FOCUSING OF ELECTRON AND POSITRON BUNCHES IN PLASMA-DIELECTRIC WAKEFIELD ACCELERATORS

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The results of analytical studies and numerical simulations of wakefield excitation in a plasma-dielectric structure are presented. In the linear theory approximation (overdense plasma) it is shown that at a certain plasma density the superposition of the plasma wave and the dielectric waves allows the acceleration of the witness bunch with simultaneous focusing. Also, we carried out a PIC simulation of the underdense ("blowout") and overdense regime of wakefield excitation in the unit.

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INTRODUCTION

Using of dielectric structures for acceleration of charged particles by the wakefields excited in them by relativistic electron bunches, is actively developing direction of new methods of acceleration [1, 2]. The experimental researches carried out, in ANL and SLAC confirmed [3, 4] performability of this method of acceleration of charged particles. Now the dielectric wakefield accelerator is considered as the promising candidate for future electron-positron colliders of TeV range of energy [5].

Despite shown both theoretically, and experimentally possibilities of obtaining a high acceleration rates, one problem, that is not solved completely, remains - difficulties with stabilization of transverse motion of the drive and accelerated bunches and, thereof, obtaining the accelerated bunches of particles with a small emittance. In this article possibility of using for this purpose of plasma filling the drift channel of dielectric structure is considered. Such plasma can be created as a result of the capillary discharge in a dielectric tube [6]. Plasma use for focusing of an accelerated bunch is not the new proposition. Focusing properties of plasma were investigated in PWFA both in the linear condition [7, 8] and in a non-linear regime [9, 10]. But in the linear condition the peak of an accelerating field corresponds the zero focusing field, and in a non-linear regime the region of acceleration is localized only near a drive bunch because of a destruction of a non-linear plasma wave. As we will demonstrate below, using of plasma-dielectric structure allows avoiding these restrictions.

1. STATEMENT OF THE PROBLEM

Let's consider the dielectric waveguide of an annular cross-section surrounded with a metal sheath of radius b, the inner radius of the dielectric bushing is equal to a. A dispersion of the dielectric tube permittivity is absent and is equal to ε_d. The interior of such dielectric waveguide $r<a$ (the accelerating channel) is completely filled with isotropic plasma of density n_p. The solid monoenergetic electron bunch with initial velocity v_0 and charge Q_0 starts being injected into the accelerating channel, parallel to a waveguide axis, at some time. The electron bunch builds up electromagnetic fields which can be used for increasing of energy of accelerated bunches in so called wake method of acceleration of the charged particles. The object of the present research is finding of longitudinal (accelerating/braking) forces, and also transverse forces which are responsible for transverse dynamics of drive and driven bunches. These forces will be used further at an simulation of a motion of test charged particles.

2. LINEAR APPROXIMATION

In the linear approximation we will assume that ions of plasma are immobile, and perturbations of plasma electrons, caused by motion of electrons of plasma, are linear. We will neglect the change of speed of bunch electrons, also as thermal motions of electrons of plasma and a bunch. In view of a linearity of plasma its dispersion properties can be described an dielectric permittivity $\varepsilon_p = 1-\omega_p^2/\omega^2$, $\omega_p^2 = 4\pi e^2 n_p/m$, where $-e$ and m are the charge and the mass of electron, n_p is a non-perturbed density of plasma, ω is an angular frequency. The set of Maxwell equations, describing excitation of the axially-symmetrical electromagnetic fields by electron bunches in a plasma-dielectric waveguide of a cylindrical configuration, looks like:

$$\frac{\partial E_r}{\partial z}-\frac{\partial E_z}{\partial r}=-\frac{1}{c}\frac{\partial H_\varphi}{\partial t}, \quad -\frac{\partial H_\varphi}{\partial z}=\frac{1}{c}\frac{\partial D_r}{\partial t},$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(rH_\varphi\right)=\frac{1}{c}\frac{\partial D_z}{\partial t}+\frac{4\pi}{c}j_z,\ D_{r,z}=\int_{-\infty}^{\infty}\varepsilon(\omega)E^{\omega}_{r,z}d\omega, \tag{1}$$

where $\varepsilon(\omega)=\begin{cases}\varepsilon_p(\omega), & \text{if } r<a,\\ \varepsilon_d, & \text{otherwise}\end{cases}$, $E^{\omega}_{r,z}$ is Fourier transform from $E_{r,z}$, j_z is a current density of bunches.

Let's find at first the electromagnetic field excited by annular infinitely thin in axial direction bunch. For such bunch the current density j_z is described by expression:

$$j_z=\frac{Q_0}{2\pi r}\delta(r-r_0)\delta(\tau-t_0), \tag{2}$$

where $\tau=t-z/v_0$, r_0 is a bunch radius, t_0 is a arrival time of bunch into waveguide ($z=0$), δ is the Dirac delta function.

