NONLINEAR PROCESSES IN PLASMA MEDIA

STOCHASTIC SIMULATION OF THE NONLINEAR KINETIC EQUATION WITH HIGH-FREQUENCY ELECTROMAGNETIC FIELDS

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A general approach to Monte Carlo methods for Coulomb collisions is proposed. Its key idea is an approximation of Landau-Fokker-Planck (LFP) equations by Boltzmann equations of quasi-Maxwellian kind. High-frequency fields are included into consideration and comparison with the well-known results are given.

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INTRODUCTION

Numerical simulation of plasma dynamics on kinetic level is a difficult problem. It is natural to use standard splitting methods, i.e. to consider separately (a) continuous motion of electrons and ions in external and self-consistent electro-magnetic fields and (b) Coulomb collisions. The splitting procedure is formally quite similar to what we do in simulation of neutral gases by Monte Carlo methods [1]. There is, however, a big difference in the simulation of the first stage (a), which is almost trivial (free molecular flow) for neutral gases. The corresponding motion of charged particles, described by Vlasov-Poisson or Vlasov-Maxwell kinetic equations [2], is much more sophisticated. The particle methods for solving these equations of "collisionless" plasma are very well developed and discussed in literature. We shall consider below only the second stage (b), related to Coulomb collisions.

The spatially homogeneous kinetic equation for Coulomb interaction were first published by L.D. Landau in 1936 [3]. Beginning with its re-discovery in the Fokker-Planck form in [4] a lot of work is done on numerical methods for the LFP equations based on finite difference schemes. Recent review on that subject can be found in [5], it contains many references. We are interested in this paper in methods, which are very close to Discrete Simulation Monte Carlo (DSMC) methods in rarefied gas dynamics [1].

The two well-known methods should be mentioned first in that field [6] and [7]. The general approach to DSMC methods for Boltzmann equation with long range potentials and for Landau equation was proposed in [8]. In the present paper we use another approach of the method (see, for example, [9]), which looks simpler for computer implementation. It looks quite natural to approximate the Landau equation by the Boltzmann equation with small-angle scattering and then to use any known DSMC scheme [1] for simulation. This approximation can be chosen in the quasi-Maxwellian way, such that the collision frequency for the auxiliary Boltzmann equation is constant. In the present paper we choose the simplest, in our view, scattering law in that class.

1. APPROXIMATION OF LANDAU EQUATIONS BY BOLTZMANN EQUATIONS

We consider an arbitrary spatially homogeneous mixture of rarefied gases. Let $\{f_i(\mathbf{v},t), i=1,...,n\}$ be distribution functions of particles with masses $\{m_i, i=1,...,n\}$, respectively. The independent variables $\mathbf{v}\in \mathrm{R}^3$ and $t\ge 0$ stand for velocity and time, respectively. Spatial densities $\{\rho_i(t), i=1,...,n\}$ are given by integrals

$$\rho_i(t)=\int\limits_{R^3} d\mathbf{v}\, f_i(\mathbf{v},t), \quad i=1,...,n. \tag{1}$$

The system of Boltzmann kinetic equations for $f_i(\mathbf{v},t)$ reads

$$\frac{\partial f_i}{\partial t}=\sum_{j=1}^{n} Q_{ij}(f_i,f_j), \quad i,j=1,...,n, \tag{2}$$

where

$$Q_{ij}(f_i,f_j)=\int d\mathbf{w}d\boldsymbol{\omega} g_{ij}(u,\mu)\Big[f_i(\mathbf{v}')f_j(\mathbf{w}')-f_i(\mathbf{v})f_j(\mathbf{w})\Big],$$
$$\mathbf{u}=\mathbf{v}-\mathbf{w},\ \mu=\frac{\mathbf{u}\cdot\boldsymbol{\omega}}{u},\ \boldsymbol{\omega}\in \mathrm{S}^2,\ i,j=1,...,n,$$
$$\mathbf{v}'=\frac{1}{m_i+m_j}\Big(m_i\mathbf{v}+m_j\mathbf{w}+m_j u\boldsymbol{\omega}\Big), \tag{3}$$
$$\mathbf{w}'=\frac{1}{m_i+m_j}\Big(m_i\mathbf{v}+m_j\mathbf{w}-m_i u\boldsymbol{\omega}\Big).$$

Functions $g_{ij}(u,\mu)$ are expressed by formulas $g_{ij}(u,\mu)=g_{ji}(u,\mu)=u\sigma_{ij}(u,\mu)$, where $\sigma_{ij}(u,\mu)$ is the differential cross section (in the center of mass system of colliding particles of sort i and sort j) of scattering at the angle $\theta=\arccos(\mu), |\mu|\le 1$.

