# SURFATRON ULTRARELATIVISTIC ACCELERATION OF PROTONS BY ELECTROMAGNETIC WAVE IN SPACE PLASMAS

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Based on numerical calculations it is investigated proton ultrarelativistic acceleration in space plasma by electromagnetic wave propagating across the external magnetic field. It is considered the conditions for the proton capture by wave, dynamics of the proton velocity components and its impulse momentum ones, the dependence of acceleration rate on the problem initial parameters, the structure of the phase plane of accelerated protons. Optimaum conditions for protons ultrarelativistic surfatron acceleration by the electromagnetic wave are formulated. It is discused the possibility of appearance of differences from the standard power-law dependence in the accelerated protons spectra obtained by processing of experimental data on the registration of cosmic rays (CR). Estimates are given for protons acceleration in the heliosphere from an initial energy of a few GeV up to TeV energies.

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## INTRODUCTION

Investigation of the of ultrarelativistic particles flows is one of the actual problems of the space plasma physics and it is of great interest, for example, for the problem of the cosmic rays origion, in particular, to understand the mechanisms of CR formation and their variability, the dependence on space weather. Surfing charges on electromagnetic waves was considered earlier, for example, in [1 - 13] for the electrons. This is an effective mechanism for generating streams of ultrarelativistic particles in space plasma and its research is needed, for example, to estimate the number of accelerated particles, the size of the acceleration region, the energy spectra of the charges, which requires a detailed analysis of the conditions of charged particles capture by waves with strong acceleration, to reveal favorable charged particle parameters for particles capture, to estimate the efficiency of particle acceleration under the influence of spatially localized wave packets, etc.

For the transverse propagation of electromagnetic wave p-polarization the square of refractive index of the plasma $N^2 = (ck / \omega)^2$ at a frequency ω is determined by expression: $N^2 = 1 - [v (1 - v)] / (1 - u^2 - v)$, $u = \omega_{He}/\omega$, $v = (\omega_{pe}/\omega)^2$, where $\omega_{He} = eH_0/m_e c$ is the gyrofrequency of nonrelativistic plasma electrons, $\omega_{pe} = (4\pi e^2 n_0/ m)^{1/2}$ is the electron Langmuir frequency, $n_0$ is the plasma density.

Below we consider the case u<1. Then the phase velocity of the electromagnetic wave is less than the speed of light for the following values of the parameter v: $1 - u^2<v<1$. The charge capture into regime of surfatron acceleration takes place if the wave field is above the some threshold value

$$\sigma = e E_0/m_e c \omega > u \gamma_p = u /(1 - \beta_p^2)^{1/2}, \beta_p = \omega/c k.$$

The following numerical calculations of protons surfing are simplified if we neglect the vortical components of wave field $E_y$, $H_z$. Nonlinear effects of the electromagnetic wave interaction with plasma are small if the wave amplitude $E_0$ significantly lower characteristic field of relativistic nonlinearity. So the condition $\sigma << 1$ must be fulfilled.

## BASIC EQUATIONS AND THE NUMERICAL CALCULATION RESULTS

Let us consider the relativistic equations of motion of the proton with mass $m_p$ in the electromagnetic wave field $E_x = E_0 \cos \Psi$, $\Psi = \omega t - k x$:

$$d (\gamma \beta_x)/dt = (e E_0/M c) \cos \Psi + (e H_0/M c) \beta_y,$$
$$d(\gamma \beta_y)/dt = - (e H_0/M c) \beta_x, \quad (1)$$
$$d \gamma / dt = (e E_0/M c) \beta_x \cos \Psi, \quad \gamma \beta_z = h = \text{const},$$

where $\beta_p = \omega/c k$, $\boldsymbol{\beta} = \mathbf{v}/c$, $\beta_x = \beta_p (1 - d\Psi / d\tau)$, $\tau = \omega t$ is the nondimensional time. Let us introduce now $\xi = \omega x/c$, then we have $\Psi = \tau - (\xi/\beta_p)$. We introduce also a small parameter $\varepsilon = (m_e/m_p)^{1/2}$ and normalized dimensionless time $s = \varepsilon \tau$. Equations (1) have the integral $J = \gamma \beta_y + u \varepsilon^2 \beta_p (\tau - \Psi) = \text{const}$. So we have for y-component of proton momentum of impulse $\gamma \beta_y = J + u \varepsilon^2 \beta_p (\Psi - \tau)$. We introduce now the notation $G = 1 + h^2 + [J + u \varepsilon^2 \beta_p (\Psi - \tau)]^2$ then it is follows $\gamma^2 = G/(1 - \beta_x^2)$. Now from (1) the nonlinear, nonstationary equation for the wave phase at proton the trajectory is following

