DISCUSSION ON IMPORTANCE OF e^+e^- PAIR EMISSION IN THE $^{12}C(\alpha,\,\gamma)^{16}O$ CAPTURE REACTION BELOW $1.9\,MeV$ ENERGY

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The cross section of the direct $E0$ pair emission has meaningful contribution to the total cross section of the $^{12}C(\alpha,\,\gamma)^{16}O$ reaction at low energy $\leq 1.9\,MeV$. $E0$ resonance emission and internal pair conversion have significant effect to the total cross section of the $^{12}C(\alpha,\,\gamma)^{16}O$ reaction. In this paper e^+e^- paired emission has been focused on taking into account the angular correlation. E0 contribution is also significant in a presence of $E1$ and $E2$ transition, therefore e^+e^- pair emission may not be neglected and has a significant effect on the total cross section in the case of the $^{12}C(\alpha,\,\gamma)^{16}O$ reaction.

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1. INTRODUCTION

Study of heavy nuclei reactions at higher energies $\geq 20\,MeV$ leads to the information about shape transitions and nuclear models. While, light nuclei interactions at low energies $\leq 5\,MeV$ gives information about the reaction mechanisms for astrophysical purposes. The $^{12}C(\alpha,\,\gamma)^{16}O$ is very important reaction and is considered as the 'Holy Grail' process in the nuclear astrophysics [1-3]. Many experimental attempts have been carried out to get better determination of $^{12}C(\alpha,\,\gamma)^{16}O$ rate, but required precision has not been achieved yet due to the lack of statistics [4]. Matei Catalin [5] has studied the cascade transition through $6.05\,MeV$ state of the ^{16}O nucleus, which has been ignored in previous studies of the $^{12}C(\alpha,\,\gamma)^{16}O$ reaction. He has observed the first excited state in ^{16}O over a wide range of energies and made the subsequent fits for the cross section (S-factor). In his experiment the $0^+ \to 0^+$ transition in the ^{16}O nucleus was analyzed for the first time in connection with the $^{12}C(\alpha,\,\gamma)^{16}O$ reaction cross section at helium burning energies. But the statistics was very weak and could not meet the required purpose [5]. They summed up the $E1$ and $E2$ transitions to get the estimation of $6.05\,MeV$ transition. The experimental set up consisted of the BGO array which does not allow for the separation of primary transitions to 6.05 and $6.13\,MeV$ states in ^{16}O. The reason for this is the fact that the energy resolution of these detectors was about $100...150\,keV$. Fig.1 is excitation function, where error bars are the data points that start from $2\,MeV$, below this there is only extrapolation that was done using the R-matrix calculation. They account a systematic error of 30%. This 30% error is not accepted well in astrophysical relevance. Also the S-factor, that they calculated, was very high.

Due to the moderate energy resolution of BGO crystals and limited acceptance of the separator, ERNA experiment was performed.

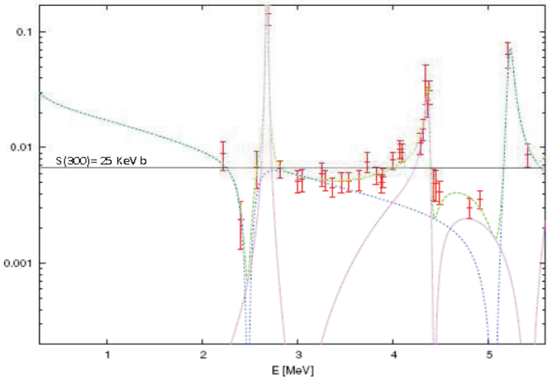

Fig.1. *The R-matrix fit of the excitation function for the* $E = 6.05\,MeV$ *transition (Error bar). Dot and line – E1 transition, small dots – E2 transition [5]*

D. Schuermann et al. [6] studied the radiative capture reaction $^{12}C(\alpha,\,\gamma)^{16}O$ in the energy range between $3.3\,MeV$ to $4.5\,MeV$. This experiment focused in particular on the cascade transition to 0^+ state at $E = 6.05\,MeV$ in ^{16}O. In this experiment γ-rays were detected. The $6.05\,MeV$ transition has been considered as a component accounting for up to 15%

