

# ION CYCLOTRON INSTABILITIES IN MULTICOMPONENT MAGNETIC FIELD-ALIGNED PLASMA FLOW WITH SHEAR VELOCITY FLOW

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The ion cyclotron instabilities of magnetic-field-aligned multi-species plasma flow with the flow velocity shear are investigated. It is assumed that plasma consist of two ion species,  $H^+$  and  $O^+$ , the most often occurring in the Earth's ionosphere. The effect of the hydrogen ions on the ion kinetic as well as hydrodynamic shear-flow-driven ion cyclotron instabilities with the frequency approximately equal to the cyclotron frequency of the  $O^+$  ions is considered.

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## INTRODUCTION

The investigations of the auroral region of the Earth's ionosphere have discovered the inhomogeneous structures of electrostatic potentials which are correlated with regions of the formation and acceleration of the magnetic field-aligned upward ion beams [1]. One of the main features of these beams is the gradient of the flow velocity across the magnetic field (flow velocity shear)  $V'_{0i}$  which can reach specifically for  $O^+$  ions the values of  $6\omega_{ci}$  [2]. The upflowing ion beams are mainly composed of  $H^+$  and  $O^+$  ions the composition of which varies significantly from beam to beam [3]. These auroral ion beams are often correlated with electrostatic ion cyclotron (EIC) oscillations having the cyclotron frequencies of hydrogen and oxygen ions [4, 5]. It was shown that the flow velocity shear along with the other mechanisms may be responsible for the excitation of EIC waves in the auroral ionosphere due to development of the shear-flow-driven EIC instability [6, 7]. The ion-parallel velocity shear-flow-driven ion cyclotron instability results from three mechanisms depending on the ratio of the phase velocity and the thermal velocity of the particles. These mechanisms dominate separately in distinct ranges of the normalized parallel-wave vector component [7]. In the short-wavelength diapason the instability is excited due to cyclotron interaction ions with the ion cyclotron waves (ion kinetic mode). The hydrodynamic effects of the instability excitation take place in more long-wavelength range when the ion cyclotron damping is negligible (hydrodynamic mode). When the wavelength along the magnetic field exceeds the critical value, the hydrodynamic mode does not exist and the ion cyclotron instability is excited by the inverse electron Landau damping (electron kinetic mode).

The shear-flow-driven EIC instabilities was studied in plasma with single ion species [6, 7]. However, the application of these results in ionosphere investigations requires taking into account the presence of several ion components, the relative concentrations of which are changed significantly with the altitude in ionospheric plasma. We have carried out the study of the shear-flow-driven ion-kinetic as well as hydrodynamic EIC instabilities in the sheared magnetic field-aligned plasma flow with two,  $H^+$  and  $O^+$ , ion species. The frequency of oscillations is assumed approximately equals to the  $O^+$  cyclotron frequency and effect of hydrogen

ions on the hydrodynamic mode of this instability is considered. We have analyzed the dispersion equation assuming that the waves are propagate nearly perpendicularly to the magnetic field but under the assumption that electrons are adiabatic. The thresholds and growth rates of these instabilities versus the relative concentration of oxygen ions for the main  $n=1$  and high  $n \ll 1$  cyclotron harmonics are obtained.

## 1. THE INSTABILITIES OF THE FIRST CYCLOTRON HARMONIC

The dispersion relation for homogeneous multi-ion component plasma with a flow velocity shear is given by [8]

$$\begin{aligned} \varepsilon(K, \omega) &= 1 + \delta\varepsilon_e + \delta\varepsilon_h + \delta\varepsilon_l = \\ &= 1 + \frac{1}{k^2 \lambda_{De}^2} + \sum_{\alpha=h,l} \frac{1}{k^2 \lambda_{D\alpha}^2} \left( 1 - \frac{k_y}{k_z} S_\alpha + i\sqrt{\pi} \times \right. \\ &\times \left. \sum_{n=-\infty}^{\infty} W(z_{\alpha n}) \Gamma_n(b_\alpha) \left( \frac{\omega - k_z V_{0\alpha}}{\sqrt{2} k_z V_{T\alpha}} - \frac{k_y}{k_z} S_\alpha z_{\alpha n} \right) \right) = 0, \quad (1) \end{aligned}$$

