# DYNAMICS OF IONS DURING DEVELOPMENT OF PARAMETRIC INSTABILITY OF LANGMUIR WAVES

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Nonlinear regimes of one-dimensional parametric instabilities of long-wave plasma waves are considered for the cases when the average field energy density is less (Zakharov's model) or greater (Silin's model) than the plasma thermal energy. The process of generation of short-wave plasma waves and perturbations of ion density is found to be similar in both cases. It is shown that the ion energy after the instability is saturated proves to be of the order of the ratio of linear growth rate to the frequency in the case when the initial field energy exceeds the plasma thermal energy. In the opposite case of hot plasma, the ions acquire a part of initial field energy equal to the ratio of initial field energy to the plasma thermal energy. The trajectory crossing of ions in the vicinity of density cavities is a reason of instability quenching in both cases.

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## INTRODUCTION

The interest in parametric instability of intense Langmuir waves, which can be easily excited in the plasma by various sources[1 - 9], was stipulated, in particular, by the new possibilities in heating of electrons and ions in plasma. The correct methods for description of parametric instability of long-waveplasma waves was developed in the pioneering works of V.P. Silin [10] and V.E. Zakharov [11]. In early one-dimensional numerical experiments on parametric decay of plasma oscillations [12], the theoretical concepts were confirmed [10] (see also [13, 14] and review [15]). However, the greatest interest has been expressed by experimenters in the mechanism of dissipation of wave energy discovered by V.E. Zakharov. The analytical studies, laboratory-based experiments and numerical simulations performed at an early stage of studying these phenomena have confirmed [16-18] the fact that in some cases a significant part of the pump field energy transfers during the instability development into the energy of short-wave Langmuir oscillations attended with bursts of fast particles [16 - 27].

In this paper, we attempt to compare the models of Silin and Zakharov by the example of one-dimensional description. The choice of one-dimensional approach, as was noted by J. Dawson [28], often keeps the main features of the processes, but simplify their description and leads to a fuller understanding of what the important phenomena are. The ion heating is of particular interest, so we use the kinetic description of ions in this work because of account of inertial effects can be significant at the nonlinear stage of the process [29].

## 1. GENERALIZED SILIN'S EQUATIONS

When the intensity of external long-wave field is much greater than the temperature of electrons in plasma $W = |E_0|^2/4\pi >> n_0 T_e$, it is reasonable to explore the approach presented by V.P. Silin [30]

$$\frac{\partial v_\alpha}{\partial t} + u_{0\alpha}\frac{\partial v_\alpha}{\partial x} - \frac{e_\alpha}{m_\alpha}E = -v_\alpha\frac{\partial v_\alpha}{\partial x}, \quad (1)$$

$$\frac{\partial n_\alpha}{\partial t} + u_{0\alpha}\frac{\partial n_\alpha}{\partial x} + n_{\alpha 0}\frac{\partial v_\alpha}{\partial x} = -\frac{\partial (n_\alpha v_\alpha)}{\partial x}, \quad (2)$$

$$\frac{\partial E}{\partial x} = 4\pi\sum_\beta e_\beta n_\beta. \quad (3)$$

Let set the wavelength of the external electric field infinite

$$E_0 = -i(|E_0|\exp\{i\omega_0 t + i\phi\} - -|E_0|\exp\{-i\omega_0 t - i\phi\})/2. \quad (4)$$

The particles oscillates under the action of this field with velocity $u_{0\alpha} = -(e_\alpha |\mathrm{E}_0|/m_\alpha\cdot\omega_0)\cos\Phi$.

Substituting into Eq.(1) the electric field obtained from Eq.(3) $E_n = -4\pi i e(n_{in} - n_{en})/k_0 n$, we find

$$\frac{\partial v_{\alpha n}}{\partial t} + u_{0\alpha}\cdot ik_0 n\cdot v_{\alpha n} + \frac{4\pi e_\alpha i}{k_0 n\cdot m_\alpha}\sum_\beta e_\beta\cdot n_{\beta m} = = -ik_0\sum_m m\cdot v_{\alpha n-m}\cdot v_{\alpha m} \quad (5)$$

Let use the following variables

$$\nu_{\alpha n} = e_\alpha\cdot n_{\alpha n}\cdot\exp\{-ia_{\alpha n}\cdot\sin\Phi\}, \quad (6)$$

$$\theta_{\alpha n} = v_{\alpha n}\cdot\exp\{-ia_{\alpha n}\cdot\sin\Phi\}, \quad (7)$$

$$a_{\alpha n} = ne_\alpha k_0 E_0/m_\alpha\cdot\omega_0^2, \quad (8)$$

where $\Phi = \omega_0 t + \phi$. After this, Eqs.(1)-(2) take the form

$$\frac{\partial \nu_{\alpha n}}{\partial t} + \theta_{\alpha n}\cdot ik_0 n\cdot e_\alpha n_{\alpha 0} = -ik_0\cdot n\cdot\sum_m \nu_{\alpha n-m}\cdot\theta_{\alpha m} \quad (9)$$

$$\frac{\partial \theta_{\alpha n}}{\partial t} + \frac{4\pi e_\alpha i}{k_0 n\cdot m_\alpha}\sum_\beta \nu_{\beta n}\cdot\exp\{i(a_{\beta n} - a_{\alpha n})\sin\Phi\} = = -ik_0\cdot\sum_m m\theta_{\alpha n-m}\cdot\theta_{\alpha m}. \quad (10)$$

