# FOCUSING OF RELATIVISTIC ELECTRON BUNCHES BY NONRESONANT WAKEFIELD EXCITED IN PLASMA

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Focusing of relativistic electron bunches by nonresonant wakefield, excited by them in plasma, is investigated by numerical simulation. For a good focusing bunches are placed optimally relatively to the excited wave. For this the specific difference of wave frequency and bunches repetition frequency is used.

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## INTRODUCTION

Focusing of bunches by radial component of wakefield is an important problem. The intensity of this focusing is larger on a few orders in comparison with used magnetic focusing [1]. However focusing, which occurs in the plasma at space charge compensation of bunches, is also not enough intense. The intensity of focusing can be increased significantly, at orientation on its use in colliders, at use of excited transverse wakefield. Focusing by excited resonant wakefield was studied in [2, 3]. Also a uniform focusing by excited wakefield has been studied in [3, 4] for a relatively long bunches and in [5] for short bunches. As in the experiment it is difficult to maintain a uniform and stationary plasma density, resonant for sequence of electron bunches, in this paper focusing of sequence of bunches of relativistic electrons by excited non-resonant wakefield is considered.

## 1. OPTIMAL NONRESONANT WAKEFIELD PLASMA LENS FOR SHORT SEQUENCE OF IDENTICAL BUNCHES OF RELATIVISTIC ELECTRONS

Numerical simulation has been performed using 2d3v-code lcode [6]. For numerical simulation вибрані Parameters:  $n_{res}=10^{11}$  cm<sup>-3</sup> is the resonant plasma density which corresponds to ratio  $\omega_{pe}=\omega_m=2\pi\cdot2.8\times10^9$ , relativistic factor of bunches equals  $\gamma_b=5$ , have been selected.  $\omega_m$  is the repetition frequency of bunches,  $\omega_{pe}=$  =  $(4\pi n_{res}e^2/m_e)^{1/2}$  is the electron plasma frequency. The density of bunches  $n_b = 6\times10^8$  cm<sup>-3</sup> is distributed in the transverse direction approximately according to Gaussian distribution,  $\sigma_r = 0.5$  cm,  $\lambda = 10.55$  cm is the wavelength,  $\xi=V_bt$ -z,  $V_b$  is the velocity of bunches. Time is normalized on  $\omega_{pe}^{-1}$ , distance – on  $c/\omega_{pe}$ , density – on  $n_{res}$ , current  $I_b$  – on  $I_{cr}=\pi mc^3/4e$ , fields – on  $(4\pi n_{res}c^2m_e)^{1/2}$ .

As it has been shown in [3], at the resonant excitation of wakefield the shorter first fronts of the bunches are defocused by smaller fields, and longer back fronts of the bunches are focused by larger fields (see Fig. 1).

I.e. focusing by resonant wakefield is strongly inhomogeneous.

Let us consider the optimum parameters for the case of nonresonant wakefield plasma lens for short sequence of identical bunches of relativistic electrons. I.e. we will show that for selected: the length of bunches, less than half the wavelength  $\xi_b < \lambda/2$ , their number  $N_b$  and repeti-

tion frequency of bunches  $\omega_m$  there is a range of suitable electron plasma frequency  $\omega_{pe}$  such that all bunches are in focusing wakefields  $F_{r}$ .



Fig. 1. Longitudinal distribution of radius r<sub>b</sub> (horns) and density n<sub>b</sub> (trapezoids) of sequence of resonant rectangular bunches and of radial wakefield F<sub>r</sub> (oscillating line), after their focusing/defocusing at the distance z=33 cm from the boundary of injection





of smaller amplitude) and of longitudinal wakefield  $E_z$ (oscillating line of larger amplitude) near boundary of injection at  $n_e=1.35$   $n_{res}$ ,  $\xi_b=0.1\lambda$ ,  $I_b=1.56\times10^{-3}$ . The arrow shows the direction of the bunch motion As it will be demonstrated below, that in the case  $\omega_{pe} > \omega_m$  all point (very short) bunches are in focusing or in a zero radial field, we use the range of parameters when  $\omega_{pe} > \omega_m$ . For determination the optimal parameters we use two conditions. Namely, that all N<sub>b</sub> are placed on the length of a beating, it is necessary  $0 < \omega_{pe} - \omega_m < \omega_{cr}$ .  $\omega_{cr}$  is some critical frequency, associated with N<sub>b</sub>. At the same time, for all the electrons of all bunches are in focusing wakefields, it is necessary  $\xi_b < \xi_{cr}$ .

