

# ENERGY CHANGE AND SPACE MOTION OF ELECTRON IN SINUSOIDAL FIELD WITH RANDOM PHASE JUMPS OR ELASTIC COLLISIONS

*V. Ostroushko* \*

*National Science Center "Kharkov Institute of Physics and Technology", 61108, Kharkov, Ukraine*

(Received April 21, 2015)

Two different types of electron motion are considered: the one-dimensional motion in the electric field, which time dependence differs from sinusoidal one through instant jumps of phase, and the three-dimensional motion in the sinusoidal field interrupted with elastic collisions. The dependencies of the characteristic electron energy and of the characteristic dimensions of the space required for electron motion on the number of phase jumps or elastic collisions are obtained.

PACS: 29.17.+w, 52.80.Pi

## 1. INTRODUCTION

When an electron moves in gas under the action of an external field its continuous motion is sometimes interrupted by collisions. Combination of sinusoidal oscillations with elastic collisions gives the phenomenon of collisional heating, which is used for electron energy increase. If electron energy exceeds relevant threshold value then the part of the energy obtained with collisional heating is spent on nonelastic processes. On the other hand, an electron acceleration by the varying field in the collisionless media (in particular, the collisionless heating studied in [1], [2], and [3]) goes without such energy losses. The processes of collisional and collisionless heating are similar, but not identical. In particular, they have different dimensions required for electron motion.

In [4], the rates of change of velocity and coordinate squares during collisional and collisionless heating were estimated. The averaging of the difference of full velocity and oscillatory one was made for each successive elastic collision or field phase jump, independently. But to present probability with the sum of products of conditional probabilities and ones of simple events the events should be disjoint. For the mentioned difference, such case is approached with decrease of the collision or jump frequency. In the present work the sums over the collisions or jumps are averaged with account of phase correlations, the frequency is arbitrary, and the collisions are considered for three-dimensional motion.

## 2. MOTION IN THE FIELD WITH PHASE JUMPS

It is assumed that field is applied in the direction  $z$  and it consists of intervals with sinusoidal time de-

pendence, but with different initial phase values in the different intervals. For the units of velocity and frequency it is chosen the oscillatory velocity amplitude and the radian frequency. Between the phase jumps with the numbers  $n$  and  $n + 1$  the electron velocity and coordinate time dependencies are

$$v(t) = v_n + \cos(t - t_n + \psi_n^+) - \cos \psi_n^+, \quad (1)$$

$$z(t) = z_n + (v_n - \cos \psi_n^+)(t - t_n) + \sin(t - t_n + \psi_n^+) - \sin \psi_n^+, \quad (2)$$

where  $t \in (t_n, t_{n+1})$ ,  $v_n = v(t_n)$ ,  $z_n = z(t_n)$ , and  $\psi_n^+$  is the field phase value just after the  $n$ -th jump.

Some assumptions should be accepted for the phase jump values  $\eta_n = \psi_n^+ - \psi_n^-$ , where  $\psi_n^-$  is the field phase value just before the  $n$ -th jump, and for the time intervals  $\tau_n = t_n - t_{n-1}$  between the jumps. In these assumptions, the words "density of probability" of the given value  $\xi$  of the quantity  $\xi'$  mean the limit of the ratio  $\Delta P / \Delta \xi$  for  $\Delta \xi \rightarrow 0$ , where  $\Delta P$  is the probability of  $\xi' \in (\xi - \Delta \xi / 2, \xi + \Delta \xi / 2)$ . It is assumed that the density of probability of  $\tau_n = \tau$  is equal to  $\nu \exp(-\nu \tau)$  (as for Poisson process with the frequency  $\nu$ ), and the density of probability of  $\eta_n = \eta$  is given by the function  $Q(\eta)$ , which obeys the equalities  $Q(\eta + 2\pi) = Q(\eta)$  and  $\int_0^{2\pi} d\eta Q(\eta) = 1$ .