The system of Boltzmann equations (2) is interesting for us merely as a starting point to pass to the Landau equations. For such transition one needs to choose a special kind of functions $g_{ij}(u,\mu)$. This choice is based on the fact which has been proved many years ago [11].

Suppose that distribution functions are infinitely differentiable and rapidly decreasing with all their derivatives at infinity. Following the original idea of Landau [3] and carrying on the Taylor expansion of integrand in (3) (with respect to $\mathbf{v}'-\mathbf{v}$ and $\mathbf{w}'-\mathbf{w}$), we obtain a formal series

$$Q_{ij}(f_i, f_j) = \sum_{k=1}^{\infty} Q_{ij}^{(k)}(f_i, f_j). \qquad (4)$$

The first term corresponds to the Landau collisional integral (for arbitrary $g_{ij}(u,\mu)$ in (3)):

$$Q_{ij}^{(1)}(f_i, f_j) = \frac{m_{ij}^2}{2m_i}\frac{\partial}{\partial v_\alpha}\int_{R^3} d\mathrm{w}\, g_{ij}^{(1)}(u) T_{\alpha\beta}(\mathbf{u}) \times \\ \times\left(\frac{1}{m_i}\frac{\partial}{\partial v_\beta} - \frac{1}{m_j}\frac{\partial}{\partial w_\beta}\right) f_i(\mathbf{v}) f_j(\mathbf{w}), \qquad (5)$$

here the summation over repeated Greek indices $\alpha, \beta = 1,2,3$ is assumed,

$$m_{ij} = \frac{m_i m_j}{m_i + m_j}, \quad T_{\alpha\beta}(\mathbf{u}) = u^2\delta_{\alpha\beta} - u_\alpha u_\beta, \\ g_{ij}^{(1)}(u) = 2\pi\int_{-1}^{1} d\mu\, g_{ij}(u,\mu)(1-\mu). \qquad (6)$$

The other terms of (4) can be symbolically written in the form

$$Q_{ij}^{(k)}(f_i, f_j) = \int_{R^3} d\mathrm{w}\, g_{ij}^{(k)}(u) A_{ij}^{(k)}(\mathbf{v}, \mathbf{w}),$$

$$g_{ij}^{(k)}(u) = 2\pi\int_{-1}^{1} d\mu\, g_{ij}(u,\mu)(1-\mu)^k, \quad k \ge 2,$$

where $A_{ij}^{(k)}(\mathbf{v}, \mathbf{w})$ is a smooth integrable functions. Then it becomes clear under which conditions the system of Boltzmann equations (2) approximates (at the formal level) the corresponding system of given Landau equations, in which $g_{ij}^{(1)}(u) = b_{ij}(u)$, where $b_{ij}(u)$ are some given functions. It is formally sufficient to this aim to choose the non-negative functions $g_{ij}(u,\mu)$ in equations (2), (3) in the form $g_{ij}(u,\mu;\varepsilon)$, where $\varepsilon > 0$ is a small parameter, and to demand that

$$\lim_{\varepsilon\to 0} 2\pi\int_{-1}^{1} d\mu\, g_{ij}(u,\mu;\varepsilon)(1-\mu) = b_{ij}(u), \\ \lim_{\varepsilon\to 0} 2\pi\int_{-1}^{1} d\mu\, g_{ij}(u,\mu;\varepsilon)(1-\mu)^k = 0, k \ge 2. \qquad (7)$$

As a simple example of such an approximation one can consider functions

$$g_{ij}(u,\mu;\varepsilon) = \frac{1}{2\pi\varepsilon}\delta\left[1-\mu-\varepsilon a_{ij}(u)\right], \qquad (8)$$

where $a_{ij}(u) = b_{ij}(u)$, if $\varepsilon b_{ij}(u) \le 2$, otherwise $a_{ij}(u) = 2\varepsilon^{-1}$. The function g_{ij} means that the scattering always occurs at fixed angle $\arccos\left[1-\varepsilon a_{ij}(u)\right]$ for collision of particles of sorts i and j. This scattering law is convenient for the Monte Carlo method. Another advantage of this approximation is that the total collision frequency is constant:

$$g_{ij}^{tot}(u,\varepsilon) = 2\pi\int_{-1}^{1} d\mu\, g_{ij}(u,\mu;\varepsilon) = \frac{1}{\varepsilon}. \qquad (9)$$