where the indices  $h, l$  refer to the heavy,  $O^+$ , and light,  $H^+$ , ions respectively,  $\lambda_{D\alpha}$  is the Debye length,  $A_n(b) = \exp(-b) I_n(b)$ ,  $I_n(b)$  is the modified Bessel function,  $b_\alpha = (k_\perp \rho_{T\alpha})^2$ ,  $\rho_{T\alpha} = V_{T\alpha} / \omega_{c\alpha}$  is the thermal Larmor radius,  $S_\alpha = V'_{0\alpha}(X) / \omega_{c\alpha}$  is the normalized flow velocity shear,

$$\begin{aligned} z_{\alpha n} &= (\omega - n\omega_{c\alpha} - k_z V_{0\alpha}) / \sqrt{2} |k_z| V_{T\alpha}, \\ W(z) &= \exp(-z^2) \left( 1 + (2i/\sqrt{\pi}) \int_0^z e^{-\xi^2} d\xi \right). \end{aligned}$$

We study the heavy-ion cyclotron mode having the frequency  $\omega(k) = n\omega_{ch} + k_z V_{0h} + \delta\omega(k)$ , where  $|\delta\omega(k)| \ll \omega_{ch}$ . Assume, that both ion species have the equal thermal  $V_{Th} = V_{Tl}$  and flow  $V_{0h} = V_{0l}$  velocities as well as equal magnitudes of velocity shear  $V'_{0h} = V'_{0l}$ .

We first analyze the instability of the fundamental,  $n=1$ , cyclotron harmonic. We assume that  $z_{hn}$ , which is the argument of the  $W$  - function, in the sum over cyclotron harmonics has an arbitrary value for the fundamental harmonic, while  $|z_{hn}| > 1$  in the remaining

sum. That is valid when the inequality  $k_z \rho_{Th} < 1$  is satisfied. Using the asymptotic form for  $W$ -function for large argument values

$$W(z_{in}) \approx \exp(-z_{in}^2) + (i/\sqrt{\pi}z_{in})(1 + 1/2z_{in}^2),$$

we carry out in Eq. (1) the summation over the cyclotron harmonics for  $n \neq 1$ :

$$\sum_{n \neq 1} W(z_{hn}) A_n(b_h) \left( \frac{\omega - k_z V_{0h}}{\sqrt{2}k_z V_{Th}} - \frac{k_y}{k_z} S_h z_{hn} \right) \approx -\frac{1 - A_0(b_h)}{b_h} - \frac{k_y}{k_z} S_h (1 - A_1(b_h)). \quad (2)$$

In the sum over cyclotron harmonics of light ions in Eq. (1) we retain only null term because of significant difference in the masses of heavy and light ions. Then the dielectric permittivity of light ions becomes

$$\delta \varepsilon_{l1} \approx \frac{1}{k^2 \lambda_{Dl}^2} \left( 1 - A_0(b_l) + \frac{k_y}{k_z} \frac{S_l A_0(b_l)}{2z_{l01}^2} \right), \quad (3)$$

where  $z_{l01} = \omega_{ch}/\sqrt{2}k_z V_{Tl}$ ,  $S_l = S_h/\mu$  and  $\mu = m_h/m_l$ .

