THE INFLUENCE OF THE DYNAMICS FEATURES OF THE TRAPPED PARTICLES ON A SPECTRUM OF THEIR OSCILLATIONS

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Some features of dynamics of ensembles of coupled linear and nonlinear oscillators are investigated. It is shown that spectral characteristics of this dynamics essentially depend not only on quantity of the oscillators making the ensemble, but also on features of motion of each oscillator. In particular, at chaotic motion – on the moments of their chaotic dynamics. Special attention is turned on appearance of LF dynamics of ensemble. In this case there is a possibility of an effective exchange of energy between HF and LF oscillations.

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INTRODUCTION

It is known that spectral characteristics of ensemble of oscillators can essentially differ from spectral characteristics of separate oscillator. In particular, the fact of appearance of normal frequencies and normal modes at system of the coupled linear oscillator is widely known. Moreover, the problem of finding of normal modes of the various complex distributed systems is one of the most important and complex problems which exists in the theory of oscillations – especially in radiophysics and in hydrodynamics. Importance of finding of normal modes and normal frequencies is caused, first of all, with possibility to describe linear dynamics of complex distributed systems in space and in time of systems with their help. Moreover, they also allow to describe nonlinear dynamics of such systems in many cases. Besides that, exactly normal frequencies are those frequencies which the considered system answers resonant responses.

The special role in a spectrum of ensemble of oscillator is played by low-frequency components. Such appearance can lead to linear and nonlinear interaction between low-frequency (LF) and high-frequency (HF) modes. Such interaction allows to organize an effective exchange between these oscillations. In particular, such interaction allows to transform energy of LF oscillations to energy of HF-oscillations. Besides, such coupling leads to appearance of regimes with chaotic dynamics [1 - 4]. It can lead to saturation of level of oscillations exited by the electronic beam [5]. In particular, to saturation of level of exited oscillations at plasma-beam instability.

The spectral characteristics of ensemble of the charged particles can be useful to diagnostics of function of distribution of the charged particles. Such information is useful, for example, for analysis the bunches of the charged particles on an accelerators output, and also on an output of generators, for example, after transmission of the charged particles through undulator in LSE.

In the paper the results of investigation of dynamics of system of the coupled linear and nonlinear oscillator are presented. The dynamics of ensemble of nonlinear oscillator is stochastic. Stochasticity is caused by existence of coupling between oscillators, or by influence of external forces.

In the second section it is shown that the system of a large number of weakly-coupled linear oscillators can have normal frequencies which are hundreds times less, than partial frequencies of separate oscillator. If to impact on such system of oscillator with the external low-frequency signal which frequency coincides with this normal frequency, energy of this low-frequency signal will be transformed to energy of oscillator. If to break coupling between oscillator after obtaining sufficient energy from an external source, they will oscillate on the partial frequencies. Amplitudes of these partial frequencies will be almost considerably large, than their initial amplitudes. Such scenario allows to transform the energy of LF oscillations in to the energy of HF-oscillations.

One of important results is the proof (section 3) that with increasing of number of nonlinear oscillators the dynamics of their centre shift becomes more and more regular, and the spectrum is displaced in low-frequency area. And the shift size is defined by the moments of distribution function.

In conclusion the obtained results are discussed.

1. REGULAR DYNAMICS OF THE ENSEMBLE OF THE COUPLED LINEAR OSCILLATORS

Essential changing of dynamics of ensemble of oscillator in comparing with dynamics of a separate oscillator can be seen even for ensemble of coupled linear oscillators. Really, it is known that the system of the coupled linear oscillator has a set of normal frequencies which differ from partial frequencies of separate oscillator. The important feature of normal frequencies is that fact that the minimum of them is much less then all partial frequencies, and the maximum – is much more then all partial frequencies. And, the minimum normal frequency can be very small. In this case, influencing such ensemble of the coupled oscillator with the external low-frequency signal which frequency is close to the minimum normal frequency, it is possible to strengthen oscillations of such ensemble resonantly. However it is necessary to keep in mind that all coupled oscillator at such excitement will oscillate too on frequency considerably smaller then their partial frequencies. However, if at some point of time we will break the coupling, the energy of oscillator received from external excitation, will pass to the energy of oscillations of separate oscillator on their partial frequencies. These frequencies (partial) can be much higher than frequency of external excitation. Thus, in such scheme of excitation the strengthening of oscillations of high-frequency oscillator due to the energy of an external low-frequency signal is possible. Let's illustrate such possibility on an example of a large number of the coupled, identical, linear oscillator.