Such an approximation can be called quasi-Maxwellian, since the total collision frequency (for any pair of sorts i and j, including the case $i=j$) is independent of velocities. Note that ε has dimensionality $[t][l]^{-3}$, we ignore this fact considering ε simply as a small parameter. From now on we consider the most important case of Landau equations for the classical plasma consisting of n sorts of charged particles with charges $\{e_i, i = 1,...,n\}$. Assuming the Coulomb logarithm L is the same constant for all interactions, we obtain (see, for example, [2]) equations (5) - (8), where

$$g_{ij}^{(1)}(u) = b_{ij}(u) = 4\pi L\frac{e_i^2 e_j^2}{m_{ij}^2 u^3}; \quad i, j = 1,...,n. \qquad (10)$$

It is clear that Boltzmann equations (2), (3), where the functions $g_{ij}(u,\mu;\varepsilon)$ are computed by formulas (7), (9), approximate (at least formally) as $\varepsilon \to 0$ the system of Landau equations (4) for n-component plasma. Note, that the formal error of above described approximation of the Landau integral $Q^{(1)}(f_i, f_j)$ by the Boltzmann integral $Q(f_i, f_j)$ has the first order $O(\varepsilon)$. More rigorous estimate gives the error not lager than $O\left(\sqrt{\varepsilon}\right)$.

2. IMPLEMENTATION OF MONTE CARLO METHOD FOR TWO COMPONENT PLASMAS

The idea of the Monte Carlo method belongs to G. Bird [1], who suggested it in 1960s, independently of earlier works of M. Kac [12] on the probabilistic nature of the Boltzmann equation. We choose as a basis the approach of Kac. The idea is to associate nonlinear equations (2) with some linear equation (Master equation), describing relatively simple stochastic process.

We consider in this section an example of electro-neutral hydrogen plasma. The Landau equations have the form (2) with collisional integral (5) and $n = 2$, $e_1^2 = e_2^2 = e^2$. We change indices 1 and 2 to e (electrons) and i (ions), correspondingly, and denote $m_e = m$, $m_i = M$ with $\gamma = m/M$. We perform a normalization with units of ρ_0, the full density of number of plasma particles; a characteristic velocity v_0 (for instance, the thermal electron velocity at the equilibrium temperature $\sqrt{T_0}$); the electron-electron collision time t_0: $2\pi L e^4 \rho_0 t_0 / m^2 v_0^3 = 1$. Equations (2), (4) will be solved with initial conditions.

$$f_{e,i}\Big|_{t=0} = f_{e,i}^{(0)}(\mathbf{v}), \quad \int_{R^3} d\mathbf{v} f_{e,i}^{(0)}(\mathbf{v}) = 1. \qquad (11)$$

For the approximate solution of the problem we choose a small real number $\varepsilon > 0$ and a large integer N_1. We model the solution of this problem through the evolution of the random vector

$$\mathbf{V}_N(t) = \{\mathbf{v}_1^{(e)}(t), ..., \mathbf{v}_{N_1}^{(e)}(t); \mathbf{v}_1^{(i)}(t), ..., \mathbf{v}_{N_1}^{(i)}(t)\} \in R^{3N}, \quad (12)$$

where $N = 2N_1$.

At $t = 0$ all electron velocities $\mathbf{v}_k^{(e)}(0)$ are distributed in R^3 independently in accordance with the distribution function $f_e^{(0)}(\mathbf{v})$ and ion velocities $\mathbf{v}_k^{(i)}(0)$ are distributed in similar way with $f_i^{(0)}$, $k = 1, ..., N_1$.

