TRANSFORMATION RATIO AT WAKEFIELD EXCITATION IN DIELECTRIC RESONATOR ACCELERATOR BY SHAPED SEQUENCE OF ELECTRON BUNCHES WITH LINEAR GROWTH OF CURRENT

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Transformation ratio has been derived in the case of wakefield excitation in dielectric resonator accelerator by sequence of electron bunches, the charge of which is shaped according to linear law, so that in every bunch (except the first one) the charge is distributed according to the rectangular trapezoid. At the use of certain bunch-precursor before the sequence the wakefield in the areas of localization of bunches-drivers does not depend on the longitudinal coordinate. For the increase of number of accelerated electrons the long asymptotics of sequence of electron bunches has been derived. In this asymptotic infinite sequence the short trains of drivers-rectangular trapezoids are interchanged by accelerated "high-current" bunches. This asymptotic infinite sequence provides the large transformation ratio and homogeneous decelerating wakefield, which results in the complete deceleration of bunches-drivers. Connection of transformation ratio with the reduction rate of the wakefield after witness and connection of the transformation ratio with the witness charge and driver charge in this asymptotic sequence have been derived.

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INTRODUCTION

The efficiency of wakefield excitation in dielectric resonator accelerator by sequence of electron bunches is determined by transformation ratio (see [1, 2]). The transformation ratio, defined as ratio $R=\frac{E_2}{E_1}$ of the wakefield E_2, which is excited in dielectric resonator accelerator by sequence of the electron bunches, to the field E_1, in which an electron bunch is decelerated, is considered with charge shaping according to linear law along sequence as well as along each bunch [3, 4]. In each bunch (except the first one) the charge is distributed accordingly to rectangular trapezoid. The bunch length equals to wave-length $\Delta\xi_b=\lambda$. The choice of such length of bunches is determined by the necessity to provide not only large R but high gradient wakefield too excited by sequence of N bunches. The porosity between bunches is multiple of wave-length $\delta\xi=p\lambda$, p=1, 2, ... A rectangular bunch as a precursor of a small charge and of length, equal to one fourth of wavelength $\Delta\xi_0=\lambda/4$, is injected before the first bunch. A next bunch is injected in the resonator, when the back wavefront of wakefield pulse, excited by previous bunches, is on the injection boundary ($z=0$). A next bunch leaves the resonator, when the first wavefront of wakefield pulse, excited by previous bunches, is on the end of the resonator. Then wakefield pulses, excited by all consistently injected bunches, are coherently added. In other words, coherent accumulation of wakefield is realized. For all major bunches the slowing down wakefield is homogeneous (in other words the same) and small. Then one can provide a large transformation ratio R. But several conditions should be correct for this purpose. The wakefield and transformation ratio have been derived after N-th bunch.

Also the optimized infinite sequence, composed of accelerated bunches-witnesses and short trains of electron bunches-drivers, exciting wakefield, has been derived. In each bunch of these short trains the charge is distributed accordingly to rectangular trapezoid with determined properties. The connection of charge of accelerated bunches with their energy and transformation ratio has been obtained analytically for this infinite sequence. Also the connection of transformation ratio with the reduction rate of the wakefield after witness and connection of the transformation ratio with the witness charge and driver charge in this asymptotic sequence have been derived analytically.

1. TRANSFORMATION RATIO AT WAKEFIELD EXCITATION IN A DIELECTRIC RESONATOR AT CHARGE RAMPING OF BUNCH SEQUENCE BY LINEAR LAW

In this paper the transformation ratio R is investigated theoretically. In many cases transformation ratio can be concluded to the ratio of maximum accelerating wakefield, experienced by witness bunch, to the maximum slowing down wakefield, experienced by driver bunches. We consider injection of bunches with length $\Delta\xi_b$, equal to the wavelength $\Delta\xi_b=\lambda$, the charge of which is ramped according to linear law, both along the sequence of bunches and along each bunch (in each bunch the charge is distributed accordingly to rectangular trapezoid), in the dielectric resonator of length L. The choice of such length of bunches is determined by the necessity to provide not only large R but high gradient excited wakefield too.

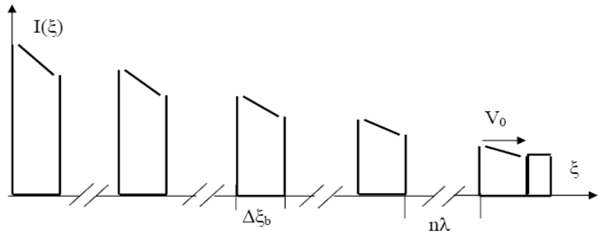

Fig. 1. The current distribution of short rectangular bunch-precursor and of sequence of rectangular trapezoid bunches

A rectangular bunch as a precursor of a small charge and of length, equal to one fourth of wavelength $\Delta\xi_0 = \frac{\lambda}{4}$, is injected before the first bunch.