We have a system with a Hamiltonian:

$$H = \sum_{i=0}^{N} \left(\frac{p_i^2}{2} + \omega_0^2 \frac{q_i^2}{2} \right) + \mu \cdot q_0 \cdot \sum_{j=1}^{N} q_j \,, \qquad (1)$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}, \; \dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \mu = const \,,$$

$$\frac{\partial H}{\partial t} = 0 \,, \; H = const \,. \qquad (2)$$

This system represents the N coupled linear oscillators. As the coefficient of coupling and natural frequencies (partial) do not depend on time, the Hamiltonian is continuous function (the energy of system should be constant). From (1) it is easy to obtain system of the equations for oscillator:

$$\begin{aligned} &\dot{q}_i = p_i \,, \quad \dot{p}_i = -\omega_0^2 q_i - \mu q_0 \,, \\ &\dot{p}_0 = -\omega_0^2 q_0 - \mu \sum_{i=1}^{N} q_i \,, \\ &\ddot{p}_i + \omega_0^2 p_i = -\mu \cdot p_0 \,, \\ &\ddot{p}_0 + \omega_0^2 p_0 = -\mu \cdot \sum_{i=1}^{N} p_i . \end{aligned} \qquad (3)$$

The system (3) describes the N coupled linear oscillator. And for simplicity we consider system in which all oscillators are coupled with each other only across the zero oscillator. It is easy to find the normal frequencies of such system. For this purpose we will solve system (3) in such form:

$$p_i = a_i \exp\left(i \cdot \omega \cdot t \right), \; a_i = const \,.$$

Substituting this solution in (3), it is easy to receive the dispersive equation:

$$\left(-\omega^2 + \omega_0^2 \right)^2 = \mu^2 N. \qquad (4)$$

The equation (4) is solved elementary:

$$\omega = \pm \omega_0 \sqrt{1 \pm \mu \cdot \frac{\sqrt{N}}{\omega_0^2}} \,. \qquad (5)$$

Signs + and – in equation (5) before the root and under the root are independent. It is seen, what even at very small coupling coefficient, but at a large number of the oscillator, one of normal modes can be very small (for the case with minus sign under a root).

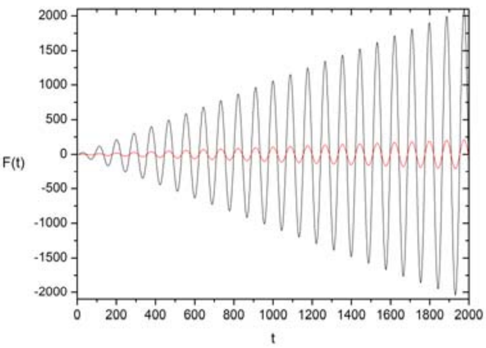

Fig. 1. The dynamics of oscillators with external periodic force

If now the system of oscillator (3) is exited by external periodic force with frequency which is equal to this minimum normal frequency of ensemble, then oscillations of ensemble will increase under the linear law in time. As an example we have taken 100 oscillator. As a result, the minimum normal frequency was 100 times less, than partial frequencies. In the Figs. 1, 2 given below the increasing of amplitudes of oscillations of separate oscillator is seen. Oscillations only two of them are presented. They differ only with initial conditions.

If now at some point of time (it is defined by existence of damping, and, respectively, saturation of strengthened oscillations) we break coupling, frequency of oscillator becomes partial frequency. Amplitudes of oscillator, of course, considerably will fall, but they will oscillate on much higher frequency, than frequency of external excitation. The amplitude of these partial frequencies will essentially exceed initial amplitudes. The illustration of this fact is on figure given below.