Let us consider the scheme with the maximal time step $\tau_N = 2\varepsilon/N$, $N = 2N_1$, then the time t takes discrete values $t_k = k\tau_N$, $k = 0, 1, ...$ Exactly one collision happens at each interval $[t_k, t_{k+1})$. Probabilities of collisions of three possible kinds are defined by $p_{ee} = p_{ii} = 1/4$, $p_{ei} = 1/2$. After it is decided, which of the three collisions, really happens, we choose a random pair of velocities of particles of corresponding sorts and "perform the collision". The velocities after collision of two electrons and two ions read

$$\begin{aligned} \mathbf{v}_r' &= \frac{1}{2}(\mathbf{v} + \mathbf{w} + |\mathbf{v} - \mathbf{w}|\boldsymbol{\omega}), \\ \mathbf{v}_s' &= \frac{1}{2}(\mathbf{v} + \mathbf{w} - |\mathbf{v} - \mathbf{w}|\boldsymbol{\omega}), \end{aligned} \qquad (13)$$

where the unit vector $\boldsymbol{\omega}$ is defined in Cartesian coordinates with the axis Oz along the vector $\mathbf{u} = \mathbf{v} - \mathbf{w}$ in the following way:

$$\boldsymbol{\omega} = \left\{\sqrt{1-\mu^2}\cos\phi, \sqrt{1-\mu^2}\sin\phi, \mu\right\},$$

$$\mu_{ee} = 1 - 2Min\left\{\frac{4\varepsilon}{u^3}, 1\right\}, \ \mu_{ii} = 1 - 2Min\left\{\frac{4\varepsilon\gamma^2}{u^3}, 1\right\},$$

where ϕ is a random angle distributed uniformly within the interval $[0, 2\pi)$. For the electron-ion collision we choose velocities $\mathbf{v}_r^{(e)} = \mathbf{v}, \mathbf{v}_s^{(i)} = \mathbf{w}$, $1 \le r, s \le N_1$, and transform them to

$$\begin{aligned} \mathbf{v}_r^{(e)\prime} &= \frac{1}{m+M}(m\mathbf{v} + M\mathbf{w} + Mu\boldsymbol{\omega}), \\ \mathbf{v}_s^{(i)\prime} &= \frac{1}{m+M}(m\mathbf{v} + M\mathbf{w} - mu\boldsymbol{\omega}), \end{aligned} \qquad (14)$$

where

$$\mu = \mu_{ei} = 1 - 2Min\left\{\frac{\varepsilon(1+\gamma)^2}{u^3}, 1\right\}.$$

Thus, starting from the initial vector $\mathbf{V}_N(0)$, we obtain a new vector $\mathbf{V}_N(\tau_N)$ after the first collision. After that the whole simulation process is repeated many times and the time counter is increased at each collision by the quantity τ_N.

We choose initial isotropic distributions $f_{e,i}^0 = 1/2\pi\delta(v^2 - 1)$, $v = |\mathbf{v}|$, $\mathbf{v} = (v_x, v_y, v_z)$. These distributions mean that the initial thermal speeds of electrons and ions are equal to one in our units. Note that average velocities of the components are equal to zero then $T_e^0 = 1/3, T_i^0 = 1/3\gamma$. We compute some moments

$$\langle v_{e,i}^{2n}\rangle(t) = \frac{1}{N_1}\sum_{k=1}^{N_1} v_{e,i}^{2n}(t), \quad n = 1, 2, ...,$$

and then make an average over K computational runs. The resulting values for various values of parameters are compared with practically exact values of integrals

$$\langle v_{e,i}^{2n}\rangle(t) = \int_{R^3} d\mathbf{v} f_{e,i}(\mathbf{v}, t) v^{2n}, \quad n = 1, 2, ...,$$

obtained by using difference scheme from [5].

3. THE EFFECT OF INVERSE BREMSSTRAHLUNG OF LASER RADIATION

Let us consider the plasma dynamics in the high-frequency weak electrical field $\mathbf{E}(t) = \mathbf{E}e^{-i\omega t}$, when $\omega \gg \nu_{ei}$ and $v_E = |eE/m_e\omega| \ll v_{Te}$ (magnetic field influence is neglectable). We suppose that $v_{Te}T \gg l$ or $v_{Te}/\omega \gg l$, where $T = 2\pi/\omega$, l is a characteristic spacial scale, then

$$\left|\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}\right| \ll \frac{v_{Te}}{l}|f| \text{ and } \left|\frac{\partial f}{\partial t}\right| \ll \omega|f|,$$

$$\text{so } \left|\frac{\partial f}{\partial t}\right| \ll \left|\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}\right|,$$

in this case the system of LFP equations reads

$$\frac{\partial f_k}{\partial t} + \frac{e_k}{m_k}\mathbf{E}(t)\frac{\partial f_k}{\partial \mathbf{v}} = Q_{ke}^{(L)}(f_k, f_e) + Q_{ki}^{(L)}(f_k, f_i),$$

$k = e, i$.