In the case of point  $\xi_b \rightarrow 0$  bunches one restriction is removed, and the relative position of bunches and  $F_r$  at  $n_e/n_{res}$ -1=0.35 ( $n_{res}$  is determined from  $(4\pi n_{res}e^2/m_e)^{1/2}$ = =  $\omega_m$ ) has the form shown in Fig. 2.

One can see that  $N_b=5$  bunches are in focusing wakefields  $F_r$ .



Fig. 3. Longitudinal distribution of radius  $r_b$  (short linear segments) of sequence of rectangular bunches of length,  $\xi_b = \lambda/4$ , of radial wake force  $F_r$  (oscillating line of smaller amplitude) and of longitudinal wakefield  $E_z$  (oscillating line of larger amplitude) near boundary of injection at  $n_e=1.35n_{res}$ ,  $I_b=10^{-3}$ 



Fig. 4. Longitudinal distribution of radius  $r_b$  (short linear segments) of sequence of rectangular bunches of length  $\xi_b = \lambda/4$ , of radial wakefield  $E_r$  (oscillating line) and of magnetic wakefield  $H_{\theta}$  (trapezoids) near boundary of injection at  $n_e = 1.35 n_{res}$ ,  $I_b = 10^{-3}$ 

In the case of bunches of finite length,  $\xi_b = \lambda/4$  at the plasma density, equal to  $n_e = 1.35n_{res}$ , the relative position *ISSN 1562-6016. BAHT. 2015. Ne4(98)* 

of bunches and  $F_r$  has the form shown in Figs. 3, 4.  $F_r$  is the total, i.e. radial field of the space charge of the bunch, wakefield and its own magnetic field of the bunch current  $H_{\theta}$ .  $E_r$  is the total, i.e. radial field of the space charge of the bunch and wakefield. As one can see, for each frequency difference  $\omega_{pe}$ - $\omega_m$  there exist the length of sequence and the length of the bunches, when all the electrons of all bunches are in focusing fields.

#### 2. FOCUSING OF LONG SEQUENCE OF RELATIVISTIC ELECTRON BUNCHES BY NONRESONANT WAKEFIELD

Let us consider the distribution of long sequence of short relativistic electron bunches (Fig. 5) relative to excited wakefield beatings at  $\omega_{pe}$ > $\omega_m$  (Fig. 6).



Fig. 5. Spatial distribution of density  $n_b$  of sequence of very short approximately Gaussian bunches  $\xi_b=0.1\lambda$  near boundary of injection at  $n_e=1.35n_{res}$ ,  $I_b=1.56 \times 10^{-3}$ 



Fig. 6. Longitudinal distribution of density  $n_b$  (vertical lines) of sequence of very short approximately Gaussian bunches  $\xi_b=0.1\lambda$  and of radial wake force  $F_r$  (oscillating line) near boundary of injection at  $n_e=1.35n_{res}$ ,  $I_b=1.56 \times 10^{-3}$ 

At  $\omega_m < \omega_{pe}$  beatings are excited. All bunches are in focusing fields of beatings except at fronts of beatings, where they are not focused.

In the case of bunches of length  $\xi_b = \lambda/4$ ,  $\lambda = 2\pi V_b/\omega_p$  one can see Fig. 7 and Fig. 8.



Fig. 7. Spatial distribution of density  $n_b$  of sequence of rectangular bunches of length  $\xi_b = \lambda/4$  near boundary of injection at  $n_e = 1.35 n_{res}$ ,  $I_b = 10^{-3}$ 



Fig. 8. Longitudinal distribution of radius  $r_b$  (short linear segments) of sequence of rectangular bunches of length  $\xi_b = \lambda/4$  and of radial wake force  $F_r$  (oscillating line) near boundary of injection at  $n_e=1.35 n_{res}$ ,  $I_b=10^{-3}$ 

Let us compare focusing in nonresonant (Fig. 9)  $\omega_m < \omega_{pe}$  and in resonant (Fig. 10)  $\omega_m = \omega_{pe}$  cases.