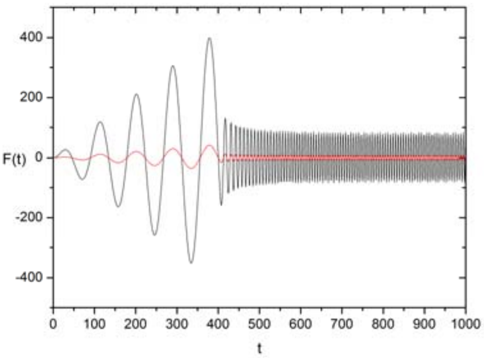

Fig. 2. The dynamics of oscillators after breaking coupling

The important feature of dynamics of such ensemble of oscillators is that fact that after braking the coupling all of them oscillate in one phase. And it is possible to organize coherent radiation.

2. COMPLEX, NONREGULAR DYNAMICS OF THE SYSTEM OF COUPLED NONLINEAR OSCILLATORS

Above we have considered rather simple system – the system consisting of linear coupled oscillators. Already this simple example has shown the essential difference in dynamics of separate oscillator from dynamics of the whole ensemble of such oscillators. The real dynamics of the charged particles which are trapped, for example, by the field of electromagnetic wave, will be nonlinear. Moreover, as a result of action on this dynamics with the external regular or random forces or as a result of interaction between these oscillators, will be chaotic in most cases. There is a question. What features of dynamics of such ensemble can be observed in this case? Below we will pay attention to the case of chaotic dynamics of separate nonlinear oscillator. Let's show that such dynamics of ensemble of noninteracting oscillator with chaotic behavior has spectral characteristics which can essentially differ from spectral characteristics of separate oscillator.

Let's consider the system representing ensemble of coupled N nonlinear oscillator with external regular periodic force. The Hamiltonian of such system can be presented in following form:

$$H = \sum_{j} \sum_{i=1}^{N} \left[\frac{\dot{x}_i^2}{2} + \Phi\left(x_i\right) + G\left(x_i, x_j\right) - \varepsilon\left(\tau\right) \cdot x_i \right] . \qquad (6)$$

For Hamiltonian (6) corresponds the following system of the equations of second order:

$$\ddot{x}_i = F_0(x_i) + F_1(x_i, x_j) + \varepsilon(\tau) . \quad (7)$$

If the last two members of the right part of system (7) are absent, this system describes dynamics of ensemble of nonlinear oscillator independent from each other. Thus the behavior of nonlinearity is defined by function $F_0(x_i) = -\partial\Phi / \partial x_i$. The second member $F_1 = -\sum_{i}^{N} \partial G / \partial x_i$ describes interaction between nonlinear oscillators. The third one – describes external periodic force. Let, for the definiteness, each of considered nonlinear oscillators represents the charged particle which moves in some nonlinear potential. Below we will consider that as a result of interaction between oscillators or as a result of influence on them with external regular force the dynamics is chaotic. In this case it is possible to present the shift of each of these oscillator in following form:

$$x_i = \bar{x} + \delta_i , \quad (8)$$

where $\bar{x} = \lim_{N\to\infty} (\sum_{i=1}^{\infty} x_i) / N$ – average coordinate of the shift of nonlinear oscillator; δ_i – random deviation. And $\langle \delta_i \rangle = 0$. In this case average sizes of ensemble for functions F_0 and F_1 are convenient for moments series expansion:

$$\langle F_0(\bar{x} + \delta_i) \rangle = F_0(\bar{x}) + \sum_{n=1}^{\infty} \frac{M_n}{n!} \left(\frac{d}{dx_i} \right)^n \cdot F_0 |_{\delta_i = 0} , \quad (9)$$

where $\mathrm{M}_n = \langle (\delta)^n \rangle$ – the moments.