As it was shown in [13], that even for weak fields $v_E \ll v_{Te}$ and $Z \gg 1$, when Langdon parameter α is big: $\alpha = Zv_E^2/v_{Te}^2 \gg 1$, or, equivalently $Zv_E^2 > v_T^2 > v_E^2$ the electron distribution function is far from the Maxwellian one. It was shown in [14] for arbitrary α, that the symmetrical part of an electron distribution function (EDF) can be written in the following form:

$$f_0(v,t) = \frac{n_e}{4\pi v_{Te}^3(t)}\varphi\left(\frac{v}{v_{Te}(t)}\right), \qquad (15)$$

$$\varphi(x) = \varphi(0)exp\left[-\left(x/x_0\right)^m\right],$$

where

$$\varphi(0) = \frac{m\left[\Gamma(5/m)\right]^{3/2}}{3^{3/2}\left[\Gamma(3/m)\right]^{5/2}}, \quad x_0 = \left[\frac{3\Gamma(3/m)}{\Gamma(5/m)}\right]^{1/2},$$

$$m = 2 + \frac{3}{1 + 1.66/\alpha^{0.724}}.$$

It can continuously vary from Maxwellian ($m = 2, \alpha \to 1$) to a super-Gaussian form with $m = 5$ for $\alpha \to 1$. Such nonequilibrium states can exist in a plasma for $\alpha > 1$ because the inverse bremsstrahlung heating rate is sufficiently fast for slow particles so that electron-electron collisions cannot restore a Maxwellian EDF. Also in [14] was obtained an equation for the time evolution of the electron temperature

$$\frac{\partial T_e}{\partial t} = \frac{4\pi ZY}{9n_e m} v_E^2 f_0(v=0,t), \qquad (16)$$

where $Y = 4\pi n_e e^4 L/m^2$. Equation (16) demonstrates that the heating rate is entirely defined by behavior of very slow electrons.

Now let us consider our particle simulation results. First, we have calculated an electron heating rate. Fig. 1 shows an electron temperature normalized to its initial value versus time for α=6.75. We have also plotted in Fig. 1 the heating rate which is predicted by Eq. (16) with f_0 given by (15) with $m = 5$ (dashed line) and the Maxwellian distribution $m = 2$ (dotted line). As one can expect the heating rate corresponds to the non-Maxwellian EDF.

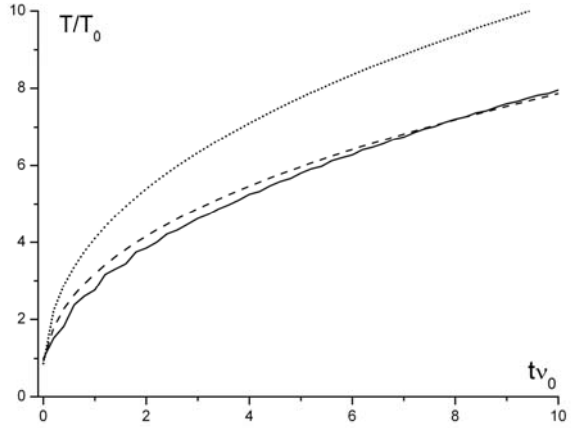

Fig. 1. Temporal dependence of the electron temperature for DSMC method (solid line), the distribution (15) (dashed line) and the Maxwellian distribution (dotted line) for $\gamma=1/1800$, N=2000, K=20, $\varepsilon = 5\cdot 10^{-4}$, Z=10, α=6.75, $\omega=10\nu_{ei}$

In Fig. 2 we plot the temporal dependence of the 6th EDF moment for α=6.75. The relativly small difference between the DSMC method results and temporal dependence expected for the distribution (15) is quite understandable since, as it was mentioned in [14], the laser energy absorbed by the fast electrons is much smaller than energy absorbed by the slow electrons and the EDF at high velocities (tail of distribution) should remain close to the Maxwellian due to the e – e collisions between fast and slow particles.