For finding of the solution of the set of equations (1), (2) we will use well-known procedure (see, for example, [8, 11]). It consists in a decomposition of the equations (1), (2) in a Fourier integrals over time t and the longitudinal coordinate z, solution of obtained ordinary differential equations set relatively a variable r by method of partial regions and a performing of an inverse Fourier transform with use of the calculus of residues. As a result we will obtain:

$$E_z = E_z^L + E_z^d + E_z^C, \quad (3)$$

$$E_r = E_r^L + E_r^d + E_r^C, \quad (4)$$

$$H_\varphi = H_\varphi^d + H_\varphi^C. \quad (5)$$

Components of fields with the "d" index correspond to a wakefield of a dielectric waveguide, they are exist in case of the vacuum drift channel $(n_p = 0)$ also, being modified when filling with its plasma. They look like:

$$E_z^d = -2E_0\,\Theta(\tau - t_0)\sum_s e_z^s(r_0)e_z^s(r)\cos\omega_s(\tau - t_0), \quad (6)$$

$$E_r^d = 2E_0\,\Theta(\tau - t_0)\sum_s e_z^s(r_0)e_r^s(r)\sin\omega_s(\tau - t_0), \quad (7)$$

$$H_\varphi^d = 2E_0\,\Theta(\tau - t_0)\sum_s e_z^s(r_0)h_\varphi^s(r)\sin\omega_s(\tau - t_0), \quad (8)$$

where eigenfunctions e_z^s, e_r^s, h_φ^s are defined as follows:

$$e_z^s(r) = \left(\frac{a}{\omega_s D'(\omega_s)}\right)^{1/2} \begin{cases} \dfrac{I_0(\kappa_p^s r)}{I_0(\kappa_p^s a)}, & \text{if } r \le a \\ \dfrac{F_0(\kappa_d^s r, \kappa_d^s b)}{F_0(\kappa_d^s a, \kappa_d^s b)}, & \text{otherwise} \end{cases}, \quad (9)$$

$$e_r^s(r) = -\left(\frac{a}{\omega_s D'(\omega_s)}\right)^{1/2} \begin{cases} \dfrac{-1}{\sqrt{1-\beta_0^2\varepsilon_p(\omega_s)}}\dfrac{I_1(\kappa_p^s r)}{I_0(\kappa_p^s a)}, & r \le a \\ \dfrac{1}{\sqrt{\beta_0^2\varepsilon_d - 1}}\dfrac{F_1(\kappa_d^s r, \kappa_d^s b)}{F_0(\kappa_d^s a, \kappa_d^s b)}, & r > a \end{cases}, \quad (10)$$

$$h_\varphi^s(r) = \beta_0\varepsilon(\omega_s)e_r^s(r). \quad (11)$$

In expressions (6)–(11) the notations are used: $\kappa_p^s = \left[1-\beta_0^2\varepsilon_p(\omega_s)\right]^{1/2}\omega_s/v_0$, $\kappa_d^s = \left(\beta_0^2\varepsilon_d - 1\right)^{1/2}\omega_s/v_0$ $F_n(x,y) = (-1)^n\left[J_n(x)Y_0(y) - Y_n(x)J_0(y)\right]$, J_n and Y_n are the Bessel and the Weber functions of n-th order, I_n are the modified Bessel function of n-th order, Θ is the Heaviside function; $E_0 = 2Q_0/a^2$, $\beta_0 = v_0/c$; ω_s are eigenfrequencies of a dielectric waveguide which are defined from the dispersion equation:

$$D(\omega) \equiv \frac{\varepsilon_p}{\kappa_p}\frac{I_1(\kappa_p a)}{I_0(\kappa_p a)} + \frac{\varepsilon_d}{\kappa_d}\frac{F_1(\kappa_d a, \kappa_d b)}{F_0(\kappa_d a, \kappa_d b)} = 0. \quad (12)$$

$D'(\omega_s)$ in expressions (9) - (10) is a derivative of dispersion function $D(\omega)$ with respect to frequency ω, calculated at $\omega = \omega_s$, a root of equation (12).

Components of fields with the "C" index in expressions (3) - (5) describe a quasistatic field of a bunch. Components of quasistatic fields are equal:

$$E_z^C = -E_0\,\mathrm{sgn}(\tau - t_0)\sum_s \bar{e}_z^s(r_0)\bar{e}_z^s(r)\exp[-\bar{\omega}_s|\tau - t_0|], \quad (13)$$

$$E_r^C = E_0\sum_s \bar{e}_z^s(r_0)\bar{e}_r^s(r)\exp[-\bar{\omega}_s|\tau - t_0|], \quad (14)$$

$$H_\varphi^C = E_0\sum_s \bar{e}_z^s(r_0)\bar{h}_\varphi^s(r)\exp[-\bar{\omega}_s|\tau - t_0|], \quad (15)$$

where:

$$\bar{e}_z^s(r) = \left(\frac{a}{\bar{\omega}_s\bar{D}'(\bar{\omega}_s)}\right)^{1/2} \begin{cases} \dfrac{J_0(\bar{\kappa}_p^s r)}{J_0(\bar{\kappa}_p^s a)}, & \text{if } r \le a \\ \dfrac{\Delta_0(\bar{\kappa}_d^s r, \bar{\kappa}_d^s b)}{\Delta_0(\bar{\kappa}_d^s a, \bar{\kappa}_d^s b)}, & \text{else} \end{cases}, \quad (16)$$