Then the dispersion relation (1) reduces to the form:

$$k^2 \lambda_{Dh}^2 \varepsilon(k, \omega) = 1 + g_1 - \frac{1 - A_0(b_h)}{b_h} - \frac{k_y}{k_z} S_h A_1(b_h) + i\sqrt{\pi} W(z_{h1}) A_1(b_h) \left( \frac{\omega_{ch}}{\sqrt{2}k_z V_{Th}} + z_{h1} - \frac{k_y}{k_z} S_h z_{h1} \right) = 0, \quad (4)$$

where  $\alpha$  is the relative concentration of heavy ions,  $g_1 = \tau/\alpha + \delta \varepsilon_{l1}(1 - \alpha)/\alpha$ ,  $\tau = T_h/T_e$ . We use in what follows the normalized wavelength along the magnetic field  $\lambda = 1/k_z \rho_{Th}$  instead of variable  $k_z$ . Considering  $z_{h1}$  as the normalized complex frequency, we find the solution  $z_{h1}(\lambda)$  of the Eq. (4) for EIC instability in the short wavelength limit, when instability is developed due to the inverse of ion cyclotron damping, and in long wavelength limit, at which ion cyclotron damping is negligible and EIC instability is developed due to hydrodynamic effects.

We find first the short wavelength threshold of the instability. The threshold values for variables  $\lambda$  and  $z_{h1}$  we obtain by equating to zero the real and imaginary parts of Eq. (4)

$$\begin{cases} \lambda/\sqrt{2} + z_{h1} - \lambda k_y \rho_{Th} S_h z_{h1} = 0, \\ 1 + g_1 - (1 - A_0(b_h))/b_h - \lambda k_y \rho_{Th} S_h A_1(b_h) = 0. \end{cases} \quad (5)$$

This system of equations has a solution when inequality  $k_y \rho_{Th} S_h > 0$  is met. For this case we obtain the short-wavelength threshold value  $\lambda_{1s}$  for the excitation of the instability, as well as the threshold value of the normalized complex frequency  $z_{1s}$  which is the real at that threshold

$$\lambda_{1s} = \frac{1}{k_y \rho_{Th} S_h A_1(b_h)} \left( 1 + g_1 - \frac{1 - A_0(b_h)}{b_h} \right), \quad (6)$$

$$z_{1s} = \frac{1}{\sqrt{2}k_y \rho_{Th} S_h} \left( 1 + \frac{A_1(b_h)}{1 + g_1 - G_{h1}} \right), \quad (7)$$

where  $G_{h1} = A_1(b_h) + (1 - A_0(b_h))/b_h$  and index 1s means the short-wavelength instability threshold of the first cyclotron harmonic. Under the condition  $\alpha=1$ , i. e. when in plasma flow only one ion species occurs, the value  $g_1$  is equal to  $\tau$  and Eqs. (6), (7) reduces to known expressions [7]. The presence in plasma flow another ion species causes decrease in relative concentration of heavy ions that leads to the shift of the boundary wavelength of instability toward larger values because in (6)  $g_1 \propto 1/\alpha$ . The dispersive part,  $\delta\omega$ , of the ion cyclotron wave frequency and the growth rate at the vicinity of the instability threshold can be obtained from Eq. (6) as [7]

$$\delta\omega \approx \delta\omega_{01} \frac{\lambda_{1s}}{\lambda} + \delta\omega_{01} \left[ \frac{\sqrt{2}k_y \rho_{Th} S_h \text{Im}W(z_{1s})}{\sqrt{\pi}|W(z_{1s})|^2} - \frac{A_1(b_h)}{(1 + g_1 - G_1)} \right] \left( 1 - \frac{\lambda_{1s}}{\lambda} \right), \quad (8)$$

$$\gamma \approx \delta\omega_{01} \frac{\text{Re}W(z_{1s})\sqrt{2}k_y \rho_{Th} S_h}{\sqrt{\pi}|W(z_{1s})|^2} \left( 1 - \frac{\lambda_{1s}}{\lambda} \right), \quad (9)$$

where  $\delta\omega_{01} = \omega_{ch} A_1(b_h)/(1 + g_1 - G_1)$  and  $\delta\omega = \delta\omega_{01}$  at the instability boundary wavelength,  $\lambda = \lambda_{1s}$ . The EIC waves, which wavelength exceeds the boundary value (6) are unstable. The phase velocity of EIC waves along the magnetic field at the vicinity of instability threshold is of the order or less than the ion thermal velocity. The EIC instability occurs due to ion kinetic effect of inverse ion cyclotron damping which caused by flow velocity shear and is the ion-kinetic shear-flow-driven EIC instability [7]. Fig. 1 shows the dispersion and the growth rate of the ion kinetic shear flow driven EIC instability versus the normalized wavelength for  $S_h = 3$ ,  $k_y \rho_{Th} = 1$  and  $\tau = 1$  which were calculated numerically from dispersion relation (1).