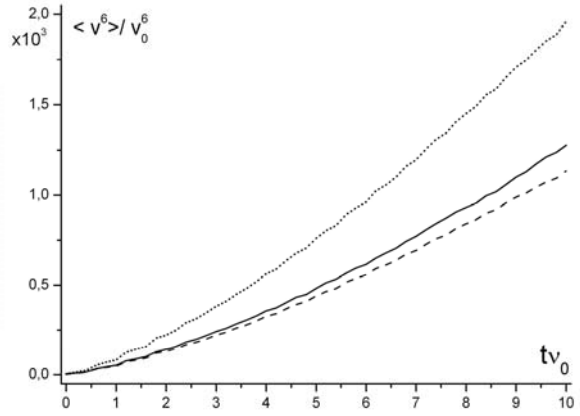

Fig. 2. Temporal dependence of the 6th electron distribution function moment for DSMC method (solid line), the distribution (15) (dashed line) and the Maxwellian (dotted line). The parameters are the same as in Fig. 1

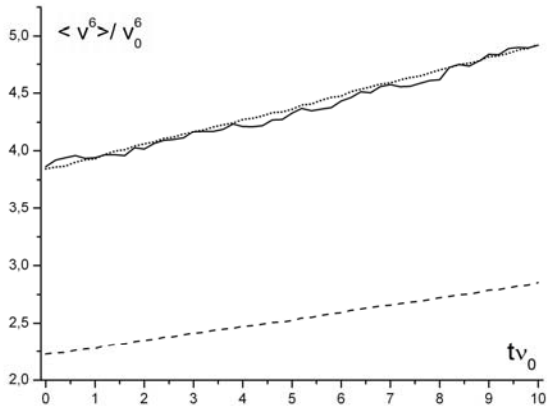

Fig. 3. Temporal dependence of the 6th electron distribution function moment for DSMC method (solid line), the distribution (15) at $m = 5$ (dashed line) and the Maxwellian (dotted line) for $\gamma=1/1800$, N=2000, $\varepsilon = 5\cdot 10^{-4}$, K=20, Z=3, α=0.01, $\omega=10\nu_{ei}$

The temporal dependence of the same 6th EDF moment but for another α=0.01 is presented in Fig. 3. In this case the EDF is Maxwellian one both for slow and fast particles.

CONCLUSIONS

In conclusion, let us summarized the main results of the paper. General approach to Monte Carlo methods for Coulomb collisions is discussed. The approach is based on a special quasi-Maxwellian way of approximation of the LFP equations by Boltzmann equations. The DSMC numerical scheme is derived for the general case of multicomponent plasmas. The order of approximation is not worse than $O\left(\sqrt{\varepsilon}\right)$, where ε is a parameter of approximation being equivalent to the time step Δt.

DSMC method is tested for the plasma dynamics in the external high-frequency weak electrical field. The numerical simulation confirmed that the non-Maxwellian distribution function is composed of a super-Gaussian bulk of slow electrons and a Maxwellian tail of energetic particles. So there is a good agreement between DSMC method results and the theory developed in Refs. [13, 14].

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СТОХАСТИЧЕСКОЕ МОДЕЛИРОВАНИЕ НЕЛИНЕЙНОГО КИНЕТИЧЕСКОГО УРАВНЕНИЯ С ВЫСОКОЧАСТОТНЫМ ЭЛЕКТРОМАГНИТНЫМ ПОЛЕМ

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Предложен общий подход к моделированию кулоновских столкновений методом Монте-Карло. Основная идея заключается в аппроксимации системы уравнений Ландау-Фоккера-Планка (ЛФП) уравнениями Больцмана квазимаксвелловского вида. Также рассматриваются высокочастотные поля, и приводится сравнение с полученными ранее результатами.

СТОХАСТИЧНЕ МОДЕЛЮВАННЯ НЕЛІНІЙНИХ КІНЕТИЧНИХ РІВНЯНЬ С ВИСОКОЧАСТОТНИМ ЕЛЕКТРОМАГНІТНИМ ПОЛЕМ

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Запропоновано загальний підхід до моделювання кулонівських зіткнень методом Монте-Карло. Основна ідея полягає в апроксимації системи рівнянь Ландау-Фоккера-Планка (ОФП) рівняннями Больцмана квазімаксвеллівського виду. Також розглядаються високочастотні поля, і наводиться порівняння з отриманими раніше результатами.