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of the $^{12}C(\alpha,\, \gamma)^{16}O$ total cross section at astrophysical energies. The experiment did not go beyond the $5\, MeV$ to avoid the high energy background. The calculated S-factor was less than $1\, keV\, b$ (Fig.2). Also they concluded that $6.05\, MeV$ cascade transition in $^{12}C(\alpha,\, \gamma)^{16}O$ has no astrophysical relevance at the stellar burning temperatures. Sum of $6.05\, MeV$ and $6.13\, MeV$ amplitudes resulted in S-factor values, which were very close to the DRAGON data.

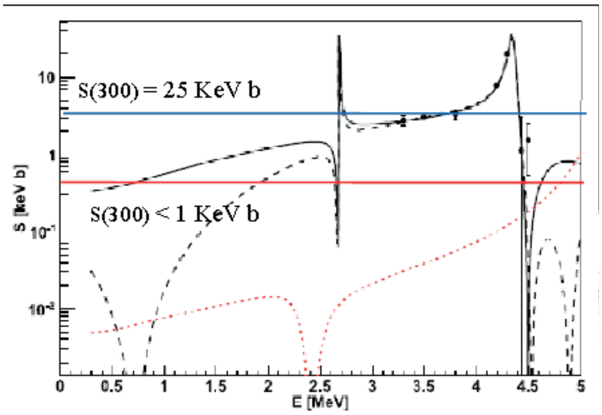

Fig.2. *R-matrix analysis of* $6.05\, MeV$ *transition. Circles are* $6.05\, MeV$ *data* ($E2$) *only, Black solid and dashed lines are two best fits; dotted line is R-Matrix calculation of* $E1$ *component [6]*

These two discrepancies can be removed while studying the emission of e^+e^- pairs at energies less than $2\, MeV$. Snover and Hurd [7] estimated the direct capture cross section for the $E0$ pair emission of reaction at low bombarding energies. They compared this transition with the $E2$ photon emission. By comparing them it was shown that $E0$ pair emission and $E2$ photon emission are relating each with other. In this case the pair emission cross section is of the order of 10^{-6} of the total cross section at low energies. This small factor of cross section is unimportant for the determination of $S_{34}(0)$. They concluded that $E0$ pair emission and internal conversion are very negligible relatively the direct single photon emission [7]. There is a theoretical analogy of $6.05\, MeV$ transition to ground state given by G. Baur et al. [8]. They did not support the E0 emission. The $E0$ emission is not seen directly up to now in the $^{12}C(\alpha,\, \gamma)^{16}O$ reaction, there are theoretical evidences that do not support the $E0$ emission.

In parallel some experimental and theoretical studies have been carried out for capturing rays in the $^{12}C(\alpha,\, \gamma)^{16}O$ reaction at low energy. Gialanella et al. [9] obtained the excitation function while observing $E1$ capture amplitude in the energy range between $1.32...2.99\, MeV$. SchÄurmann et al. [10] have measured the total cross section of $^{12}C(\alpha,\, \gamma)^{16}O$ reaction in energy range between $1.9...4.9\, MeV$. They also studied the ground state emission $E1$ and $E2$.

In this paper, a discussion on the $0^+ \to 0^+$ transition via e^+e^- pair transition at energy $< 1.9\, MeV$ has been carried out. A miniscule work has been carried out at low energy level because experimental facilities have not been introduced yet to carry out the experimental studies. Therefore it is significant to focus on the interactions at low energy level. This paper aims to describe the importance of $0^+ \to 0^+$ transitions via e^+e^- pair transition. This transition still has been neglected but it has worth in total cross section measurements of $^{12}C(\alpha,\, \gamma)^{16}O$ reaction. So it is desirable to explain the transition at low energy below $1.9\, MeV$ that may be helpful to avoid extrapolation.