For example we will consider ensemble of oscillator, each of which represents the mathematical pendulum. In this case $F_0(x_i) = -\sin(x_i)$. The average size from this function is:

$$\langle F_0(x_i) \rangle = -\langle \sin(x_i) \rangle = -\left[1 - \sum_{m=1}^{\infty} \frac{\mathrm{M}_{2m}}{(2m!)} \right] \sin \bar{x} . \quad (10)$$

For equation (10), it is possible to receive at once the one of the most important results. This result is that ensemble characteristics even noninteracting mathematical pendulums can essentially differ from characteristics of separate oscillator. In order to illustrate this fact from system (7) we will find the equation which describes dynamics of average deviation. The external force and the coupling between oscillator can be neglected for the simplicity.

$$\ddot{\bar{x}} + \left[1 - \sum_{m=1}^{\infty} \frac{\mathrm{M}_{2m}}{(2m!)} \right] \sin \bar{x} = 0 . \quad (11)$$

The equation (11) describes the dynamics of a mathematical pendulum. However the potential of this mathematical pendulum, and respectively, and oscillatory characteristics of this pendulum essentially depend on statistical characteristics of the separate oscillators making considered ensemble. Thus, oscillatory properties of ensemble even of independent nonlinear oscillator appear essentially dependent on chaotic dynamics of separate oscillator. It is necessary to keep in mind that chaotic dynamics of separate oscillator appears only as result or interactions between these oscillators, or, as a result of external influence on these oscillators. In equation (11) this fact in not represented explicitly. Let's pay attention that the second addend in square brackets in a equation (11) has the minus sign. It means that the random nature of the dynamics of separate oscillator always leads to reduction of effective potential where the ensemble moves. In particular, frequency of small linear oscillations of such ensemble decreases also.

Above we have considered the case when nonlinear oscillators are not coupled and external force does not impact on them. However we assumed that dynamics of each oscillator is random. Random dynamics can be caused either interaction between oscillator or influence of external force. The analysis of the general case (when there is a interaction and external forces) is possible only with numerical methods. Such investigation has been carried out. Some results of such investigation for ensemble of mathematical pendulums are presented below.

For numerical investigations the interaction between oscillators which corresponds to one-dimensional coulomb interaction between oscillator has been chosen:

$$F_1(x_i, x_j) = -\mu \sum_{j=1}^{N} \frac{sign(x_i - x_j)}{(x_i - x_j - a)^2} . \quad (12)$$

In a equation (12) size a characterizes the minimum distance between oscillator. External force we choose as: $\varepsilon(\tau) = A \cdot \cos\omega\tau$. The set of the equations (7) with such external force and with coupling (12) describes ensemble of N the charged particles which moves in external periodic potential and which affects by external periodic force.

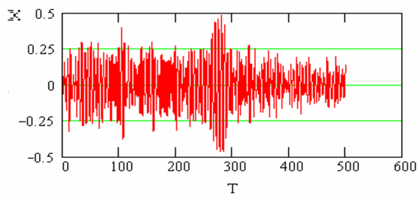

Fig. 3. Dependence of the shift of separate oscillator on time

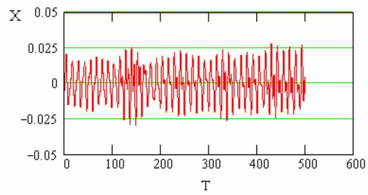

Fig. 4. Dependence of the mean position of oscillators on time

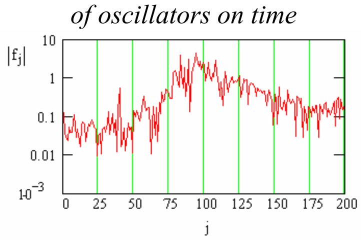

Fig. 5. Characteristic spectrum of the separate oscillator

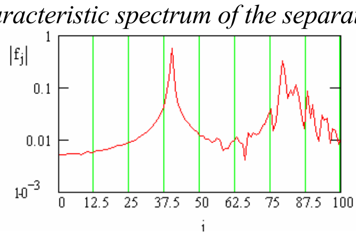

Fig. 6. Spectrum of the mean position of oscillators

The main results of numerical calculations are in a good qualitative agreement with the situation described above, i.e. the existence of interaction and external force leads to chaotic dynamics of each separate oscillator, and the dynamics of the whole ensemble essentially depends on statistical characteristics of the dynamics of separate oscillator. Below in Figs. 3-6 some of characteristic results are presented.