$$d^2\Psi / ds^2 + [ \sigma\cdot(1 - \beta_x^2)^{3/2} / G^{1/2}\cdot\beta_p] \cos \Psi +$$
$$(u/G \cdot\beta_p)\cdot(1 - \beta_x^2)\cdot[J + u \varepsilon \beta_p (\varepsilon \Psi - s)] = 0, \quad (2)$$

with $\beta_y = [J + u \beta_p (\tau - \Psi) + \sigma\chi \cos \Psi] / G$.

The nonlinear equation (2) is solved numerically, we take the initial data in the form of $\Psi(0) = \Psi_0$, $\Psi_\tau(0) \equiv a$. In this case $\beta_x(0) = \beta_p (1 – a)$ where the condition must be fulfilled $1 – (1/\beta_p) < a < 1 – (1/\beta_p)$. Proton capture into the surfing mode acceleration by wave occurs if $\sigma > \sigma_C$, $\sigma_C = u \gamma_p$, $\gamma_p = 1/(1 - \beta_p^2)^{1/2}$ is the relativistic factor of accelerating wave. At sufficiently long times the numerical solution should go to the following asymptotics $\beta_x \approx \beta_p$, $\beta_y \approx 1/\gamma_p$. It is important to note that the rate of change in the energy of accelerated proton does not depend on the amplitude of the wave field $E_0$ which determines the average rate of acceleration parameter $< \cos \Psi(\tau) > \approx \sigma_c/\sigma$ related to asymptotic of the bottom positon for effective potential well.

To find the range of the initial phases of Ψ(0) at which the capture of proton in the strong acceleration by the wave the phase velocity $\beta_p$ was fixed and it is supposed that $0 < \beta_p < 1$ and $|\Psi(0)| < \pi$. The wave amplitude σ was choosen somewhat higher threshold $\sigma_c$ namely $\sigma = 1.4 \sigma_c$. Then, numerical simulations on a relatively small time s ≤ 7000 determine the range of the initial phase which took place in the proton capture by wave with its following ultrarelativistic acceleration. If for some phases Ψ(0) proton capture did not take place the calculation has performed up to $\tau \le 7{\cdot}10^4$.

Let us to present now the calculation results for the following choices of the problem parameters h = 0.35, $\gamma(0)\beta_y(0) = -0.47$, $\beta_p = 0.52$, u = 0.31, a = 0 corresponding to weakly relativistic proton $\gamma(0) \approx 1.357$ at s = 0. The initial phase Ψ(0) was chosen in the next interval (-3.1…3.1). The results of calculations of the proton capture by wave with surfatron acceleration have showed that for the initial phases in the interval (-1.2…0.2) with a phase step δΨ(0) = 0.1 capture occurs immediately. In the case of Ψ(0) in range (0.3…1.2) proton capture occurs for a short time but then the proton emitted from the effective potential well. So if Ψ(0) equal to 0.3 the proton is captured by wave if s < 200 and in the case of Ψ(0) = 0.6 it is trapped by wave for s < 63 but then for up to the time s = 7000 proton is passing particle. For Ψ(0) in the interval (1.3…3.1) proton capture at the interim account s < 7000 is absent. Let us present results of numerical calculations for the case Ψ(0) = - 1.2. Fig. 1 shows a plot of the phase Ψ(s) for s < 7000.