It is clear that $a_{in} - a_{en} \approx n(ek_0 E_0/m_e\cdot\omega_0^2) = a_n$, where, $\omega_{pe} = \sqrt{4\pi e^2 n_0/m_e}$, $\Omega_i = \sqrt{4\pi e^2 n_0/M}$, $m_i \equiv M$ and $k_n = nk_0$ defines a set of wave numbers. Equations (9)-(10) for electrons becomes

$$\frac{\partial \nu_{en}}{\partial t} - \theta_{en}\cdot ik_0 n\cdot en_0 = -ik_0\cdot n\cdot\sum_m \nu_{en-m}\cdot\theta_{em} \quad (11)$$

$$\frac{\partial \theta_{en}}{\partial t} - \frac{4\pi ei}{k_0 n\cdot m_e}(\nu_{en} + \nu_{in}\cdot\exp\{ia_n\cdot\sin\Phi\}) = = -ik_0\cdot\sum_m m\theta_{en-m}\cdot\theta_{em}. \quad (12)$$

Then, we use

$$\nu_{en}=\sum_s u_n^{(s)}\cdot\exp\{is\omega_0\cdot t)=u_n^{(0)}+u_n^{(1)}\cdot e^{i\omega_0 t}+ +u_n^{(-1)}\cdot e^{-i\omega_0 t}+u_n^{(2)}\cdot e^{i2\omega_0 t}+u_n^{(-2)}\cdot e^{-i2\omega_0 t} \quad (13)$$

$$\theta_{en}=\sum_s v_n^{(s)}\cdot\exp\{is\omega_0\cdot t)=v_n^{(0)}+v_n^{(1)}\cdot e^{i\omega_0 t}+ +v_n^{(-1)}\cdot e^{-i\omega_0 t}+v_n^{(2)}\cdot e^{i2\omega_0 t}+v_n^{(-2)}\cdot e^{-i2\omega_0 t} \quad (14)$$

and well known expansion

$$\exp\{ia\cdot\sin\Phi\}=\sum_{m=-\infty}^{\infty}J_m(a)\cdot\exp\{im\Phi\}\,, \quad (15)$$

where $J_m(x)$ is the Bessel function, and $J_0(x)=J_0(-x)$, $J_1(x)=-J_1(-x)=J_{-1}(-x)$, $J_2(x)=J_{-2}(x)=J_2(-x)$ [31]. Below, we find the non-resonance terms for perturbation of density $u_n^{(0)},u_n^{(2)},u_n^{(-2)}$ and velocity $v_n^{(0)},v_n^{(2)},v_n^{(-2)}$ in the oscillating reference frame [32 - 34]:

$$v^{(0)}{}_n=\frac{k_0}{\omega_0}\sum_m(n-m)[v^{(+1)}{}_{n-m}\cdot v^{(-1)}{}_m-v^{(-1)}{}_{n-m}\cdot v^{(1)}{}_m)= =\frac{1}{i\omega_0}[\frac{\partial v^{(1)}}{\partial x}v^{(-1)}-\frac{\partial v^{(-1)}}{\partial x}v^{(1)}]_n, \quad (16)$$

$$u^{(0)}{}_n=-\nu_{in}\cdot J_0(a_n)+\frac{k_0{}^2n^2\cdot m_e}{4\pi e}\cdot\sum_m v^{(1)}{}_{n-m}\cdot v^{(-1)}{}_m= =-\nu_{in}\cdot J_0(a_n)-\frac{m_e}{4\pi e}\frac{\partial^2}{\partial x^2}[v^{(1)}v^{(-1)}]_n, \quad (17)$$

$$v^{(\pm2)}{}_n=\frac{\pm2\omega_0}{3k_0n\cdot en_0}\nu_{in}\cdot J_{\pm2}(a_n)e^{\pm2i\phi}\mp\frac{k_0}{\omega_0}\sum_m mv_{n-m}^{(\pm1)}\cdot v_m^{(\pm1)} =\frac{\pm2\omega_0}{3k_0n\cdot en_0}\nu_{in}\cdot J_{\pm2}(a_n)e^{\pm2i\phi}\mp\frac{1}{i\omega_0}[\frac{\partial v^{(\pm1)}}{\partial x}v^{(\pm1)}]_n, \quad (18)$$

$$u^{(\pm2)}{}_n=\frac{1}{3}\nu_{in}\cdot J_{\pm2}(a_n)e^{\pm2i\phi}-\frac{k_0{}^2n\cdot en_0}{\omega^2{}_{pe}}\sum_s sv_s^{(\pm1)}\cdot v_{n-s}^{(\pm1)} =\frac{1}{3}\nu_{in}\cdot J_{\pm2}(a_n)e^{\pm2i\phi}+\frac{en_0}{\omega^2{}_{pe}}\frac{\partial}{\partial x}[\frac{\partial v^{(\pm1)}}{\partial x}v^{(\pm1)}]_n. \quad (19)$$