So, in Figs. 3, 4 the dependence of shift of separate oscillator (see Fig. 3) and the dependence of shift of mean position of ensemble of oscillator (see Fig. 4) on time is presented. First of all, it is seen, that the dynamics of the separate oscillator is chaotic, and the dynamics of ensemble is much more regular. It is seen also that characteristic frequencies of oscillations of ensemble are more lower, than characteristic frequencies of separate oscillator. This fact is proved in Figs. 5 and 6. In Fig. 5 – the spectrum of oscillation of a separate oscillator, and in Fig. 6 – the spectrum of oscillations of ensemble is presented.

Besides the presented Figs. 3-6 correlation functions and Lyapunov's main exponents have been obtained. The correlation functions of oscillations of a separate oscillator quickly fall down, and correlation functions of oscillations of all ensembles oscillate without damping. Lyapunov's main exponents for separate oscillator practically in all phase plane are positive, and similar values for ensemble practically do not differ from zero.

CONCLUSIONS

Thus, the ensemble of linear identical coupled oscillators can has in the spectrum of normal frequencies, frequency which can be many times smaller (hundreds times) then partial frequencies of separate oscillator. This feature is necessary to take into account in the analysis of dynamics of such ensemble. Besides, this feature can be used for transformation of energy of low-frequency oscillations to energy of high-frequency oscillations. The scheme of such transformation can be such, for example. Let we have a large number of the coupled high-frequency vibrators. As it is seen above, the ensemble of such vibrators can have a low-frequency normal mode. If to influence now on such ensemble with external signal which frequency is equal to this low-frequency normal mode, such ensemble will gain energy from an external low-frequency source. If, as it is shown in the second section, we will break coupling between vibrators, they will oscillate on the high-frequency partial frequencies. Amplitude of such oscillations will be essential more than their initial amplitudes. It is important that phases of oscillations of all these separate vibrators will be identical similar irrespectively of initial phases of these vibrators. Such synchronisation of phases allows to organize coherent radiation of this ensemble of vibrators on their high-frequency partial frequencies.

The important was found that dynamics of ensemble of nonlinear oscillator can essentially depend on statistical characteristics of dynamics of separate oscillator. This feature is shown even in that case when oscillator are independent and do not interact with each other. The influence of the statistical moments of separate oscillator on size of effective potential in which makes oscillations considered ensemble is also important. This feature of dynamics of nonlinear oscillator can be function of distribution of the charged particles grasped in various potentials useful for definition.

It is necessary to note that these results could be expected a priori. Really, if we have large number of oscillators, and their dynamics is described by stationary casual process, then after averaging, the first summand in (11) will become zero (as derivative of a constant). And with inevitability will become zero a square bracket in second summand in this equation. In this extreme case the frequency of oscillation of averaged displacement becomes zero too.

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ВЛИЯНИЕ ОСОБЕННОСТЕЙ ДИНАМИКИ ЗАХВАЧЕННЫХ ЧАСТИЦ НА СПЕКТР ИХ КОЛЕБАНИЙ

В.А. Буц, Д.М. Ваврив, Д.В. Тарасов

Исследованы особенности динамики ансамблей связанных линейных и нелинейных осцилляторов. Показано, что спектральные характеристики этой динамики существенно зависят не только от количества осцилляторов, составляющих ансамбль, но и от особенностей движения каждого осциллятора. В частности, при хаотическом движении – от моментов их хаотической динамики. Особое значение обращено на возникновение НЧ-динамики ансамбля. В этом случае появляется возможность эффективного обмена энергией между ВЧ- и НЧ-колебаниями.

ВПЛИВ ОСОБЛИВОСТЕЙ ДИНАМІКИ ЗАХОПЛЕНИХ ЧАСТИНОК НА СПЕКТР ЇХ КОЛИВАНЬ

В.О. Буц, Д.М. Ваврів, Д.В. Тарасов

Досліджені особливості динаміки ансамблів зв'язаних лінійних і нелінійних осциляторів. Показано, що спектральні характеристики цієї динаміки суттєво залежать не тільки від кількості осциляторів, складових ансамблю, але й від особливостей руху кожного осцилятора. Зокрема, при хаотичному русі – від моментів їх хаотичної динаміки. Особливе значення звернено на виникнення НЧ-динаміки ансамблю. У цьому випадку з'являється можливість ефективного обміну енергією між ВЧ- і НЧ-коливаннями.