2. $E0$ TRANSITION

$E0$ transitions in light nuclei might have a large impact in nuclear astrophysics, since they may connect 0^+ states that cannot be connected by radiative transitions. $E0$ transitions occur between two states of the nucleus both of which have spin $I = 0$. As we know from electromagnetism, there are no multipoles of order $l = 0$ in the radiation field. Hence no radiative transition can take place. The transition $0^+ \to 0^+$ is strictly forbidden for electromagnetic radiation. However, a transition is possible in which the K electrons take over the energy [11, 12]. In this case we must take into account the region of configuration space for which the electron is within the nucleus. Its contribution is negligible in general, but it is the most important in the case considered here. Since a transition of the $0^+ \to 0^+$ type does not produce any electromagnetic field outside the nucleus, the energy transfer must take place inside. If the energy difference between the nuclear states is larger than $2m_ec^2$, a new type of internal conversion can occur. The energy can be transmitted to electrons in the negative-energy states near the nucleus. These states occur in the relativistic wave equation of the electron. It is well known that Dirac's theory of the positron assumes that the states of negative kinetic energy are occupied by electrons. The lifting of one of these negative-energy electrons into a positive-energy state appears as the creation of an electron-positron pair. This process has been suggested by Oppenheimer et al. [13] and calculations have been done by various authors [14, 15]. The probability of internal pair formation is larger than the probability of ejection of a K shell electron if the available energy is appreciably larger than $2m_ec^2$ Internal pair formation supplements ordinary internal conversion in that the pair formation rate is the largest where the internal conversion rate is the smallest, namely in the region of low atomic number and high transition energies.

3. THE CAPTURE REACTION $^{12}C(\alpha,\gamma)^{16}O$

The capture reaction $^{12}C(\alpha,\gamma)^{16}O$ ($Q = 7.16\, MeV$) takes place in the helium burning stage of red giant stars and represents a key reaction of nuclear astrophysics as it strongly influences the production of all elements heavier than $A = 16$ as well as the stellar evolution from the helium burning phase to the late explosive stages. It is the regulator of C/O abundance in the universe and influences the composition of CO White dwarfs [10, 6]. The cross section of this reaction at the Gamow energy $E0 \sim 300\, KeV$ determines the He burning time scale together with the convection mechanism. The carbon abundance at that stage has important consequences for the subsequent evolution of various astrophysical scenarios, e.g. a direct influence on type II supernova nucleosynthesis, the maximum luminosity, kinetic energy of type I supernova nucleosynthesis and the cooling sequence of CO White dwarfs [4, 8]. The possible role of $E0$ emission has already been addressed by Baur-Snover, as $E0$ emission is important in $^{12}C(\alpha, \gamma)^{16}O$ capture measurement since they are made by detecting the emitted e^+e^- pairs. The major factor that enhances the importance of $E0$ emission is that it occurs by s-wave capture [8].

4. EXPLANATION

Many experimental works have been done on the capture cross section of $^{12}C(\alpha, \gamma)^{16}O$ reaction considering the γ-ray emission but at the energies more than $1.9\, MeV$ for the contribution in the total cross section. We will show theoretically the importance of e^+e^- pair emission at low energy. A comparison of the $E0$ and $E2$ emissions for the transitions between individual levels of finite spin is given in [7]. While the ratio of cross sections is given as [8]:

$$\frac{\sigma_{E0}}{\sigma_{E2}} = \frac{4\pi}{5} \frac{f_{E0}}{f_{E2}} \frac{|R_{00}|^2}{|R_{02}|^2}, \tag{1}$$

where f_{E0} and f_{E2} are given by [14, 15]:

$$\begin{aligned} f_{E0}(E) &= \frac{e^4}{27(\hbar c)^6} b(S)(E - mc^2)^3 (E + mc^2)^2, \\ f_{E2}(E) &= \frac{4\pi e^2}{27(\hbar c)^5} E^5. \end{aligned} \tag{2}$$