So the charge density of short rectangular bunch-precursor and of sequence of rectangular trapezoid bunches is distributed according to (Fig. 1)

$$n_b(z,\,t)=n_{b0}\,,\ 0<V_0t\text{-}z<\Delta\xi_0\,,\ 0<t<\frac{\left(L+\Delta\xi_0\right)}{V_0}\,, \quad (1)$$

$$n_b(z,\,t)=n_{b0}\left[1-\frac{\pi}{2}+\left(V_0t\text{-}z\right)\frac{2\pi}{\lambda}\right],\ N=1,$$

$$\Delta\xi_0<V_0t\text{-}z<\Delta\xi_0+\Delta\xi_b\,,\ \Delta\xi_0/V_0<t<\frac{\left(L+\Delta\xi_0+\Delta\xi_b\right)}{V_0};$$

$$n_b(z,\,t)=n_{b0}\left[1+\left(N-1\right)2\pi+\left[V_0\left(t\text{-}T\left(N\text{-}1\right)\right)\text{-}z\right]\frac{2\pi}{\lambda}\right],$$

$$N>1,\ 0<V_0\left(t\text{-}T\left(N\text{-}1\right)\right)\text{-}z<\Delta\xi_b\,,$$

$$T\left(N\text{-}1\right)<t<T\left(N\text{-}1\right)+\frac{\left(L+\Delta\xi_b\right)}{V_0}\,. \quad (2)$$

Then the ratio of charge Q_N of the N-th bunch to charge Q_1 of the 1-st bunch equals

$$Q_N/Q_1=\frac{\left[1+\left(2N-1\right)\pi\right]}{\left(1+\pi\right)}\,. \quad (3)$$

At large N>>1 this ratio increases along sequence according to $\frac{Q_N}{Q_1}\propto N$. The general charge Q_Σ of sequence is equal to $Q_\Sigma = Q_1\frac{(N+N^2\pi)}{(1+\pi)}$.

A next bunch is injected in the resonator, when the back wavefront of wakefield pulse, excited by previous bunches, is on the injection boundary ($z=0$) (Fig. 2).

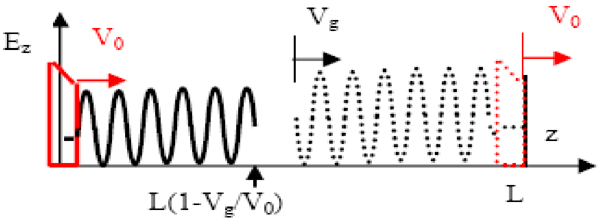

Fig. 2. A schematic of the wakefield pulse (continuous), excited by previous two bunches, when 3-rd bunch is injected in the resonator. A schematic of the wakefield pulse (dotted), excited by three bunches, when 3-rd bunch leaves the resonator

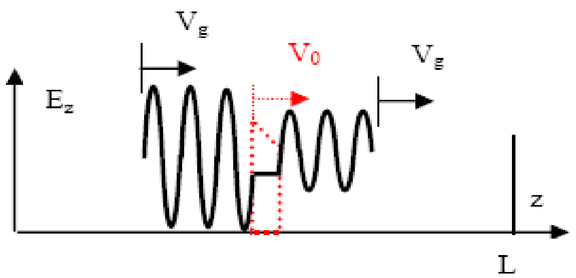

Fig. 3. An approximate view of the wakefield pulse, excited by previous two bunches and excited by 3-rd bunch, when 3-rd bunch is in the middle of the resonator

A next bunch leaves the resonator, when the first wavefront of wakefield pulse, excited by previous bunches, is on the end of the resonator ($z=L$) (see Fig. 2). An approximate view of the wakefield pulse, excited by previous two bunches and excited by 3-rd bunch, when 3-rd bunch is in the middle of the resonator, is shown in Fig. 3.