The obtained equations should be supplemented by equations for resonant values

$$\pm2i\omega_0\{\frac{\partial u_n^{(\pm1)}}{\partial t}\mp i\Delta u_n^{(\pm1)}\mp\frac{i\nu_{in}\omega_{pe}^2J_{\pm1}(a_n)\cdot e^{\pm i\phi}}{2\omega_0}\} =k^2{}_0n\cdot en_0\sum_m m[v_{n-m}^{(0)}\cdot v_m^{(\pm1)}+v_{n-m}^{(\pm1)}\cdot v_m^{(0)}]- -ik_0\cdot n\cdot(\pm i\omega_0)\sum_m[u_{n-m}^{(0)}\cdot v_m^{(\pm1)}+u_{n-m}^{(\pm1)}\cdot v_m^{(0)}]+ +k_0n[en_0\sum_m mv_{n-m}^{(\mp1)}\cdot v_m^{(\pm2)}\pm\omega_0\sum_m[u_{n-m}^{(\pm2)}\cdot v_m^{(\mp1)}], \quad (20)$$

where $\Delta=(\omega_{pe}^2-\omega_0{}^2)/2\omega_0$. Authors of [32 - 34] have used the following representation $u_n^{(\pm1)}=\pm k_0n\cdot en_0v_n^{(\pm1)}/\omega_0=ik_0n\cdot E_n^{(\pm1)}/4\pi$. In this case, gathering in the r.h.s. of Eq.(20) the terms responsible for electronic non-linearity we rewrite this equation for short-wave perturbations as follows

$$\pm2i\omega_0\{\frac{\partial u_n^{(\pm1)}}{\partial t}\mp i\Delta u_n^{(\pm1)}\mp i\nu_{in}\cdot\frac{\omega_{pe}^2J_{\pm1}(a_n)\cdot e^{\pm i\phi}}{2\omega_0}\}+ \frac{\omega_0^2n}{en_0}\sum_m\frac{\nu_{in-m}}{m}\cdot[u_m^{(\mp1)}J_{\pm2}(a_{n-m})e^{\pm2i\phi}+u_m^{(\pm1)}J_0(a_{n-m})]= =(k_0n\cdot en_0/\omega_0)\cdot I. \quad (21)$$

Obviously, that the electronic non-linearity (r.h.s. of Eq. (21)) is equal to zero as well as in [35]. Than the Eq. (21) can be rewritten [32 - 34] as

$$\frac{\partial u_n^{(\pm1)}}{\partial t}\mp i\Delta u_n^{(\pm1)}\mp i\nu_{in}\cdot\frac{\omega_{pe}^2J_{\pm1}(a_n)\cdot e^{\pm i\phi}}{2\omega_0}= \pm i\frac{\omega_0n}{2en_0}\sum_m\frac{\nu_{in-m}}{m}\cdot[u_m^{(\mp1)}\cdot J_{\pm2}(a_{n-m})e^{\pm2i\phi}+ +u_m^{(\pm1)}\cdot J_0(a_{n-m})]. \quad (22)$$

If the electric field will be presented in the form [35]

$$E_n=\sum_s E_n^{(s)}\cdot\exp\{is\omega_0\cdot t)=\bar{E}_n+ +\frac{1}{2}(E_n^{(1)}\cdot e^{i\omega_0 t}+E_n^{(-1)}\cdot e^{-i\omega_0 t})+ +E_n^{(2)}\cdot e^{i2\omega_0 t}+E_n^{(-2)}\cdot e^{-i2\omega_0 t}, \quad (23)$$

than $E_n^{(\pm1)}\to E_n^{(\pm1)}/2=-4\pi iu_n^{(\pm1)}/k_0n$, and the Eq.(22) takes the form

$$\frac{\partial E_n^{(\pm1)}}{\partial t}\mp i\Delta E_n^{(\pm1)}\mp\frac{8\pi\omega_{pe}\nu_{in}}{2k_0n}J_{\pm1}(a_n)\cdot\exp(\pm i\phi)= \pm i\frac{\omega_0}{2en_0}\sum_m\nu_{in-m}\cdot[E_m^{(\mp1)}\cdot J_{\pm2}(a_{n-m})e^{\pm2i\phi}+ +E_m^{(\pm1)}\cdot J_0(a_{n-m})]. \quad (24)$$

Going to the representation of pumping wave $E_0$ corresponding to fixed velocity of oscillations $u_{0\alpha}=-(e_\alpha E_0/m_\alpha\cdot\omega_0)\cdot\cos\Phi$, we find $E_0\to-iE_0$ and $E_0^*\to iE_0^*$. The equation for $E_0$ can be written as [32 - 34]

$$\frac{\partial E_0}{\partial t}=\frac{8\pi i\omega_0}{2en_0k_0}\sum_m\frac{\nu_{i,-m}}{m}\cdot[u_m^{(-1)}\cdot J_2(a_{-m})e^{2i\phi}+ +u_m^{(+1)}\cdot J_0(a_{-m})] \quad (25)$$

or expressing the perturbations of density through the components of electric field

$$\frac{\partial E_0}{\partial t}=-\frac{\omega_0}{2en_0}\sum_m\nu_{i,-m}\cdot[E_m^{(-1)}\cdot J_2(a_m)e^{2i\phi}+ +E_m^{(+1)}\cdot J_0(a_m)]. \quad (26)$$

