_j a_j^2$*

Further researches of this process have found the following dynamics of quadratic form $I = \sum_j a_j^2$ with time (see Fig. 2). Exact after the first peak of the derivative, the metastable structure – a system of convective rolls is formed, and up to the moment when the second burst have appeared with value of $I \approx 1$ it has remain unchanged. The next burst of $\partial I / \partial t$ indicates the emergence of a secondary metastable structure with a new value of $I \approx 1.07$ .

After the second burst of the quadratic form derivative a stable structure of squared convective cells is started to build up. Such behavior proves the existence of structural-phase transitions in the system.

***Convection with temperature dependent viscosity.*** The term $\gamma \cdot \nabla(\Phi \nabla \Phi)$ appears in Eq. (3) when we take into account the temperature dependence of viscosity.

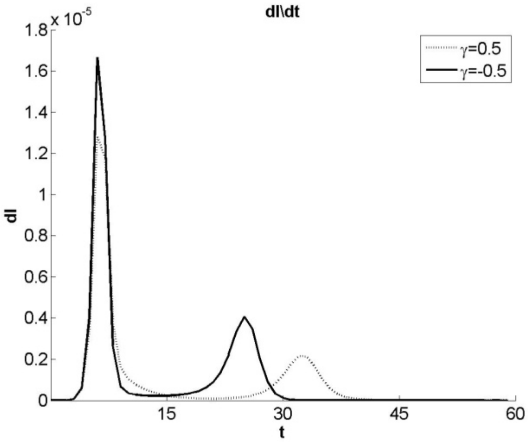


*Fig. 3. The evolution of the derivative* $\partial I / \partial t$ *for* $|\gamma| = 0.5$

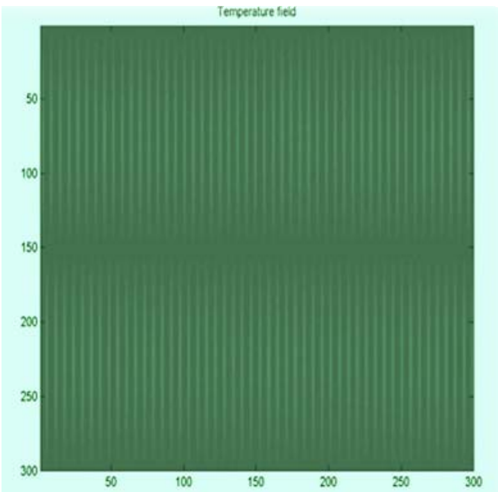


*Fig. 4. Temperature field corresponding to the roll structure,* $|\gamma| = 0.5$

For $\gamma > 0$ the gas (this case corresponds to the gas convection) flows up to the center of the cell, for $\gamma < 0$ (which corresponds to the movement of the liquid) the liquid flows outward and down from the center of the cell (see for example [35]). At $|\gamma| << 1$, the effect of this term on the dynamics of the process is negligible. The convection develops in accordance with above-described scenario. However, when the parameter $\gamma$ approaches the unit value, one further mechanism of energy transfer between each triplet of interacting modes appears which destroys the previous mechanism of mode interaction arising due to vector cubic nonlinearity. The consequences of this destruction are almost identical for $\gamma$ of different signs.

First of all, the rapid growth of the modes spectrum at the linear stage of instability forms a quasi-stabile structure with rather intricate topology, depending on the initial conditions. However, after a short time there is a second structural transition (see Fig. 3) as a result of which the stableandwell-definedelongated rolls are formed, which structure is shown in Fig. 1. The spatial distribution of temperature field of the structure is demonstrated in Fig. 4.

Fig. 5 demonstrates the specific features of structural transitions, where one can see the regularity of the function $I = \sum_j a_j^2$ , which characterizes the state of the system.

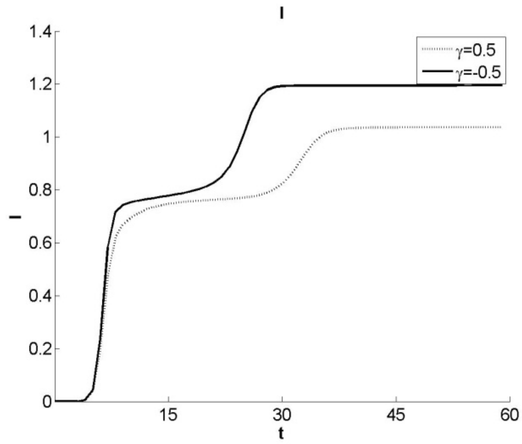


*Fig. 5. Dynamics of the quadratic form* $I = \sum_j a_j^2$ *, which characterizes the state of the system at* $|\gamma| = 0.5$

Thus, an appreciable temperature dependence of viscosity can lead to formation of stable convective rolls. Such convective rollscanbe observed in the thin cloud cover (see Fig. 6).

*Fig. 6. Formation of convective rolls, extending for hundreds of kilometers in the north of Australia at the beginning of rainy season*

## CONCLUSIONS

Thus, the temperature dependence of viscosity included in the Proctor-Sivashinsky model, which describes the convection in a thin layer of liquid (or gas) with poorly heat conducting boundaries, results in suppression of structural phase transition, which previously led to formation of square cell pattern. As in the absence of temperature dependent viscosity, the long-livedquasi-stablestates with a topology that is defined by the boundaries of the system and the initial conditions are observed. Some differences between the gas and liquid media consist only in small differences in the amplitude of the final structure of the convective rolls, without changing the nature of structural phase transitions.

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## ПРОСТРАНСТВЕННЫЕ СТРУКТУРЫ В КОНВЕКТИВНО-НЕУСТОЙЧИВОЙ ВЯЗКОЙ СРЕДЕ

***И.В. Гущин, А.В. Киричок, В.М. Куклин***

Рассмотрены модели описания конвекции в слое жидкости (газа) с плохо проводящими тепло границами с учетом зависящей от температуры вязкости. Обсуждается корректная модель Проктора-Сивашинского в рамках которой можно описать как развитие пространственных структур, так и структурно-фазовые переходы второго рода между состояниями, обладающими разной топологией с разной степенью нарушений трансляционной инвариантности. Обсуждается влияние температурной зависимости вязкости на развитие процесса формирования структур и структурных трансформаций.

## ПРОСТОРОВІ СТРУКТУРИ В КОНВЕКТИВНО-НЕСТІЙКОМУ В'ЯЗКОМУ СЕРЕДОВИЩІ

***І.В. Гущін, О.В. Киричок, В.М. Куклін***

Розглянуто моделі опису конвекції в шарі рідини (газу) з границями, що погано проводять тепло, з урахуванням залежності в'язкості від температури. Обговорюється коректна модель Проктора-Сівашінского в рамках якої, можна описати як розвиток просторових структур так і структурно-фазові переходи другого роду між станами, що мають різну топологію та різний ступень порушення трансляційної інваріантності. Обговорюється вплив температурної залежності в'язкості на розвиток процесу формування структур і структурних трансформацій.