Excited longitudinal wakefield E_z is non-uniform along the bunch-precursor $\xi\le\Delta\xi_0$, and for all major bunches the slowing down wakefield E_z is homogeneous (in other words the same) and small. Then one can provide a large transformation ratio R. But several conditions should be correct for this purpose. Namely, we choose the length of the resonator L, the group velocity V_g and the frequency of bunch repetition rate ω_m and the wave frequency, which satisfy the following equality

$$T=\frac{2L}{V_g}=\frac{2\pi}{\omega_m}=\frac{2\pi n}{\omega_0}\,,\ n=1,2,\ldots\,. \quad (4)$$

Then for the selected length of the resonator and n, equal $\frac{L}{\lambda}=4$ and $n=10$ group velocity should be equal $\frac{V_g}{V_0}=0.8$. V_0 is the beam velocity. For $\frac{L}{\lambda}=5$ and $n=16$ group velocity should be equal $\frac{V_g}{V_0}=0.625$.

Thus, all the next bunches after the first one begin to be injected in the resonator (on the boundary $z=0$), when the trailing edge of the wakefield pulse, created by the previous bunches, is located at the point $z=0$. At this moment the leading edge of the wakefield pulse, located at the distance from the injection boundary, equal to $L\left(1-\frac{V_g}{V_0}\right)$ (see Fig. 2), is located at the distance $L\left(\frac{V_g}{V_0}\right)$ from the end of the resonator ($z=L$).

Again injected bunch reaches the end of the resonator together with the leading edge of the wakefield pulse, created by the previous bunches. Then wakefield pulses, excited by all consistently injected bunches, are coherently added. In other words, coherent accumulation of wakefield is realized.

At wakefield pulse excitation by the 1-st bunch the wakefield in the whole resonator within the time $0<t<\frac{\left(L+\Delta\xi_0+\Delta\xi_b\right)}{V_0}$ accordingly to [2, 5] equals to

$$E_z(z,\,t)=$$

$$=-\left[\frac{R}{Q}\right]\left(\frac{I_0}{2}\right)\{\,\theta(V_0t\text{-}z)\theta(z\text{-}V_0t\text{+}\Delta\xi_0)\sin\left(k\xi\right)+$$

$$+\,\theta(V_0t\text{-}\Delta\xi_0\text{-}z)\theta(z\text{-}V_0t\text{+}\Delta\xi_0+\Delta\xi_1)+ \quad (5)$$

$$+\left[\left(1\text{+}2\pi\right)\cos\left(k\xi\right)+\sin\left(k\xi\right)\right]\theta(V_0t\text{-}\Delta\xi_1\text{-}\Delta\xi_0\text{-}z)\theta(z\text{-}V_gt)\,\},$$

$\xi=V_0t\text{-}z$. The 1-st term is the field inside of the bunch-precursor, the 2-nd term is the field inside the 1-st bunch, the 3-rd term is the wakefield after the 1st bunch. The field is uniform inside the 1-st bunch.

$$E_z(\xi)\Big|_{\xi=\lambda/4}=-\left[\frac{R}{Q}\right]\left(\frac{I_0}{2}\right)\sin(k\xi)\Big|_{\xi=\lambda/4}=-\left[\frac{R}{Q}\right]\left(\frac{I_0}{2}\right). \quad (6)$$

Such small and constant field $E_z(\xi)\Big|_{\xi=\lambda/4}=-\left[\frac{R}{Q}\right]\left(\frac{I_0}{2}\right)$ is inside all the other bunches.

At wakefield pulse excitation by the 2-nd bunch the wakefield in whole resonator within the time $T\le t\le T+\frac{(L+\Delta\xi_b)}{V_0}$, $T=\frac{2L}{V_g}$, equals to

$$E_z(z,t)=-\left[\frac{R}{Q}\right]\left(\frac{I_0}{2}\right)\times\{\theta[V_0(t-T)-z]\theta[z+\lambda-V_0(t-T)]+$$
$$+[(1+2\pi)\cos(k\xi)+\sin(k\xi)]\times$$
$$\times\{\theta[V_g(t-T)+L(1-\frac{V_g}{V_0})-z]\,\theta[z-V_g(t-T)]-$$
$$-\theta[V_0(t-T)-z]\theta[z+\lambda-V_0(t-T)]\}+$$
$$+2\pi\cos(k\xi)\theta[V_0(t-T)-\lambda-z]\theta[z-V_g(t-T)]\}. \quad (7)$$

Here the last term is the wakefield, excited by the 2-nd bunch, the first term is the slowing down field in the 2-nd bunch (uniform and small) and the second term is the wakefield, excited by 1-st bunch.