Let the functions  $\Phi_n^\pm(\psi)$  be the densities of probabilities of  $\psi_n^\pm = \psi$ . Assuming  $\Phi_n^\pm(\psi + 2\pi) = \Phi_n^\pm(\psi)$ , one gets the equalities

$$\Phi_n^+(\psi) = \int_0^{2\pi} d\psi' Q(\psi') \Phi_n^-(\psi - \psi'), \quad (3)$$

$$\Phi_{n+1}^-(\tau) = \int_0^\infty d\tau' \nu \exp(-\nu \tau') \Phi_n^+(\tau - \tau'). \quad (4)$$

For an integer  $m$ , let us put  $Q_m = \int_0^{2\pi} d\eta e^{im\eta} Q(\eta)$ ,  $\Phi_{n,m}^\pm = \int_0^{2\pi} d\psi e^{im\psi} \Phi_n^\pm(\psi)$ , and  $p_m = \nu(\nu - im)^{-1}$ . From (3) and (4), it is followed  $\Phi_{n,m}^+ = Q_m \Phi_{n,m}^-$ ,  $\Phi_{n+1,m}^- = p_m \Phi_{n,m}^+$ ,  $\Phi_{n+1,m}^+ = Q_m p_m \Phi_{n,m}^+$ .

\*E-mail address: ostroushko-v@kipt.kharkov.ua

As  $Q(\eta) \geq 0$ , for any real  $\eta$ , the relationships  $|Q_m| \leq \int_0^{2\pi} d\eta Q(\eta) = Q_0 = 1$  take place. For  $m \neq 0$ , the mapping  $\Phi_{n,m}^+ \Rightarrow \Phi_{n+1,m}^+$  leads to decrease of  $\Phi_{n,m}^+$  with  $n$  increase, and so, each phase jump is accompanied with decay of higher harmonics of probability distribution and with approach of phase distribution to homogeneous one.

From (1) and (2) one can obtain the equalities

$$v_{n+1} = v_n + \cos \psi_{n+1}^- - \cos \psi_n^+, \quad (5)$$

$$z_{n+1} = z_n + (v_n - \cos \psi_n^+) \tau_{n+1} + \sin \psi_{n+1}^- - \sin \psi_n^+. \quad (6)$$

Summation of (5) for  $n = 0, 1, \dots, N-1$  gives

$$v_N - v_0 = \sum_{n=1}^N (\cos \psi_n^- - \cos \psi_{n-1}^+), \quad (7)$$

and  $\psi_n^+ - \psi_l^+ = \sum_{j=l+1}^n (\tau_j + \eta_j)$  for  $n > l$ . Let us take the squares of the different sides of (7) and average them over  $\tau_n$  and  $\eta_n$  with denoting the averaging by the angle brackets. In the squares the products of cosines may be replaced by the sums with use of the equality  $2 \cos \alpha \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$ . For  $N \gg 1$ , when full velocity becomes much greater than oscillatory one,  $v_N^2 \gg 1$ , the main contribution to the value of  $\langle (v_N - v_0)^2 \rangle$  give the summands with cosines of the difference of phases, and with  $N$  increase, the contribution corresponding to each fixed difference of indexes becomes approximately proportional to  $N$ . Taking into account the equalities  $\langle \exp(i\tau_n) \rangle = p_1$ ,  $\langle \exp(i\eta_n) \rangle = Q_1$ , for  $n \geq l$  one gets the equalities

$$\begin{aligned} \langle \cos(\psi_n^- - \psi_l^-) \rangle &= \text{Re} [(p_1 Q_1)^{n-l}], \\ \langle \cos(\psi_{n-1}^+ - \psi_{l-1}^+) \rangle &= \text{Re} [(p_1 Q_1)^{n-l}], \\ \langle \cos(\psi_n^- - \psi_{l-1}^+) \rangle &= \text{Re} [p_1 (p_1 Q_1)^{n-l}], \\ \langle \cos(\psi_n^+ - \psi_l^-) \rangle &= \text{Re} [Q_1 (p_1 Q_1)^{n-l}], \end{aligned}$$

and summation of geometric progression gives

$$\langle v_N^2 \rangle \approx fN, \quad (8)$$

where  $f = \text{Re} [(1 - p_1)(1 - Q_1)(1 - p_1 Q_1)^{-1}]$ .