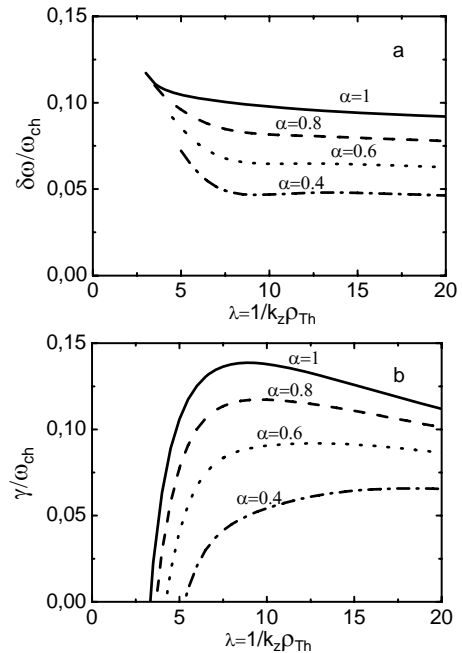


Fig. 1. The dispersion (a) and growth rate (b) of ion-kinetic instability versus the normalized wavelength

The numerical solution confirms the increase in boundary wavelength of instability (see Fig. 1,b). The value  $\delta\omega_{01}$  decreases also with the decreasing  $\alpha$ . This leads to a decrease in the dispersion (8), as well as in the growth rate (9) as can be seen from Fig. 1.

With increasing of wavelength of the ion cyclotron waves the phase velocity of these waves along the magnetic field increases and may greatly exceeds the thermal velocity of the heavy ions. In this case the effect of ion cyclotron damping is negligible both for light and heavy ions and instability is excited due to hydrodynamic effects. In Eq. (4) inequality  $|z_{i1}| > 1$  holds and the asymptotic form of  $W$  – function for large argument can be used. The dielectric permittivity of heavy ions can be written as

$$\delta\varepsilon_h \approx \frac{1}{k^2 \lambda_{Dh}^2} \left( 1 + g_1 - G_{h1} - \frac{\omega_{ch}}{\delta\omega} A_1(b_h) + \frac{k_y \rho_{Th}}{2\lambda} \frac{\omega_{ch}^2}{\delta\omega^2} S_h A_1(b_h) \right). \quad (10)$$

Using the expressions (3) and (10) the dispersion equation (1) can be reduced to the form

$$\delta\omega^2(K) - p\delta\omega(K) + q = 0, \quad (11)$$

where

$$p = \delta\omega_{1h} \left[ 1 + g_1 - G_{h1} + k^2 \lambda_{Dh}^2 \delta\varepsilon_l(k, \omega(k)) \right]^{-1},$$

$$q = \sigma_h^2 \left[ 1 + g_1 - G_{h1} + k^2 \lambda_{Dh}^2 \delta\varepsilon_l(k, \omega(k)) \right]^{-1},$$

$\delta\omega_{1h} = \omega_{ch} A_1(b_h)$ ,  $\sigma_h^2 = \omega_{ch}^2 k_y \rho_{Th} S_h A_1(b_h) / \lambda$ . The solution of Eq. (11) has the form

$$\delta\omega = (\delta\omega_{1h} \pm \Omega_{1h}) / 2\beta_{1h}, \quad (12)$$

where  $\Omega_{1h} = (\delta\omega_{1h}^2 - 4\sigma_h^2 \beta_{1h})^{1/2}$ ,  $\beta_{1h} = 1 + g - G_{1h}$ .