The slowly varying in time electric field [30]

$$\bar{E}_n=(-\frac{4\pi i}{k_0n})(\langle\{\nu_{en}\cdot\exp\{-ia_n\cdot\sin\Phi\}\rangle+\nu_{in})= =[u_n^{(1)}J_1(a_n)\cdot e^{-i\phi}+u_n^{(-1)}\cdot J_{-1}(a_n)\cdot e^{i\phi}]- -\frac{n^2J_0(a_n)}{en_0}\sum_m\frac{u_{n-m}^{(1)}\cdot u_m^{(-1)}}{(n-m)m}- -\frac{nJ_2(a_n)}{en_0}\cdot\sum_m\frac{1}{m}[u_{n-m}^{(1)}\cdot u_m^{(1)}e^{-2i\phi}+u_{n-m}^{(-1)}\cdot u_m^{(-1)}e^{2i\phi}]\}, \quad (27)$$

can be represented in another form

$$\bar{E}_n=\frac{1}{2}[E_n^{(1)}J_1(a_n)\cdot e^{-i\phi}+E_n^{(-1)}\cdot J_{-1}(a_n)\cdot e^{i\phi}]- -\frac{ink_0}{16\pi en_0}J_0(a_n)\sum_m E_{n-m}^{(1)}\cdot E_m^{(-1)}- -\frac{ik_0J_2(a_n)}{16\pi en_0}\sum_m(n-m)[E_{n-m}^{(1)}\cdot E_m^{(1)}e^{-2i\phi}+ +E_{n-m}^{(-1)}\cdot E_m^{(-1)}{}_m e^{2i\phi}], \quad (28)$$

that permits description of ions by particle-in-cell method. Their equations of motion can be written as follows

$$\frac{d^2x_s}{dt^2}=\frac{e}{M}\sum_n \overline{E}_n\cdot\exp\{ik_0 n x_s\}, \qquad (29)$$

and the ion density can be defined as

$$n_{in}=n_0\cdot\frac{k_0}{2\pi}\int_{-\pi/k_0}^{\pi/k_0}\exp[-ink_0\cdot x_s(x_0,t)]\cdot dx_{s0}. \qquad (30)$$

Note that the use of particle-in-cell method for description of ion dynamics allows, what is more, to improve the computational stability [29].

The use of Eqs. (9) - (10), where the right-hand sides can be neglected in view of their smallness, permits the hydro dynamical description of ions. The equation for ion density at this takes the form [32 - 34]:

$$\begin{aligned}\frac{\partial^2\nu_{in}}{\partial t^2}&=-\Omega_i^2\{\nu_{in}[1-J_0^2(a_n)+\frac{2}{3}J_2{}^2(a_n)]+\\&+[u_n^{(1)}J_1(a_n)\cdot e^{-i\phi}+u_n^{(-1)}\cdot J_{-1}(a_n)\cdot e^{i\phi}]-\\&-\frac{n^2J_0(a_n)}{en_0}\sum_m\frac{u_{n-m}^{(1)}\cdot u_m^{(-1)}}{(n-m)m}-\\&-\frac{nJ_2(a_n)}{en_0}\cdot\sum_m\frac{1}{m}[u_{n-m}^{(1)}\cdot u_m^{(1)}e^{-2i\phi}+u_{n-m}^{(-1)}\cdot u_m^{(-1)}e^{2i\phi}]\}.\end{aligned} \qquad (31)$$

Or after neglecting small terms

$$\begin{aligned}\frac{\partial^2\nu_{in}}{\partial t^2}&=\frac{ik_0n}{8\pi}[E_n^{(1)}J_1(a_n)e^{-i\phi}+E_n^{(-1)}J_{-1}(a_n)\cdot e^{i\phi}]-\\&+\frac{n^2k_0^2}{64\pi^2en_0}\sum_m J_0(a_n)\cdot E^{(1)}{}_{n-m}\cdot E^{(-1)}{}_m+\\&+\frac{nk_0^2J_2(a_n)}{64\pi^2en_0}\cdot\sum_m(n-m)[E_{n-m}^{(1)}\cdot E_m^{(1)}e^{-2i\phi}+\\&+E_{n-m}^{(-1)}\cdot E_m^{(-1)}e^{2i\phi}]\}.\end{aligned} \qquad (32)$$

One can verify that the complex conjugate of Eq.(3.24) with the lower sign becomes (the dummy index in sums can be inverted $m\to -m$)

$$\begin{aligned}&\frac{\partial E_{-n}^{(-1)*}}{\partial t}-i\Delta E_{-n}^{(-1)*}-\frac{4\pi\omega_{pe}\nu_{i,-n}^*J_1(a_n)}{k_0n}\cdot e^{i\phi}-\\&-i\frac{\omega_0}{2en_0}\sum_m\nu_{i,m-n}^*\cdot[(E_{-m}^{(1)*}\cdot J_{-2}(a_{m-n})e^{2i\phi}+\\&+E_{-m}^{(-1)*}\cdot J_0(a_{m-n})]=0.\end{aligned} \qquad (33)$$

At the same time this equation for positive indexes can be written as

$$\begin{aligned}&\frac{\partial E^{(1)}{}_n}{\partial t}-i\Delta E^{(1)}{}_n-\frac{4\pi\omega_{pe}\nu_{in}}{k_0n}J_{\pm1}(a_n)\cdot e^{i\phi}]\}-\\&-i\frac{\omega_0}{2en_0}\sum_m\nu_{in-m}\cdot[E_m{}^{(-1)}\cdot J_2(a_{n-m})e^{2i\phi}+\\&+E_m{}^{(1)}\cdot J_0(a_{n-m})]=0.\end{aligned} \qquad (34)$$