One can see that in nonresonant case all bunches are focused except at fronts of beatings, where they are not focused.



Fig. 10. Longitudinal distribution of radius  $r_b$  of resonant sequence of rectangular bunches of length  $\xi_b = \lambda/4$  at  $I_b = 10^{-3}$  after their focusing/defocusing at the distance z=50 cm from the boundary of injection

Now we consider the long sequence of short Gaussian bunches. The sequence is shaped according to linear dependence. The space interval between bunches equals to the wavelength (see Fig. 11).



Fig. 11. Longitudinal distribution of radius  $r_b$  (points) and density (vertical lines) of long shaped according to linear dependence sequence of very short bunches, of radial wake force  $F_r$  (oscillating line of smaller amplitude) and of longitudinal wakefield  $E_z$  (oscillating line of larger amplitude) near boundary of injection at  $I_b=10^{-3}$ 



Fig. 12. Longitudinal distribution of radius  $r_b$  (points) and density (vertical lines) of long shaped according to linear dependence sequence of approximately Gaussian bunches, the length of which equals  $\xi_b = \lambda/5$ , of radial wake force  $F_r$  (oscillating line of smaller amplitude) and of longitudinal wakefield  $E_z$  (oscillating line of larger amplitude) near boundary of injection at  $I_b=2.5 \times 10^{-3}$ 



Fig. 13. Longitudinal distribution of radius  $r_b$  (points) and density (vertical lines) of long sequence with precursor shaped according to linear dependence along sequence as well as along each bunch, of radial wake force  $F_r$  (oscillating line of smaller amplitude) and of longitudinal wakefield  $E_z$  (oscillating line of larger amplitude) near boundary of injection at  $I_b=10^{-3}$ 

One can see that all bunches are in maxima of focusing field and thus they are decelerated slowly, as they are in zero decelerating field, excited by previous bunches.

Now we consider the long sequence of Gaussian bunches, shaped according to linear dependence. The space interval between bunches is equal to the wavelength, and the bunch length equals  $\xi_b = \lambda/5$  (Fig. 12). Also we consider a long sequence of short bunches with precursor, shaped according to linear dependence along the sequence and along each bunch (Fig. 13). The space interval between bunches equals wavelength. One can see that in both cases all bunches are in maximal focusing fields and in a small  $E_z$ .

Thus, bunches of sequence, shaped according to linear dependence, and bunches of sequence, shaped according to linear dependence with precursor, are in maximal focusing fields.

### CONCLUSIONS

Focusing of relativistic electron bunches by nonresonant wakefield, excited by them in the plasma, has been investigated by numerical simulation. For an efficient focusing the bunches are placed optimally with respect to the excited wave. For that it uses a determined difference of wave frequency and repetition frequency of bunches.

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#### ФОКУСИРОВКА РЕЛЯТИВИСТСКИХ ЭЛЕКТРОННЫХ СГУСТКОВ НЕРЕЗОНАНСНЫМ КИЛЬВАТЕРНЫМ ПОЛЕМ, ВОЗБУЖДАЕМЫМ В ПЛАЗМЕ

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Исследуется численным моделированием фокусировка релятивистских электронных сгустков нерезонансным кильватерным полем, возбуждаемым ими в плазме. Для хорошей фокусировки сгустки размещаются оптимальным образом относительно возбуждаемой волны. Для этого используется определенная разность частоты волны и частоты следования сгустков.

#### ФОКУСУВАННЯ РЕЛЯТИВІСТСЬКИХ ЕЛЕКТРОННИХ ЗГУСТКІВ НЕРЕЗОНАНСНИМ КІЛЬВАТЕРНИМ ПОЛЕМ, ЩО ЗБУДЖУЄТЬСЯ В ПЛАЗМІ

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Досліджується чисельним моделюванням фокусування релятивістських електронних згустків нерезонансним кільватерним полем, яке збуджується ними в плазмі. Для хорошого фокусування згустки розміщуються оптимальним чином щодо хвилі, яка збуджується. Для цього використовується певна різниця частоти хвилі і частоти слідування згустків.