At wakefield pulse excitation by the N-th bunch the wakefield in whole resonator within the time $T(N-1)\le t\le T(N-1)+\frac{(L+\Delta\xi_b)}{V_0}$, $T=\frac{2L}{V_g}$, equals to

$$E_z(z,t)=-\left[\frac{R}{Q}\right]\left(\frac{I_0}{2}\right)\times$$
$$\times\{\theta[V_0(t-T(N-1))-z]\theta[z+\lambda-V_0(t-T(N-1))]+$$
$$+[(1+2\pi(N-1))\cos(k\xi)+\sin(k\xi)]\times$$
$$\times\{\theta[V_g(t-T(N-1))+L(1-\frac{V_g}{V_0})-z]\,\theta[z-V_g(t-T(N-1))]-$$
$$-\theta[V_0(t-T(N-1))-z]\theta[z+\lambda-V_0(t-T(N-1))]\}+$$
$$+2\pi\cos(k\xi)\theta[V_0(t-T(N-1))-\lambda-z]\theta[z-V_g(t-T(N-1))]\}. \quad (8)$$

Here the last term is the wakefield, excited by the N-th bunch, the first term is the slowing down field in the N-th bunch (uniform, small and the same for all bunches) and the second term is the wakefield, excited by $N-1$ bunches. Thus, if the lengths of all bunches are equal $\Delta\xi_b=\lambda$, after the N-th bunch the transformation ratio is equal to

$$R=\sqrt{1+(1+2\pi N)^2}\approx 2\pi N. \quad (9)$$

Thus the choice of such ramped sequence of bunches (1), (2) provides not only large R (9), but also large amplitude of excited wakefield $E_{0N}\approx NE_{01}$ (8). Thus the total charge Q_Σ of sequence is proportional $Q_\Sigma\approx N^2Q_1$, i.e. it is the much larger in comparison with nonramped sequence, for which $Q_\Sigma=NQ_1$.

2. ASYMPTOTIC INFINITE SEQUENCE OF SHORT TRAINS OF DRIVERS-RECTANGULAR TRAPEZOIDS, INTERCHANGED BY ACCELERATED «HIGH-CURRENT» BUNCHES

We consider the case, when after N bunches-trapezoids the sequence continues as asymptotic infinite sequence of interchanging short (from K bunches-trapezoids) trains of the shaped drivers and separate "high-current" witnesses (Fig. 4). Then after every K-th bunch-driver a "high-current" witness follows, so that it takes away considerable energy. Thus after witness the amplitude of the wakefield decreases from $E_{z0}=NE_1$ to $\chi E_{z0}=(N-K)E_1$, $\chi=\frac{(N-K)}{N}<1$, $K=N(1-\chi)$. In this case the transformation ratio, at the use of the averaged over accelerating time of accelerating field, equals

$$R\approx\frac{[NE_1+(N-K)E_1]}{2E_{sl}}=(N-\frac{K}{2})\frac{E_1}{E_{sl}}. \quad (10)$$

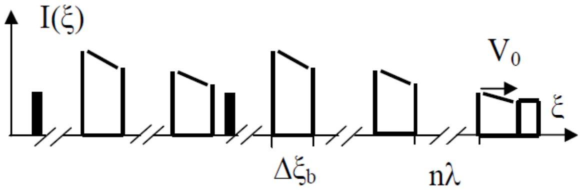

Fig. 4. The current distribution of short rectangular bunch-precursor and of asymptotic sequence of short trains of rectangular trapezoid bunches and of witnesses

We use that the transformation ratio $R=2\pi N$ known at K=0. Then $\frac{E_1}{E_{sl}}=2\pi$ and we obtain the connection of R with χ and with number of bunches N of the sequence, after which the periodic quasi-stationary asymptotic wakefield is set,

$$R\approx\pi N(1+\chi). \quad (11)$$

Here E_{sl} is the decelerating wakefield in the region of being of bunch-trapezoid.

We specify that the words "high-current" witness mean. From balance of energies one can derive

$$Rq_W=Kq_N\left[1+\pi(2N-K)\right]/\left[1+\pi(2N-1)\right]. \quad (12)$$
$$q_N=I_0\lambda\left[1+\pi(2N-1)\right].$$

Here q_N is the charge of N-th bunch. Then the ratio of witness charge to driver charge equals

$$q_W/q_N=K/\pi(2N-1)=(1-\chi)/\pi(2-1/N). \quad (13)$$

One can see that the maximal ratio q_W/q_N equals $q_W/q_N=1/\pi(2-1/N)$ at K=N, i.e. at χ=0. Thus $R=\pi N$ for infinite sequence. As $K\ge1$, then $q_W/q_N\ge1/\pi(2N-1)$. The transformation ratio, equal to $R=1+\pi(2N-1)$, is achieved at $q_W \ll q_N$.