a) It is clear that for $E_{-n}^{(-1)}=(E_n^{(1)})*$ and $\nu_{i,-n}=(\nu_{i,n})*$ Eqs. (3.31) and (3.32) are identical. Just as, it is easy to verify that $E_n^{(-1)}=(E_{-n}^{(1)})*$ and $\nu_{i,n}=(\nu_{i,-n})*$, i.e. the perturbations of ion charges possess the symmetry $n_{i,-n}=(n_{i,n})*$. At this, for correct description of the instability it is sufficiently to use the high-frequency components $E_n^{(1)}$, $E_{-n}^{(1)}$ and $E_0^{(1)}$, as well as perturbations of ion charge $\nu_{i,n}$ for positive values of index $n$. Other variables can be expressed through them. So, we can stop using the superscript. In this case we can rewrite Eqs. (3.23), (3.28)

$$\begin{aligned}&\frac{\partial E_n}{\partial t}-i\Delta E_n-\frac{4\pi\omega_{pe}\nu_{in}}{k_0n}J_1(a_n)\cdot e^{i\phi}-\\&-i\frac{\omega_0}{2en_0}\sum_m\nu_{in-m}\cdot[E_{-m}^*J_2(a_{n-m})e^{2i\phi}+\\&+E_m\cdot J_0(a_{n-m})]=0,\end{aligned} \qquad (35)$$

$$\begin{aligned}\frac{\partial^2\nu_{in}}{\partial t^2}&=-\Omega_i^2\{\nu_{in}[1-J_0^2(a_n)+\frac{2}{3}J_2{}^2(a_n)]\\&+\frac{ik_0n}{8\pi}J_1(a_n)[E_n\cdot e^{-i\phi}-E_{-n}^*\cdot e^{i\phi}]-\\&+\frac{n^2k_0^2}{64\pi^2en_0}\sum_m J_0(a_n)\cdot E_{n-m}\cdot E_{-m}^*+\\&+\frac{nk_0^2J_2(a_n)}{64\pi^2en_0}\cdot\sum_m(n-m)[E_{n-m}\cdot E_m\cdot e^{-2i\phi}+\\&+E_{m-n}^*\cdot E_{-m}^*e^{2i\phi}]\}.\end{aligned} \qquad (36)$$

In addition, when the particle-in-cell method is used, one can use the motion equation (30) and expression for ion density (31)with slowly varying electric field

$$\begin{aligned}\overline{E}_n&=(-\frac{4\pi i}{k_0n})\nu_{in}[1-J_0^2(a_n)+\frac{2}{3}J_2{}^2(a_n)]+\\&+\frac{1}{2}J_1(a_n)[E_n\cdot e^{-i\phi}-E_{-n}^*\cdot e^{i\phi}]-\\&-\frac{ink_0}{16\pi en_0}J_0(a_n)\sum_m E_{n-m}\cdot E_{-m}^*-\\&-\frac{ik_0J_2(a_n)}{16\pi en_0}\sum_m(n-m)[E_{n-m}\cdot E_m\cdot e^{-2i\phi}+\\&+E_{m-n}^*\cdot E_{-m}^*e^{2i\phi}].\end{aligned} \qquad (37)$$

These equations should be supplemented by the equation for pumping field $E_0$

$$\begin{aligned}\frac{\partial E_0}{\partial t}&=-\frac{\omega_0}{2en_0}\sum_m\nu_{i,-m}\cdot[E_{-m}^*\cdot J_2(a_m)e^{2i\phi}+\\&+E_m\cdot J_0(a_m)].\end{aligned} \qquad (38)$$

Note that the values corresponding to subscripts with different signs are independent that results in spatial distortion of integral perturbations not only owing to variation of amplitudes but also because of spatial displacement of different components of the wave packet.

b) In the case $n_{i,-n}=n_{i,n}=(n_{i,n})*$, i.e. when the perturbations of ion density don't change their location, the high-frequency electric field also remains spatially symmetrical $E_n^{(1)}=E_{-n}^{(1)}=(E_n^{(-1)})*=(E_{-n}^{(-1)})*$. Then, the variables $E_n^{(1)}$ and $n_{i,n}$ are sufficient for description of the process, that is stipulated by strong relation between values with different signs. The structure of the field and density in this case, such as in the case of Zakharov's model (that will be presented later), represents the motionless spatial formation which amplitude increases and half-width decreases, at least in some region.

c) Growing in time perturbations of ion density of the type $n_{i,-n}=-(n_{i,n})*$ are not realized.

## 2. ZAKHAROV'S EQUATIONS

When $a_n << 1$ and $J_1(a_n) \approx a_n/2$, $J_0(a_n) \approx 1$, $J_2(a_n) \approx a_n^2/8$, the equations (35) - (38) are identical to equations derived in [35] under condition [37] $W = |E_0|^2/4\pi << n_0 T_e$ within the detuning $(\omega_{pe}^2 - \omega_0{}^2)/2\omega_0 \to (\omega_{pe}^2 - \omega_0{}^2 + k^2{}_0 n^2 v^2{}_{Te})/2\omega_0$ and replacement $E_0 \to -iE_0$ and $E_0^* \to iE_0^*$.

$$\frac{\partial E_n}{\partial t} - i\frac{\omega_{pe}^2 - \omega_0{}^2 + k^2{}_0 n^2 v^2{}_{Te}}{2\omega_0} E_n - \quad (39)$$

$$-i\frac{\omega_0}{2n_0}\cdot\{n_{in}E_0 + \sum_{m\neq 0} n_{in-m}E_m\} = 0$$

$$\frac{\partial^2 n_{in}}{\partial t^2} = -\frac{k_0^2 n^2}{16\pi M}\{E_n E_0^* + E_0 E_{-n}^* + \sum_{m\neq 0,n} E_{n-m}E_{-m}^*\}. \quad (40)$$