Multiplying together the different sides of the equalities (5) and (6) gives the expression for the difference  $z_{n+1}v_{n+1} - z_nv_n$ . For  $n \gg 1$ , the main contribution to it gives the summand  $v_n^2 \tau_{n+1}$ , for which  $\langle v_n^2 \tau_{n+1} \rangle \approx (fn)/\nu$ . For  $N \gg 1$ , taking the sum, one gets  $\langle z_N v_N \rangle \approx (fN^2)/(2\nu)$ . Taking the squares of the different sides of (6) gives the expression for the difference  $z_{n+1}^2 - z_n^2$ . The main contribution to it gives the summand  $2z_nv_n\tau_{n+1}$ , for which  $\langle 2z_nv_n\tau_{n+1} \rangle \approx (fn^2)/\nu^2$ . As a result, one gets

$$\langle z_N^2 \rangle \approx (fN^3)/(3\nu^2). \quad (9)$$

For purely oscillatory electron motion (with zero average velocity) the averaged value of the velocity square is equal to 1/2 (in the chosen velocity units). For low frequency of phase jumps ( $\nu \ll 1$ ), one has  $|p_1| \ll 1$  and  $f \approx 1$ , and from (8) it is followed that average increase of the velocity square per jump is equal to the doubled average value of the oscillatory velocity square.

### 3. MOTION IN SINUSOIDAL FIELD WITH ELASTIC COLLISIONS

Here it is assumed that the field with sinusoidal time dependence is applied in the direction  $z$ , but an electron motion is considered as three-dimensional, and the designations  $\theta$  and  $\varphi$  for relevant angles in spherical coordinates are used. Let  $\psi_n$  be time instant of the elastic collision with the number  $n$  (and the value of phase, due to the choice of time unit). Let  $\tau_n = \psi_n - \psi_{n-1}$ , and so,  $\psi_n = \psi_0 + \sum_{j=1}^n \tau_j$  for  $n \geq 1$ . Let  $x_n, y_n$ , and  $z_n$  be coordinates of the point of  $n$ -th collision. Let  $v_{x,n}^\pm, v_{y,n}^\pm$ , and  $v_{z,n}^\pm$  be velocity component just before (index  $-$ ) or just after (index  $+$ ) the collision. Let  $\theta_n^+$  and  $\varphi_n^+$  be relevant angles just after the collision, and  $v_n$  be absolute value of velocity (which is not changed in elastic collision). It is assumed that motion directions just after collision have isotropic distribution, as in the model of hard spheres, and the density of probability for all these directions is equal to  $1/(4\pi)$ , with respect to solid angle. The density of probability of  $\tau_n = \tau$  is assumed to be equal to  $\nu \exp(-\nu\tau)$ . The introduced quantities are connected through the equalities

$$\begin{aligned} v_{x,n+1}^- &= v_{x,n}^+ = v_n \sin \theta_n^+ \cos \varphi_n^+, \\ v_{y,n+1}^- &= v_{y,n}^+ = v_n \sin \theta_n^+ \sin \varphi_n^+, \\ v_{z,n+1}^- &= v_n \cos \theta_n^+ + \cos \psi_{n+1} - \cos \psi_n, \\ x_{n+1} &= x_n + v_{x,n}^+ \tau_{n+1}, \\ y_{n+1} &= y_n + v_{y,n}^+ \tau_{n+1}, \\ z_{n+1} &= z_n + (v_n \cos \theta_n^+ - \cos \psi_n) \tau_{n+1} + \sin \psi_{n+1} - \sin \psi_n. \end{aligned}$$

Taking the sum of squares, one can get the equality

$$\begin{aligned} v_{n+1}^2 - v_n^2 &= \\ &= 2v_n \cos \theta_n^+ (\cos \psi_{n+1} - \cos \psi_n) + \\ &+ 1 - \cos(\psi_{n+1} + \psi_n) - \cos \tau_{n+1} + \\ &+ [\cos(2\psi_{n+1}) + \cos(2\psi_n)]/2. \end{aligned} \quad (10)$$

The stage of the process is considered, at which  $N \gg 1$  and  $v_N \gg 1$ . The sum of the equalities (10) over  $n = 0, 1, \dots, N$  is being taken and averaged over the values of  $\theta_n^+$  and  $\tau_{n+1}$  with denoting the averaging with the angle brackets. The main contribution to the sum give the averaged differences  $1 - \cos \tau_{n+1}$ , and one gets the relationship

$$\langle v_N^2 \rangle \approx N/(\nu^2 + 1), \quad (11)$$

which corresponds to (8) with  $Q_1 = 0$ . Similarly, the sum of the equalities for  $r_{n+1}^2 - r_n^2$ , where  $r_n^2 = x_n^2 + y_n^2 + z_n^2$ , is being taken and averaged, also, over the values of  $\varphi_n^+$ . The main contribution to the sum give the averaged values of  $v_n^2 \tau_{n+1}^2$ , and one can get the relationship