From (11), (13) one can derive the connection R with N and with q_W/q_N

$$R=2N\pi\left[1-(1-1/2N)\pi q_W/q_N\right]. \quad (14)$$

Thus $\pi N\le R\le1+\pi(2N-1)$, because

$$1/\pi(2N-1)\le q_W/q_N\le1/2\pi(1-1/2N).$$

CONCLUSIONS

So it has been shown that in the case of wakefield excitation in dielectric resonator accelerator by sequence of electron bunches, the charge of which is shaped according to linear law, so that in every bunch (except the first one) the charge is distributed according to the rectangular trapezoid, the transformation ratio can achieve large value. At the use of certain bunch-precursor before the sequence the wakefield in the areas of localization of bunches-drivers does not depend on the longitudinal coordinate.

For the increase of number of accelerated electrons the long asymptotics of sequence of electron bunches has been derived. In this asymptotic infinite sequence the short trains of drivers-rectangular trapezoids are interchanged by accelerated "high-current" bunches. This asymptotic infinite sequence provides the large transformation ratio and homogeneous decelerating wakefield, which results in the complete deceleration of bunches-drivers. The connection of transformation ratio with the reduction rate of the wakefield after witness and connection of the transformation ratio with the witness charge and driver charge in this asymptotic sequence have been derived.

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КОЭФФИЦИЕНТ ТРАНСФОРМАЦИИ ПРИ ВОЗБУЖДЕНИИ КИЛЬВАТЕРНОГО ПОЛЯ В ДИЭЛЕКТРИЧЕСКОМ РЕЗОНАТОРНОМ УСКОРИТЕЛЕ ПРОФИЛИРОВАННОЙ ПОСЛЕДОВАТЕЛЬНОСТЬЮ ЭЛЕКТРОННЫХ СГУСТКОВ С ЛИНЕЙНО НАРАСТАЮЩИМ ТОКОМ

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Получен коэффициент трансформации в случае возбуждения кильватерного поля в диэлектрическом резонаторном ускорителе последовательностью электронных сгустков, заряд которых профилирован по линейному закону, так что в каждом сгустке (кроме первого) заряд распределен согласно прямоугольной трапеции. При использовании определенного сгустка-предвестника перед последовательностью кильватерное поле в областях локализации сгустков-драйверов не зависит от продольной координаты. Для увеличения числа ускоренных электронов построена длинная асимптотика последовательности электронных сгустков. В этой асимптотической бесконечной последовательности короткие цепочки драйверов-прямоугольных трапеций чередуются ускоряемыми «сильноточными» сгустками. Эта асимптотическая бесконечная последовательность обеспечивает большой коэффициент трансформации и однородное тормозящее кильватерное поле, которое приводит к полному торможению сгустков-драйверов. Определена связь коэффициента трансформации с коэффициентом уменьшения поля после витнеса и связь коэффициента трансформации с зарядом витнеса и зарядом драйвера в этой асимптотической последовательности.

КОЕФІЦІЄНТ ТРАНСФОРМАЦІЇ ПРИ ЗБУДЖЕННІ КІЛЬВАТЕРНОГО ПОЛЯ В ДІЕЛЕКТРИЧНОМУ РЕЗОНАТОРНОМУ ПРИСКОРЮВАЧІ ПОСЛІДОВНІСТЮ ПРОФІЛЬОВАНИХ ЕЛЕКТРОННИХ ЗГУСТКІВ З ЛІНІЙНО НАРОСТАЮЧИМ СТРУМОМ

В.І. Маслов, І.М. Оніщенко

Отримано коефіцієнт трансформації у разі збудження кільватерного поля в діелектричному резонаторному прискорювачі послідовністю електронних згустків, заряд яких профільований за лінійним законом, так що в кожному згустку (окрім першого) заряд розподілений згідно прямокутної трапеції. При використанні певного згустка-передвісника перед послідовністю кільватерне поле в областях локалізації згустків-драйверів не залежить від подовжньої координати. Для збільшення числа прискорених електронів побудована довга асимптотика послідовності електронних згустків. У цій асимптотичній нескінченній послідовності короткі ланцюжки драйверів-прямокутних трапецій чергуються прискорюваними "сильнострумовими" згустками. Ця асимптотична нескінченна послідовність забезпечує великий коефіцієнт трансформації і однорідне гальмівне кільватерне поле, яке призводить до повного гальмування згустків-драйверів. Визначено зв'язок коефіцієнта трансформації з коефіцієнтом зменшення поля після вітнеса і зв'язок коефіцієнта трансформації із зарядом вітнеса і зарядом драйвера в цій асимптотичній послідовності.