We also give the expression for slowly varying electric field with account of the pump wave

$$\bar{E}_n = -\frac{ik_0 ne}{4m\omega_p{}^2}(E_n E_0^* + E_0 E_{-n}^* + \quad (41)$$

$$+\sum_{m\neq 0,n} E_{n-m}E_{-m}^*),$$

that makes possible the description of ions with the use of particle-in-cell method and Eqs. (28) and (29). The amplitude of the pump wave $E_0$ can be found from equation

$$\frac{\partial E_0}{\partial t} - i\frac{\omega_0}{2n_0}\cdot\sum_m n_{i,-m}E_m = 0. \quad (42)$$

## 3. LINEAR THEORY

We restrict our consideration to the most interesting case of the long-wave pumping. The dispersion equation for the high-temperature case in supersonic limit $\partial^2 n_{in}/n_{in}\partial t^2 >> k_0^2 c_s^2 n^2$ follows from linear approximation of Zakharov's equations (2.32) and (2.33) with the use of representation $E^{-1}\partial E/\partial t = i\Omega$:

$$-\Omega^2\{\Omega^2 - \Delta^2\} + \Delta\cdot A = 0. \quad (43)$$

In Zakharov's model, the normalized to the Langmuir frequency correction $\delta = \Omega/\omega_{pe}$, in general, should be written in the form

$$\delta^2 = \frac{\Delta^2}{2} \pm \sqrt{\frac{\Delta^4}{4} + B\Delta^2}, \quad (44)$$

where

$$B = \frac{1}{2}\frac{m_e}{M}\frac{|E_0|^2}{4\pi n_0 T_e}. \quad (45)$$

Since the value $(\Delta^4 + 4B\Delta^2)^{1/2} - \Delta^2$ increases monotonically with $\Delta$, having no a distinct maximum, the instability increment for small $\Delta^2 << B$, $\delta^2 \approx -\Delta\sqrt{B}$ and $|\delta^2| < B$ is equal

$$\mathrm{Im}\,\Omega = |\Omega| \approx \left(\frac{k_0^2 n^2 v^2{}_{Te}}{2\omega_{pe}^2}\right)^{1/2}\left(\frac{1}{2}\frac{|E_0|^2}{4\pi n_0 T_e}\frac{m_e}{M}\right)^{1/4}\omega_{pe}. \quad (46)$$

For the case of large $\Delta^2 >> B$, $\Omega^2 \approx -B$, it has the form

$$\mathrm{Im}\,\Omega = |\Omega| \approx \left(\frac{1}{2}\frac{|E_0|^2}{4\pi n_0 T_e}\frac{m_e}{M}\right)^{1/2}\omega_{pe}. \quad (47)$$

This means that the increment increases with the wave-number of perturbations, reaching the maximum (47).

In Silin's model, the growth rate normalized to the plasma frequency reaches the value

$$\delta = \pm\frac{i}{\sqrt[3]{2}}\cdot A^{1/3} = \pm\frac{i}{\sqrt[3]{2}}\left(\frac{m_e}{M}\right)^{1/3} J_1^{2/3}(a_n) \quad (48)$$

for the detuning value $\Delta^3 = A/2$ or which the same for $\Delta = (m_e/2M)^{1/3} J_1^{2/3}(a_{n_m})$

The perturbations with wave-number $k_m = k_0 n_m$ for which $a_{n_m} = 1.84$, the Bessel function has a maximum and the growth rate reaches the maximal value

$$\delta_{\max} = \pm 0.44 i\left(\frac{m_e}{M}\right)^{1/3}. \quad (49)$$

With the development of the instability, the pump wave amplitude decreases and the increment maximum moves to shorter wavelengths.

It is significant that the values of the maximal growth rate of parametric instability increase with decreasing of perturbation amplitudes. Moreover, Zakharov's model shows that decrease in the amplitude of the pump field results in decrease in the growth rates within the entire unstable region, while Silin's model demonstrates that similar process shifts the maximum growth rate in the short-wave band, without reducing its value (49). Thus, the process of energy transfer to the short-wave part of the spectrum in the two models is largely determined by the linear mechanism of perturbation growth.

## 4. SIMULATION RESULTS

Consider the case when ion density perturbations are spatially symmetric and $\Phi_n = 0$. In addition, there is no spatial shift between perturbations of different scales during instability development. It can be shown that the energy transfer from long-wave plasma wave to electrons and ions will be most effective in this case. For low initial amplitude values (low noise level), the main energy of the growing instability spectrum have been concentrated in short-wave region at the close of the linear stage of the process. The perturbations in this spectrum region have a maximum growth rate and have significantly kept ahead the neighbor modes during the linear stage of the instability development. The range of instability is found to berelativelynarrow. Therefore, severallong-livedsmall-scaledensity perturbations may arise on the length of the pumping wave. These cavities arise during all linear stage of the process due to phase synchronization of high-frequency modes. The phenomenon of phase synchronization of growing modes at the linear stage of the process was mentioned in earlier works [33]. For large initial mode amplitude RF spectrum(i.e. at high levels of Langmuir noise), the number of density cavities arising on the length of the pumping wave decreases to only one or two [29]. Herewith, the spectra of the high-frequency field perturbations and ion density perturbations broaden.