The solution (12) gives the hydrodynamic shear-flow-driven EIC instability if inequality  $4\sigma_h^2 \beta_{1h} > \delta\omega_{1h}^2$  is met. For the wave numbers such as  $k_y \rho_{Th} \ll 1$  and respectively  $k_z \rho_{Tl} \ll 1$  this condition can be written as  $\lambda < \lambda_{1l}$ , where  $\lambda_{1l} \ll k_y \rho_{Th} S_h \beta_{1h} / A_1(b_h)$  is the long-wavelength threshold of instability for  $n=1$  harmonic.

Now we investigate the effect of  $\alpha$  and  $S_h$  on the growth rate of the hydrodynamic shear-flow-driven EIC instability. The growth rate of instability obtained from Eq. (11) is approximately

$$\gamma \ll [(\lambda_{1l}/\lambda) - 1]^{1/2} / 2\beta_{1h}. \quad (13)$$

With a decrease of  $\alpha$  the growth rate away from the long-wavelength threshold decreases approximately as  $\sqrt{\alpha}$ , however, the magnitude of threshold wavelength increases as  $\alpha$ , so that the longer waves become unstable. The dependence of the growth rate on the normalized shear  $S_h$  is expressed by the similar relation, i.e.  $\gamma \propto \sqrt{S_h}$  and  $\lambda_{1l} \propto S_h$ . Thus the effects of relative concentration of oxygen ions on the growth rate and long-wavelength threshold is identical with the flow velocity shear.

We also have solved numerically the dispersion equation (1) for the different values of relative concentration of oxygen ions and have obtained the dependence of the dispersion as well as growth rate versus the normalized wavelength along the magnetic field. The results of calculations for  $S_h = 3$ ,  $k_y \rho_{Th} = 1$  and  $\tau = 1$  are shown in Fig. 2.

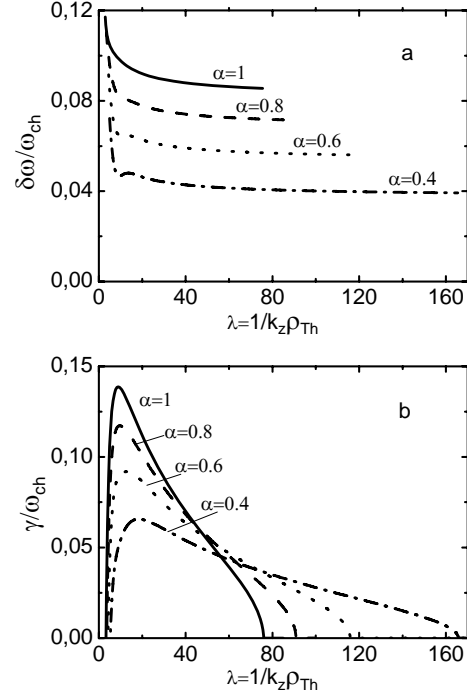


Fig. 2. The dispersion (a) and growth rate (b) of instability versus the normalized wavelength

The maximum of the growth rate occurs at  $|z_{1h}| \ll 1$  that is a boundary of ion-hydrodynamic mode which is located to the right of the point of maximum. The Fig. 2,b shows a decrease of the growth rate as well as an increase of the long-wavelength threshold with the decrease of  $\alpha$  that is in a good agreement with analytical results.