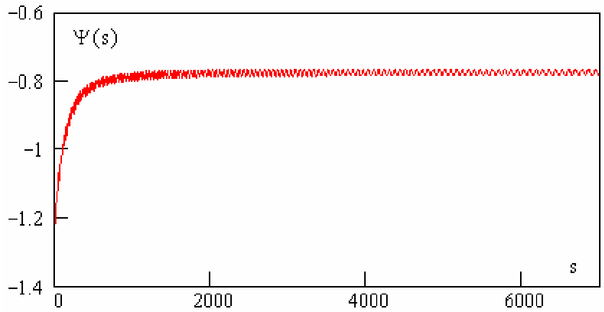


*Fig. 1. Graph of wave phase at the trajectory of proton*

According to Fig. 1 the wave phase is moving fast to the asymptotic value of 0.8 and subsequently oscillates around this value. According to calculations, the oscillation amplitude decreases very slowly with time and the period of the oscillations increases also slowly. In the phase plane (dΨ/ds, Ψ) particle trajectory has a singular point of type stable focus which is quite similar to the previously discussed for the electrons acceleration by wave.

Graphs of the proton relativistic factor γ(s) and its analytical approximation M(s) are shown in Fig. 2.

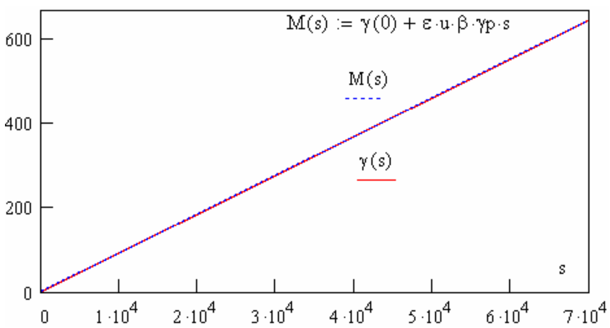


*Fig. 2. Graphs of the proton relativistic factor and its analytical approximation*

As we can see there is good agreement of γ(s) with a linear function M(s). So the rate of proton acceleration is almost constant in accordance with the asymptotic solution of equation (1) for a strong particle acceleration. It is interesting to note that such agreement takes place even on a relatively small acceleration times. For $s = 7{\cdot}10^4$ relativistic factor of the accelerated proton has reached the value $\gamma \approx 646.5$.

Graphs of transverse to the external magnetic field, the dimensionless component of the proton impulse momentum are given at Fig. 3. As it seen the values of the growth rate for $g_x$, $g_y$ are practically constant and correspond to the asymptotics $g_x \approx \gamma{\cdot}\beta_p$, $g_y \approx -\gamma/\gamma_p$.

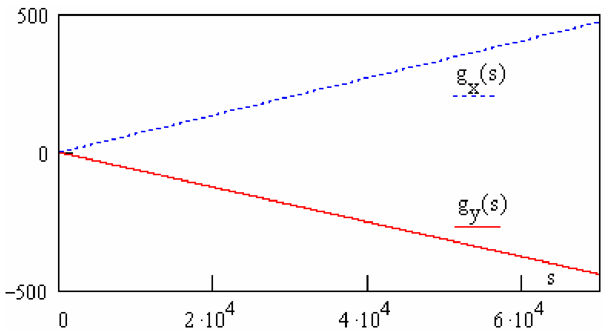


*Fig. 3. The dynamics of transverse components of accelerated proton momentum of impulse*

The transverse component of the accelerated proton velocity quickly go to the above asymptotic values.

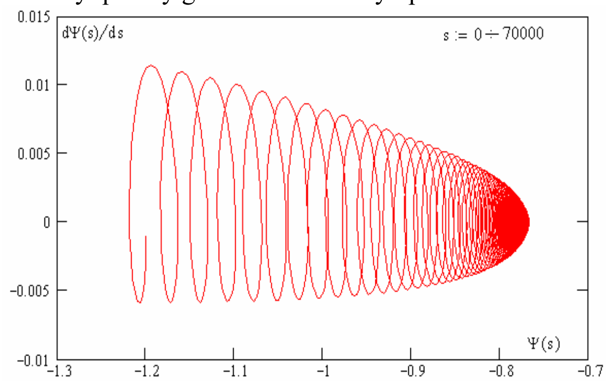


*Fig. 4. The structure of the phase plane for the accelerated proton*

For the time interval $0<s<7{\cdot}10^4$ structure of the phase plane (dΨ/ds, Ψ) is shown in Fig. 4. The yield gives visible path of the image point to the particular type of stable focal point corresponding to the bottom of effective potential well. It was calculated shift of the proton in the direction of the wave propagation ξ(s) = ω x/c which was close to the asymptotic behavior of $\xi(s) \approx \beta_p\, s/\varepsilon$. The trapped protons shift along the wave front (y-axis) is also observed in almost linear dependence on time. The proton velocity along the external magnetic field during particle acceleration decreases. At the initial moment of time it was $\beta_z(0) = 0.227$ but for $s = 3.5{\cdot}10^4$ it has became $\beta_z = 1.267{\cdot}10^{-3}$.