Estimation on $\frac{|R_{00}|^2}{|R_{02}|^2}$ ratio tells that the capture takes place at the nuclear radius, so at low interaction energies the effective radius is large. Thus to get the maximum advantage from the E0 transition we must add a factor obtained in the following manner to approach to the angular correlation term. From the above estimates and working on the potential model calculation of Woods-Saxon potential the $S_{E0}(0.3)$ factor calculated was:

$$S_{E0}(0.3) = 0.02\, keV\, b. \tag{3}$$

This factor may be improved by taking into account the angular correlation of emitted particles. The $E0$ direct capture occurs between the identical quantum numbers $j = 0$ and even parity. According to the selection rule there is no $E0$ transition, when $l = 0$ as given by the following relation [16]:

$$\begin{aligned} H_{L,M,i} = \frac{q_i}{m_i} \frac{E_0}{\sqrt{2i_\omega}} \left\langle \Psi_L \left| e^{-i\vec{k}\vec{r}} i_\varepsilon \cdot p_i \right| \Psi_H \right\rangle - \\ -g_i \frac{q_i}{2m_i} \frac{E_0}{\sqrt{2c}} \left\langle \Psi_L \left| e^{-i\vec{k}\vec{r}} i_B \cdot S_i \right| \Psi_H \right\rangle. \end{aligned} \tag{4}$$

Solving (4) only for the electric component gives us:

$$H^{El}_{L,M,i} = -\frac{q_i E_0}{\sqrt{2}} \frac{(-ik)^{l-1}}{(l-1)!} \left\langle \Psi_L \frac{1}{l} \left| r_{ik}^{l-1} r_{i\varepsilon} \right| \Psi_H \right\rangle. \tag{5}$$

Here $r_{i,k}$ is the component of position of particle in the direction of motion, $r_{i,\varepsilon}$ is the component of particle in the direction of electric field and angular momentum components are in the direction of magnetic field. In order to estimate the E0 transition probability, a convenient formulation has been proposed in [17]. $E0$ is possible through the internal conversion process [18, 19]. The electromagnetic processes also contribute to the astrophysical rate for $^{12}C + \alpha$ capture reaction at low energy. The phase space factor for the emission of e^+e^- pairs is small but we can improve the results by working under angular correlation to get the better contribution of the cross section at low energy. If the angular distribution between electron and positron is isotropic then, for a high energy pair transition in a nuclide of low atomic number, the energy distribution is given by:

$$N(E)dE \propto E^2 (E_0 - E)^2 dE, \tag{6}$$

where $N(E)dE$ is the number of electrons (positrons) emitted with some kinetic energy between E and $E + dE$ and $E_0 = ET - 2m_e c^2$, where ET is the nuclear transition energy [20]. If the transition is, in addition, $0^+ \to 0^+$ decay, then angular correlation is given by:

$$W(\theta) \propto 1 + \cos(\theta). \tag{7}$$

Equations (6) and (7) tell us that the probability of pair emission is maximum for an equal sharing of energy between electron and positron. By using the Dirac equation and Coulomb distorted wave approximation we get the angular correlation for $E0$ conversion as

$$\frac{d^2\eta}{dk\, d\cos(\theta)} = \frac{1}{2} \frac{d\eta}{dk} [1 + \varepsilon \cos(\theta)], \tag{8}$$

where ε the anisotropy factor, and k is the kinetic energy. Simplification gives us:

$$\frac{d\eta}{d\cos(\theta)} = [1 + \cos(\theta)]. \tag{9}$$

This relation shows that the emission of electron and positron pairs can be enhanced by working under angular correlation and this is possible at smaller values which favor the $6.05\, MeV$ e^+e^- pairs at low energy. If the interaction plane is set in such a way as to get the jet of particles at some lower angle rather than transverse to the beam axis, then the probability may be increased to favor the pair emission Fig.1. For this reason a strong experimental setup that surrounds the 4π angle of the detection system is required to get the better contribution of electron and positron pairs. To increase the S factor for $E0$ pair emission at Gamow energy, it will be reasonable to take into account the angular correlation factor, so as to increase the pair emission probability.