## 2. THE INSTABILITIES OF THE HIGH CYCLOTRON HARMONICS

Now we investigate the ion kinetic and hydrodynamic EIC instabilities of multicomponent plasma flow excited due to shear velocity flow for high cyclotron harmonics,  $\omega(k) = n\omega_{ch} + k_z V_{0h} + \delta\omega(k)$  with  $|n| \geq 2$  and  $\delta\omega(k) \ll \omega_{ci}$ . Assume that  $z_{hn}$  in the sum over cyclotron harmonics in Eq. (1) has an arbitrary value for the  $n = n'$  term, while in remaining sum  $|z_{hn}| > 1$ , for which the asymptotic form of  $W$  – function for large argument values may be used. The summation over cyclotron harmonics for heavy ions at  $k_y \rho_{Th} \ll 1$  gives [7]

$$\sum_{n \neq n'} W(z_{hn}) A_n(b_h) \left( \frac{\omega - k_z V_{0h}}{\sqrt{2} |k_z| V_{T\alpha}} - \frac{k_y}{k_z} S_h z_{hn} \right) \approx \psi(z_{\perp}) - \frac{k_y}{k_z} S_h (1 - A_{n'}(b_h)), \quad (14)$$

where  $z_{\perp} = (n'\omega_{ch} + \delta\omega)/\sqrt{2}k_y V_{Th} \approx n'/\sqrt{2}k_y \rho_{Th}$  and  $\psi(z_{\perp}) = -2z_{\perp} \exp(-z_{\perp}^2) \int_0^{z_{\perp}} \exp(t^2) dt$ . In the sum over cyclotron harmonics of light ions in Eq. (1) as in the previous section we retain only null term so that the dielectric permittivity of light ions becomes

$$\delta\varepsilon_{ln'} \approx \frac{1}{k^2 \lambda_{Dl}^2} \left( 1 - A_0(b_l) + \frac{k_y}{k_z} \frac{S_l A_0(b_l)}{2z_{l0n'}} \right), \quad (15)$$

where  $z_{l0n'} = n'\omega_{ch}/\sqrt{2}k_z V_{Tl}$ . Then the dispersion relation (1) has the form

$$k^2 \lambda_{Dh}^2 \varepsilon_m(K, \omega) = 1 + g_{n'} + \psi(z_{\perp}) - \frac{k_y}{k_z} S_h A_{n'}(b_h) + i\sqrt{\pi} \times \\ \times W(z_{in'}) A_{n'}(b_h) \left( \frac{n'\omega_{ci}}{\sqrt{2}k_z V_{Th}} + z_{hn'} - \frac{k_y}{k_z} S_i z_{hn'} \right) = 0, \quad (16)$$

where  $g_{n'} = \tau/\alpha + \delta\varepsilon_{ln'}(1-\alpha)/\alpha$ . Further we use index  $n$  instead of index  $n'$ .

For the determine of the short-wavelength boundary for ion-kinetic mode of the shear-flow-driven EIC instability we as in the previous section consider the system of equations which we obtain by equating to zero the real and imaginary parts of Eq. (16),

$$\begin{cases} \lambda/\sqrt{2} + z_{hn} - \lambda k_y \rho_{Th} S_h z_{hn} = 0, \\ 1 + g_n + \psi(z_{\perp}) - \lambda k_y \rho_{Th} S_h A_n(b_h) = 0. \end{cases} \quad (17)$$

This system has a solution when the inequality  $k_y \rho_{Th} S_h > 0$  is met. For this case we obtain the short-wavelength threshold value  $\lambda_{ns}$  for the excitation of the instability, at which normalized complex frequency  $z_{ns}$  becomes real

$$\lambda_{ns} = \frac{(1 + g_n + \psi(z_{\perp}))}{S_h k_y \rho_{Th} A_n(b_h)}, \quad (18)$$

$$z_{ns} = \frac{n(1 + g_n - G_n + A_n(b_h))}{\sqrt{2}k_y \rho_{Th} S_h (1 + g_n - G_n)}, \quad (19)$$

where  $G_n = A_n(b_h) - \psi(z_{\perp})$  and index  $ns$  means the short-wavelength threshold of instability for  $n$ -th cyclotron harmonic. Value  $\lambda_{ns}$  as well as for main cyclotron harmonic increases when  $\alpha$  decrease. The value  $\delta\omega = \delta\omega_{0n}$  at the instability threshold is  $\delta\omega_{0n} = n\omega_{ch} A_n(b_h)/(1 + g_n - G_n)$ . The growth rate for ion cyclotron instability at the vicinity of stability threshold can be obtained by use the same approach as for the first cyclotron harmonic

$$\gamma = \delta\omega_{0n} \frac{\sqrt{2}S_h |k_y| \rho_{Th} \text{Re}W(z_{ns})}{\sqrt{\pi n} |W(z_{ns})|^2} \left( 1 - \frac{\lambda_{ns}}{\lambda} \right). \quad (20)$$