We used the following parameters for simulation. The number of simulated ion particles $0 < s \le S = 2.500...5000$, $-N < n < N$, $N = 50...100$, $a_0(0) = ek_0 E_0(0) / m_e \omega_{pe}^2 = 0.06$, $\xi_s = k_0 x_s / 2\pi$, $\tau = \delta t$, $d\xi_s / d\tau |_{\tau=0} = v_s |_{\tau=0} = 0$, $ek_0 |n| e_n(0) / m_e \omega_{pe}^2 = 0.0001...0.0025$, $\Delta = 1$, $m_e / M = 10^{-3}$ and $W / n_0 T_e = 0.1$ for Zakharov's model.

The ions acquire kinetic energy in the potential wells of the secaverns. At the nonlinear stage of the instability, the trajectories of ions cross each other, the ion density perturbations are smoothed out and their amplitude increases. Relationship between ionic perturbations andhigh-frequency field is weakened and the instability saturates. The amplitude of the pumping wave is stabilized after some oscillation at a rather low level (Fig. 1).

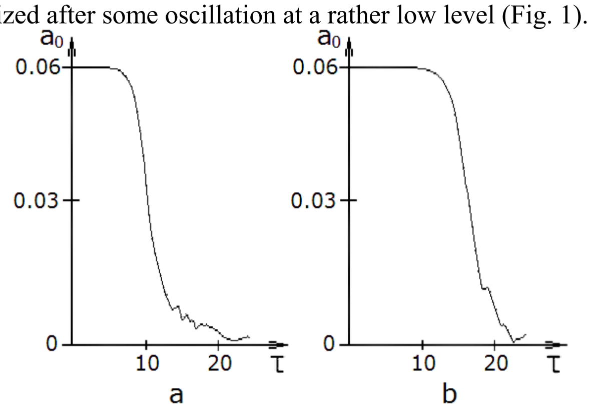


*Fig. 1. Evolution of amplitude of the pumping wave (a – Zakharov's model; b – Silin's model)*

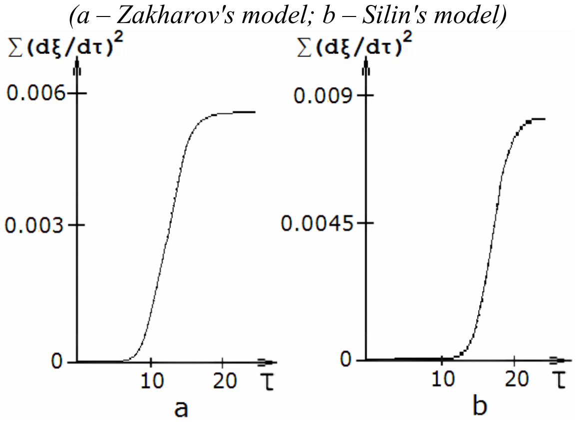


*Fig. 2. Evolution of the value $\sum_s (d\xi_s / dt)^2$ (a – Zakharov's model; b – Silin's model)*

The main energy is now contained in the short-wave Langmuir spectrum. Some small part of the initial energy is converted into kinetic energy of the ions (Fig. 2).

The kinetic energy of ions positioned on a length of the pumping wave can be expressed through the estimated value of the sum of the squared dimensionless velocity $I = \sum_s (d\xi_s / d\tau)^2$ and the number of simulated particles $S$

$$\frac{2\pi}{k_0} \cdot \frac{1}{2} n_0 M \langle \left( \frac{dx_s}{dt} \right)^2 \rangle = I \cdot \frac{4\pi^2 \delta^2 M n_0}{2 k_0{}^2 S} \frac{2\pi}{k_0}, \qquad (50)$$

where $\langle (dx_s/dt)^2 \rangle$ is the ensemble average. The ratio of the ion kinetic energy to the initial energy of intense long-wavelength Langmuir waves can be written as

$$\frac{2\pi}{k_0} [n_0 M \langle \left( \frac{dx_s}{dt} \right)^2 \rangle ] / \{ (|E_0|^2 / 4\pi) \cdot \frac{2\pi}{k_0} \} = \\ = \frac{E_{kin}}{W_0} = 4\pi^2 \frac{I}{a_0{}^2 S} \frac{M}{m} \delta^2, \qquad (51)$$

where $E_{kin}$ is the density of the ion kinetic energy, $W_0 = |E_0|^2 / 4\pi$ is the initial energy density of long-wavelength Langmuir waves. When the ion density perturbations are spatially symmetric and $\Phi_n = 0$, the simulation shows that the maximum possible value $I_s \approx 1.2 \cdot 10^{-2}$ for $S = 5000$ and $I_s \approx 4,5 \cdot 10^{-1}$ for $S = 2.500$ are reached during the instability development for Silin's and Zakharov's models correspondingly. The ratio of time scales for these two models is equal to $1.6 (m_e / M)^{1/6} (W / n_0 T_e)^{1/2} = 0.16$. Taking this into account, it is easy to see the energy of ions are of the same order in both models.

The ratio of ion kinetic energy to the initial energy of long-wave oscillations occurs equal to $E_{kin} / W_0 \approx 3.6 \cdot 10^{-3} (M / m_e)^{1/3}$ for Silin's model and $E_{kin} / W_0 \approx 1.2 \cdot 10^{-2} W_0 / n_0 T_e$ for Zakharov's model. This means that in Silin's model the ions derive a portion of field energy of the order of $\delta_{max}$. This effect was predicted in [19] and confirmed in [29]. A portion of transferred energy in Zakharov's model is of the order of $W_0 / n_0 T_e$.