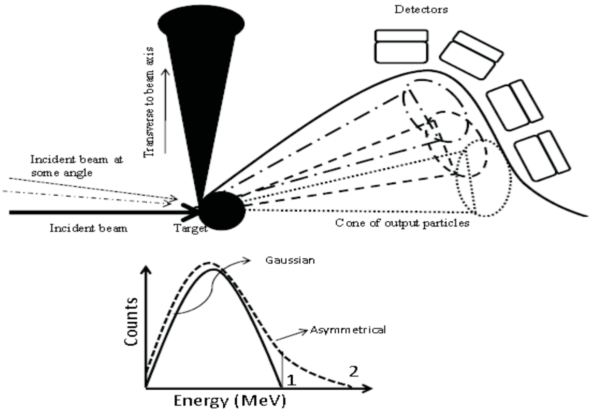

Fig.3. *Schematic diagram to achieve enhanced probability of e^+e^- pairs under angular correlation*

Fig.3 explains that the cone of particles in transverse direction to the plane of interaction gives less probability to get an event as represented by Gaussian distribution. While if the interaction plane is inclined to certain angle no matter what the geometry is, then the probability to get events has increased as clearly indicating by asymmetrical shape. In this asymmetrical shape the points 1 and 2 show the coincidence to get the desired events. It means if we count one event transverse to beam axis then it may increase to two when interaction is at some angle.

5. CONCLUSIONS

So we can conclude that the electromagnetic processes do contribute significantly to the astrophysical rate for the $^{12}C(\alpha,\gamma)^{16}O$ reaction. Extrapolation of data to lower c.m. energies for the reaction may be avoided using relation (9), which shows the significance of the e^+e^- pair emission below $1.9\,MeV$ energy.

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ДИСКУССИЯ О ВАЖНОСТИ ЭМИССИИ e^+e^- - ПАР В РЕАКЦИИ ЗАХВАТА $^{12}C(\alpha,\gamma)^{16}O$ ПРИ ЭНЕРГИЯХ ≤ 1.9 МэВ

У. Табассам, К. Мехбуб

Поперечное сечение прямой $E0$ эмиссии e^+e^- - пар дает значительный вклад в полное сечение для реакции $^{12}C(\alpha,\gamma)^{16}O$ при низких энергиях ≤ 1.9 МэВ. $E0$ резонансная эмиссия и внутренняя конверсия пар оказывают значительное влияние на полное сечение реакции $^{12}C(\alpha,\gamma)^{16}O$. В настоящей работе мы сконцентрировались на учете угловых корреляций эмиссии e^+e^- - пар. $E0$ вклад также является существенным наряду с $E1$- и $E2$-переходами, и поэтому эмиссия e^+e^- - пар не может принебрегаться, она имеет значительное влияние на полное сечение в случае $^{12}C(\alpha,\gamma)^{16}O$ - реакции.

ДИСКУСІЯ ПРО ВАЖЛИВІСТЬ ЕМІСІЇ e^+e^- - ПАР В РЕАКЦІЇ ЗАХВАТУ $^{12}C(\alpha,\gamma)^{16}O$ ПРИ ЕНЕРГІЯХ ≤ 1.9 МеВ

У. Табассам, К. Мехбуб

Поперечний переріз прямої $E0$ емісії e^+e^- - пар дає значний вклад у повний переріз для реакції $^{12}C(\alpha,\gamma)^{16}O$ при низьких енергіях ≤ 1.9 МеВ. E0 резонансна емісія і внутрішня конверсія пар мають значний вплив на повний переріз реакції $^{12}C(\alpha,\gamma)^{16}O$. У даній роботі ми сконцентрувалися на врахуванні кутових кореляцій емісії e^+e^- - пар. $E0$ вклад також є суттевим поряд з $E1$- і $E2$-переходами, і тому емісією e^+e^- - пар не можна нехтувати, вона має значний вплив на повний переріз у випадку $^{12}C(\alpha,\gamma)^{16}O$ - реакції.