The growth rate (20) is affected by threshold value  $z_{ns} \approx n/\sqrt{2}k_{\perp} \rho_{Th} S_h$ , because  $\gamma \propto \text{Re}W(z_{ns}) = \exp(-z_{ns}^2)$ . The growth rate of instability is exponentially small when the flow velocity shear and the transverse wave

number are such that  $\sqrt{2}|k_y| \rho_{Ti} S_i < n$ , whereas, at opposite inequality the growth rate is not exponentially small. The magnitude of the growth rate is also affected by the factor  $k_y \rho_{Ti} A_n(b_h)$ . The function  $A_n(b_h)$  at  $k_y \rho_{Th} \ll n \ll 1$  has asymptotic form

$$A_n(b_h) \approx \left( 1/\sqrt{2\pi} k_y \rho_{Th} \right) \exp\left(-n^2/2k_y^2 \rho_{Th}^2\right), \quad (21)$$

which implies that long waves with  $k_y \rho_{Th} < n$  have exponentially small growth rate. Thus, waves with longitudinal wavelength  $\lambda > \lambda_{ns}$  and transverse wave numbers such as  $k_y \rho_{Th} \ll n$  and  $\sqrt{2}k_y \rho_{Th} S_h > n$  are unstable. We note also that the threshold wavelength  $\lambda_{ns}$  (18) as well as  $z_{ns}$  (19) with these transverse wave numbers is approximately equal to the corresponding magnitudes for the first cyclotron harmonic (6), (7) with transverse wave numbers  $k_y \rho_{Th} \ll n$ .

It follows from Eqs. (20), that the dependence of the growth rate on the relative concentration of the heavy ions is identical to that for the main cyclotron harmonic.

Now we consider the excitation of the hydrodynamic EIC instability. Using the same assumptions as for the first harmonic we sum over cyclotron harmonics in Eq. (1) at  $k_y \rho_{Th} \ll n \ll 1$  and obtain approximately the dielectric permittivity of heavy ions as

$$\delta\varepsilon_h \approx \frac{1}{k^2 \lambda_{Dh}^2} \left( 1 - G_{hn} - \frac{n\omega_{ch}}{\delta\omega} A_n(b_h) + \frac{k_y}{k_z} \frac{k_z^2 V_{Th}^2}{\delta\omega^2} S_h A_n(b_h) \right). \quad (22)$$

In the dielectric permittivity of light ions (15) we take into account that inequality  $k_y \rho_{Th} \ll 1$  holds, so that  $k_y \rho_{Tl} > 1$  and then  $\delta\varepsilon_l \approx 1/k^2 \lambda_{Dl}^2$ . In this case the dispersion equation (1) takes the similar form as for the first harmonic (11). Its solution is

$$\delta\omega = (\omega_{nh} \pm \Omega_{nh})/2\beta_{nh}, \quad (23)$$

where  $\delta\omega_{nh} = n\omega_{ch} A_n(b_h)$ ,  $\Omega_{nh} = (\delta\omega_{nh}^2 - 4\sigma_h^2 \beta_{nh}^2)^{1/2}$ ,  $\beta_{nh} \approx 1 - G_{nh} + \tau/\alpha_h + \alpha_l/\alpha_h$ . The solution (23) gives the shear-flow-driven EIC instability if inequality  $4\sigma_h^2 \beta_{nh}^2 > \delta\omega_{nh}^2$  is met. This condition can be also written as  $\lambda < \lambda_{ln}$ , where  $\lambda_{ln} \approx k_y \rho_{Th} S_h \beta_{nh} / n^2 A_n(b_h)$  is the long-wavelength threshold of instability for  $n \ll 1$  harmonics. Because  $A_n(b_h)$  at  $k_y \rho_{Th} \ll n \ll 1$  has the asymptotic form (21) and for  $k_y \rho_{Th} = n$  we have  $A_n(b_h) \approx 0.2/n$  that gives  $\lambda_{ln} \approx \lambda_{l1}$ . Thus the long-wavelength threshold is the same as for the first and high cyclotron harmonics.