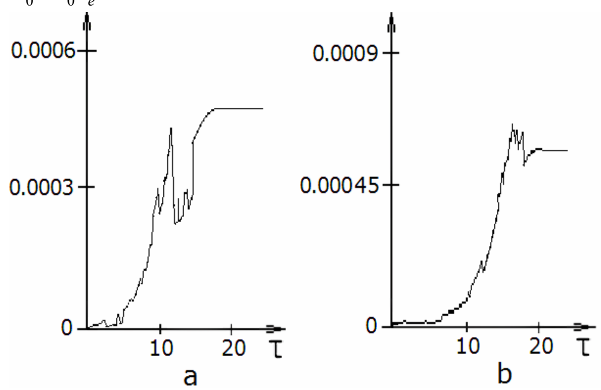


*Fig. 3. Evolution of speed distribution function half-width for simulated particles (a – Zakharov's model; b – Silin's model)*

In Zakharov's model only a fraction of a percent of the initial energy can be transferred to the ions during development of the long-wave parametric instability of plasma waves. Nevertheless, the stabilization of the instability and its saturation are mostly determined by the trapping of ions to the potential wells of caverns in both models. During the trapping, the mixing of ions and the destruction of cavities occur that evidenced by the sharp increase in the ion density.

If the speed distribution of ions was Maxwellian, the half-width of such distribution (Fig. 3) would have been associated with the thermal velocity by the relation $\bar{v} = 1,18 v_T$.

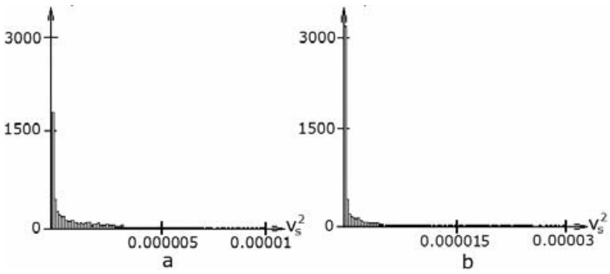


*Fig. 4. Distribution function of the value* $(d\xi_s / dt)^2$
*(a – Zakharov's model; b – Silin's model)*

However, the simulation shows that the half-width reaches at the nonlinear stage of the process the value of $\bar{v} = 0,005$ for Zakharov's model and $\bar{v} = 0,006$ for Silin's model. In addition, if the speed distribution of ions was Maxwellian, the value of $I_M = S \cdot v_T^2 \approx 0.72 S \cdot \bar{v}^2$, will be of the order of $0,22$ for Zaharov's model and $0,73 \cdot 10^{-3}$ for Silin's model, while the simulation gives the value two times greater in Zakharov's model and an order greater in Silin's model. In the hot plasma the speed distribution of ions is close to the normal and one can say about the ion temperature. In Silin's model the difference in more than 15 times is caused by the existence of a large group of fast ions (Fig. 4) that was observed in the experiments [36].

In conclusion, we note that the process of parametric instability for Silin's and Zakharov's models are similar mostly due to the similarity of the systems of equations [37].

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## ДИНАМИКА ИОНОВ ПРИ РАЗВИТИИ ПАРАМЕТРИЧЕСКОЙ НЕУСТОЙЧИВОСТИ ЛЕНГМЮРОВСКИХ ВОЛН

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Рассмотрены нелинейные режимы развития одномерных параметрических неустойчивостей длинноволновых ленгмюровских волн в случаях, когда энергия поля меньше (модель Захарова) и больше (модель Силина) тепловой энергии плазмы. Процесс генерации коротковолнового спектра плазменных волн и возмущений ионной плотности оказывается подобным в обеих моделях описания параметрических неустойчивостей. Показано, что энергия ионов при насыщении неустойчивостей оказывается порядка отношения линейного инкремента к частоте в случае, когда начальная энергия поля заметно превышает тепловую энергию плазмы. В условиях горячей плазмы ионам передается доля энергии, равная половине отношения начальной энергии поля к тепловой энергии плазмы. Пересечение траекторий ионов вблизи каверн плотности является причиной срыва неустойчивости в обоих случаях.

## ДИНАМІКА ІОНІВ ПРИ РОЗВИТКУ ПАРАМЕТРИЧНОЇ НЕСТІЙКОСТІ ЛЕНГМЮРІВСЬКИХ ХВИЛЬ

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Розглянуто нелінійні режими розвитку одновимірних параметричних нестійкостей довгохвильових ленгмюрівських хвиль у випадках, коли енергія поля менша (модель Захарова) і більша (модель Сіліна) за теплову енергію плазми. Процес генерації короткохвильового спектра плазмових хвиль і збурень іонної густини виявляється подібним в обох моделях опису параметричних нестійкостей. Показано, що енергія іонів при насиченні нестійкостей виявляється дорівнює за порядком відношенню лінійного інкремента до частоти у випадку, коли початкова енергія поля помітно перевищує теплову енергію плазми. В умовах гарячої плазми іонам передається частка енергії, що дорівнює половині відношення початкової енергії поля до теплової енергії плазми. Перетин траєкторій іонів поблизу каверн густини є причиною зриву нестійкості в обох випадках.