Unlike electrons the protons surfatron acceleration for some values of the initial phases are found the ability to capture and acceleration of particles on a relatively small initial time interval s of the order of tens to hundreds but then proton escapes from the effective potential well. For of the initial phase $\Psi(0) = 0.3$ when the particle is captured for the time $s < 200$, the $\Psi(s)$ plot is shown in Fig. 5. Numerical calculations to $s = 70\,000$ showed the absence of proton capture by the wave. This is probably due to the non-stationarity of effective potential well at times s in the hundreds. According to calculations after the departure of proton from the potential well the cyclotron rotation is realized in an external magnetic field. The proton displacement for cyclotron rotation case is shown in Fig. 6.

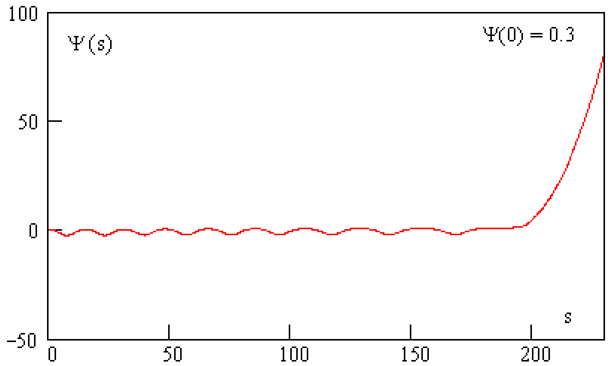


*Fig. 5. The dynamics of the wave phase at proton trajectory in the case Ψ(0) = 0.3*

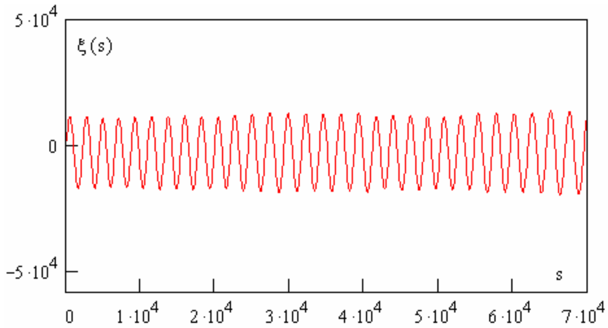


*Fig. 6. Proton shift graph after particle leaving from effective potential well*

Let us give scaling factors for protons accelerated in the heliosphere $\gamma = 621\cdot\gamma_P\cdot(\beta_P/0.9)\cdot(u/0.3)\cdot(s/10^5)$, where f - frequency of the accelerating wave in Hz, and also for the displacement of particle in the direction of wave propagation $\delta x = (\beta_P/0.9)\cdot(10^2$ Гц$/f\cdot(s/10^5)\cdot 12.46\cdot AU$. Let us note that one astronomical unit is 1 AU=$1.5\cdot 10^8$ km and radius of the solar heliosphere is around 200 AU. Thus protons trapped by electromagnetic wave will accelerated in the heliosphere (depending on the capture location and the value of wave phase velocity $\beta_p$) to an energy of the order of tens to hundreds of Gev at distances (10…100) AU. Consequently, surfatron acceleration mechanism is a local source of additional generation of cosmic rays in the energy range given above. It is quite obvious that the effectiveness of this source will depend on the heliospheric weather which means a CR spectrum variability observed experimentally. Calculations for other values of the problem parameters leads to similar results.

## CONCLUSIONS

In this paper on the basis of the numerical calculations the dynamics of capture and subsequent strong surfatron acceleraion of protons with initial energies of the order of $m_p c^2$ by electromagnetic wave propagating in space plasmas across a weak magnetic field is investigated. For the realization of surfatron acceleration the amplitude of wave electric field was taken above the threshold value. The problem is reduced to the study of nonlinear, time-dependent second-order equations for the wave phase on the particle trajectory $\Psi(s)$. The dynamics of the impulse momentum components, velocity ones, the proton energy and the structure of phase plane $(d\Psi/ds, \Psi)$ on the value of initial phase $\Psi(0)$ for condition $\beta_x(0) = \beta_p$ corresponding to the Cherenkov resonance particles with an electromagnetic wave in a magnetized plasma were studied.