Evaluating the effect of the relative concentration on the condition  $|z_{hn}| > 1$ , we conclude that for  $k_y \rho_{Th} = n$  the condition on the  $\alpha$  coincides with that of the main harmonic. Now we estimate the effect of  $\alpha$  and  $S_h$  on the growth rate of high cyclotron harmonic of the shear-

flow-driven EIC instability. The growth rate of instability obtained from Eq. (23) approximately equals

$$\gamma \approx [(\lambda_{nl}/\lambda) - 1]^{1/2} / 2\beta_{nh}. \quad (24)$$

Since  $\lambda_{l1} \approx \lambda_{nl}$  we obtain that the dependence of growth rate on the concentration and the shear is the same as for the main cyclotron harmonic.

### CONCLUSIONS

In the magnetic-field-aligned plasma flow with the flow velocity shear which is composed of  $O^+$  and  $H^+$  ions the ion-kinetic and hydrodynamic shear-flow-driven EIC instabilities with the cyclotron frequency of  $O^+$  have been investigated. The effect of presence of the  $H^+$  ions in plasma flow and decreasing of the relative concentration  $\alpha$  of  $O^+$  ions is as follows:

1. The growth rates as well as the dispersion additions to the ion cyclotron frequency as well as to their harmonics are decrease proportionally with the decreasing of  $\alpha$ .

2. The regions of unstable wavelengths of these instabilities is almost independent on the number of the ion cyclotron harmonic at  $\alpha = 1$ .

3. The regions of unstable wavelengths of these instabilities for  $n=1$  cyclotron harmonic with decreasing  $\alpha$  are expanding whereas for the high cyclotron harmonics these regions are slightly narrowed.

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## ИОННЫЕ ЦИКЛОТРОННЫЕ НЕУСТОЙЧИВОСТИ ПОТОКА МНОГОКОМПОНЕНТНОЙ ПЛАЗМЫ ВДОЛЬ МАГНИТНОГО ПОЛЯ СО СДВИГОМ ПОТОКОВОЙ СКОРОСТИ

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Исследованы ионные циклотронные неустойчивости многокомпонентного потока плазмы вдоль магнитного поля со сдвигом потоковой скорости. Предполагается, что в состав плазмы входят ионы двух видов,  $H^+$  и  $O^+$ , наиболее распространенных в ионосфере Земли. Рассматривается влияние ионов водорода на возбуждаемые широм потоковой скорости ионную кинетическую и гидродинамическую ионные циклотронные неустойчивости с частотами колебаний приблизительно равными циклотронной частоте ионов  $O^+$ .

## ІОННІ ЦИКЛОТРОННІ НЕСТІЙКОСТІ ПОТОКУ БАГАТОКОМПОНЕНТНОЇ ПЛАЗМИ ВЗДОВЖ МАГНІТНОГО ПОЛЯ ЗІ ЗСУВОМ ПОТОКОВОЇ ШВИДКОСТІ

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Досліджено іонні циклотронні нестійкості багатоконпонентного потоку плазми вздовж магнітного поля зі зсувом потокової швидкості. Припускається, що до складу плазми входять іони двох видів,  $H^+$  і  $O^+$ , найбільш поширених в іоносфері Землі. Розглядається вплив іонів водню на іонну кінетичну і гидродинамічну іонні циклотронні нестійкості, що збуджуються широм потокової швидкості, з частотами коливань приблизно рівними циклотронній частоті іонів  $O^+$ .