$$\langle r_N^2 \rangle \approx N^2 [\nu^2 (\nu^2 + 1)]^{-1}. \quad (12)$$

It is worthy to compare the increase of velocity square per collision in three-dimensional and one-dimensional cases. In 1-D case, starting from the equality  $v_{n+1} = \cos \psi_n - \cos \psi_{n+1} - v_n$ , and introducing the variables  $v'_n = (-1)^n v_n$  and  $\psi'_n = \psi_n + \pi n$ , one comes to the equality  $v'_{n+1} = v'_n + \cos(\psi'_{n+1} - \pi) - \cos \psi'_n$ , which also corresponds to the collisionless motion in

the field with phase jumps on  $\pi$ . In this case,  $Q(\eta) = \sum_{n=-\infty}^{+\infty} \delta(\eta - \pi - 2\pi n)$ ,  $Q_1 = -1$ ,  $\langle v_N^2 \rangle \approx (2N)/(4\nu^2 + 1)$ . So, the ratio of the values of energy increase per collision for 1-D and 3-D cases at  $\nu \ll 1$  is 2.

#### 4. CONCLUSIONS

Comparison of the relationships (8), (9), (11), and (12) shows, that the rates of electron energy change through field phase jumps and elastic collisions are similar, but the rates of coordinate change somewhat differ, in connection with change of the electron motion direction for the collisions and absence of its change for the jumps. It should be noted, that collisional and collisionless heating may be used in the different cases. In contrary to the collisional heating, the collisionless one, as any acceleration in the given field, may take place at anyhow small gas pressure. But if the pressure is not too small then a combination of the phase jumps with the collisions gives combined features of the process development. Perhaps, in particular, sparse collisions give considerable decrease of the motion space dimensions without considerable change of the energy increase rate.

#### ACKNOWLEDGEMENT

The author is grateful to V.A. Buts, V.I. Karas', and O.V. Manuilenko for the detailed explanations of mistakes in some previously obtained results.

#### References

1. F.G. Bass, Ja.B. Fainberg, V.D. Shapiro. Quasi-linear theory of turbulent plasma with account of electric fields correlation // *Soviet Phys. JETP*. 1965, v.22, p.230–238.
2. V.I. Karas', Ja.B. Fainberg, A.F. Alisov, et al. Interaction with plasma and gases of microwave radiation with stochastically jumping phase // *Plasma Physics Reports*. 2005, v.31, p.748–760.
3. V.I. Karas', A.F. Alisov, O.V. Bolotov, et al. Optical radiation special features from plasma of low-pressure discharge initiated by microwave radiation with stochastic jumping phase // *PAST. Series "Plasma Electronics and New Methods of Acceleration"*. 2013, N4(86), p.183–188.
4. V. Ostroushko. Estimation of dimensions of interaction space necessary for particle acceleration in stochastic field // *PAST. Series "Nuclear Physics Investigations (63)"*. 2014, N5(93), p.140–142.

#### ИЗМЕНЕНИЕ ЭНЕРГИИ И ДВИЖЕНИЕ ЭЛЕКТРОНА В ПРОСТРАНСТВЕ В СИНУСОИДАЛЬНОМ ПОЛЕ СО СЛУЧАЙНЫМИ СКАЧКАМИ ФАЗЫ ЛИБО УПРУГИМИ СТОЛКНОВЕНИЯМИ

*В. Остроушко*

Рассмотрено два разных типа движения электрона: одномерное движение в электрическом поле, зависимость которого от времени отличается от синусоидальной мгновенными скачками фазы, и трехмерное движение в синусоидальном поле, прерываемое упругими столкновениями. Получены зависимости характерной энергии электрона и характерных размеров пространства, требуемого для движения электрона, от количества скачков фазы либо упругих столкновений.

#### ЗМІНА ЕНЕРГІЇ ТА РУХ ЕЛЕКТРОНА У ПРОСТОРІ У СИНУСОЇДАЛЬНОМУ ПОЛІ З ВИПАДКОВИМИ СТРИБКАМИ ФАЗИ АБО ПРУЖНИМИ ЗІТКНЕННЯМИ

*В. Остроушко*

Розглянуто два різних типи руху електрона: одновимірний рух в електричному полі, залежність якого від часу відрізняється від синусоїдальної через миттєві стрибки фазы, та тривимірний рух у синусоїдальному полі, який переривається пружними зіткненнями. Отримано залежності характерної енергії електрона та характерних розмірів простору, потрібного для руху електрона, від кількості стрибків фазы або пружних зіткнень.