It was founded that for much of the possible values of $\Psi(0)$ the proton capture by wave in the effective potential well occurs immediately with following particle ultrarelativistic acceleration. In the absence of a proton capture by wave at the considered range of numerical calculations the particle cyclotron rotation was observed. It can be expected that after a sufficiently large number of Larmor rotations the Cherenkov resonance $\beta_x \approx \beta_p$ will take place and the phase $\Psi$ becomes favorable. So it will be realized the proton capture by wave and subsequent strong particle acceleration will occurs. Trapped proton acceleration rate is constant. Therefore the maximum energy of a particle is proportional to the time of his confinement in the effective potential well. Let us note that the value of energy gain rate is reduced by decreasing of parameters u, $\beta_p$.

The following condition should be noted here. To implement the protons ultrarelativistic acceleration with their energy growth on many orders of magnitude it is natural to assume that the total number of trapped particles must be small enough. So we may neglect by the electromagnetic wave damping during time of particles acceleration. Otherwise the problem of generation of ultrarelativistic particles flows in space plasmas in the application to the cosmic rays origion is not meaningful due to the relatively rapid decay of the wave.

We have examined the cases of surfatron acceleration of protons with a negative initial value of the particle impulse momentum along the wave front when after the proton capture by wave its acceleration takes pace. In the opposite case - positive initial values of the proton impulse momentum along the wave front the following dynamic is observing. Particle with $\beta_x \approx \beta_p$ is trapped by the wave and the proton velocity component $\beta_y$ is reducing. Being trapped the particle changes sign of its velocity component $\beta_y$ on the negative one and only then a strong proton surfatron acceleration takes place. For large initial energy of particles on the stage of their braking-acceleration the protons relativistic factor $\gamma(s)$ may be approximated by the linear analytical functions.

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## УЛЬТРАРЕЛЯТИВИСТСКОЕ СЕРФОТРОННОЕ УСКОРЕНИЕ ПРОТОНОВ ЭЛЕКТРОМАГНИТНОЙ ВОЛНОЙ В КОСМИЧЕСКОЙ ПЛАЗМЕ

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На основе численных расчетов исследовано ультрарелятивистское ускорение протонов в космической плазме электромагнитной волной, распространяющейся поперек внешнего магнитного поля. Рассмотрены условия захвата протонов волной, динамика компонент скорости и импульса, зависимость темпа ускорения от исходных параметров задачи, структура фазовой плоскости ускоряемого протона. Сформулированы оптимальные условия для ультрарелятивистского серфотронного ускорения протонов электромагнитной волной, обсуждается возможность возникновения за счет серфотронного механизма отличий в спектрах ускоренных протонов, полученных при обработке экспериментальных данных по регистрации космических лучей, от стандартных степенных зависимостей. Даны оценки доускорения протонов в гелиосфере, имевших начальную энергию порядка нескольких гигаэлектронвольт, до энергий порядка тераэлектронвольт.

## УЛЬТРАРЕЛЯТИВІСТСЬКЕ СЕРФОТРОННОЕ ПРИСКОРЕННЯ ПРОТОНІВ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ В КОСМІЧНІЙ ПЛАЗМІ

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На основі чисельних розрахунків досліджено ультрарелятивістське прискорення протонів в космічній плазмі електромагнітною хвилею, що розповсюджується поперек зовнішнього магнітного поля. Розглянуто умови захоплення протонів хвилею, динаміка компонент швидкості і імпульсу, залежність темпу прискорення від вихідних параметрів завдання, структура фазової площини прискорюємого протона. Сформульовано оптимальні умови для ультрарелятивістського серфотронного прискорення протонів електромагнітною хвилею, обговорюється можливість виникнення за рахунок серфотронного механізму відмінностей у спектрах прискорених протонів, отриманих при обробці експериментальних даних по реєстрації космічних променів, від стандартних ступенних залежностей. Дано оцінки доускорення протонів в геліосфері, що мали початкову енергію порядку декількох гігаелектронвольт, до енергій порядку тераелектронвольт.