$$\bar{e}_r^s(r) = \left(\frac{a}{\bar{\omega}_s\bar{D}'(\bar{\omega}_s)}\right)^{1/2} \begin{cases} \dfrac{1}{\sqrt{1-\beta_0^2\bar{\varepsilon}_p}}\dfrac{J_1(\bar{\kappa}_p^s r)}{J_0(\bar{\kappa}_p^s a)}, & \text{if } r \le a \\ \dfrac{1}{\sqrt{\beta_0^2\varepsilon_d - 1}}\dfrac{\Delta_1(\bar{\kappa}_d^s r, \bar{\kappa}_d^s b)}{\Delta_0(\bar{\kappa}_d^s a, \bar{\kappa}_d^s b)}, & \text{else} \end{cases}, \quad (17)$$

$$\bar{h}_\varphi^s(r) = \beta_0\bar{\varepsilon}(\bar{\omega}_s)\bar{e}_r^s(r) \quad (18)$$

and

$\bar{\kappa}_p^s = \left[1-\beta_0^2\bar{\varepsilon}_p\right]^{1/2}\bar{\omega}_s/v_0$, $\bar{\kappa}_d^s = \left[\beta_0^2\varepsilon_d - 1\right]^{1/2}\bar{\omega}_s/v_0$, $\Delta_n(x,y) = I_n(x)K_0(y) - (-1)^n K_n(x)I_0(y)$, K_n are Macdonald functions; sgn(x) is a function, equal to 1, 0 and –1, if x>0, x=0 and x<0, correspondingly; $\bar{\varepsilon}_p = 1 + \omega_p^2/\bar{\omega}_s^2$, $\bar{\omega}_s$ are eigenvalues which are defined from the equation:

$$\bar{D}(\bar{\omega}) \equiv \frac{\bar{\varepsilon}_p}{\bar{\kappa}_p}\frac{J_1(\bar{\kappa}_p a)}{J_0(\bar{\kappa}_p a)} - \frac{\varepsilon_d}{\bar{\kappa}_d}\frac{\Delta_1(\bar{\kappa}_d a, \bar{\kappa}_d b)}{\Delta_0(\bar{\kappa}_d a, \bar{\kappa}_d b)} = 0. \quad (19)$$

$\bar{D}'(\bar{\omega}_s)$ in expressions (16)–(17) is a derivative of function $\bar{D}(\bar{\omega})$ with respect to $\bar{\omega}$, calculated at $\bar{\omega} = \bar{\omega}_s$.

It should be noted that the equation (19) can be obtained from the equation (12) if in the last to make replacement $\omega = i\bar{\omega}$, and to impose a condition

$$\bar{\omega}_s > \omega_p\beta_0\gamma_0,\ \gamma_0 = (1-\beta_0^2)^{-1/2}. \quad (20)$$

In consequence of this condition, for ultrarelativistic electron bunch and high densities of plasma (namely such cases are of interest for perspective wake accelerators), the quasistatic fields (13) - (15) very quickly fall down from a bunch and are small inside a bunch. The numerical analysis of these fields is given in other paper [12].

And at last, components with the "L" index in expressions (3) - (4) describe plasma (Langmuir) wakefield with a frequency $\omega = \omega_p$ which is localized in the plasma channel. The electric field of a plasma wakefield is described by expressions:

$$E_z^L = -2Q_0k_p^2\cos[\omega_p(\tau - t_0)]\Theta(\tau - t_0) \times \begin{cases} \Delta_0(k_p a, k_p r_0)I_0(k_p r)/I_0(k_p a), & \text{if } r \le r_0 \\ I_0(k_p r_0)\Delta_0(k_p a, k_p r)/I_0(k_p a), & \text{if } r_0 < r \le a \end{cases}, \quad (21)$$

$$E_r^L = 2Q_0k_p^2\sin[\omega_p(\tau - t_0)]\Theta(\tau - t_0) \times \begin{cases} \Delta_0(k_p a, k_p r_0)I_1(k_p r)/I_0(k_p a), & \text{if } r < r_0 \\ -I_0(k_p r_0)\Delta_1(k_p a, k_p r)/I_0(k_p a), & \text{if } r_0 < r < a \end{cases}. \quad (22)$$

The field of Langmuir wave has no component of a magnetic field, unlike the wakefield of dielectric waveguide (6) - (8) where the magnetic field decreases transverse defocusing force by $[1-\beta_0^2\varepsilon_p(\omega_s)]^{-1}$ times. As the plasma wave period in the general case does not coincide with the period of a dielectric wakefield, the plasma wave gives possibility for independent focusing of accelerated particles. At that we assume that acceleration is provided by the wakefield of a dielectric wave.

Expressions (6) - (22) describe the electromagnetic field excited by a thin annular bunch. To obtain the fields excited by a bunch of the finite sizes, it is necessary to integrate these expressions over ring's arrival time into the waveguide t_0 and by ring's positions with the corresponding charge-density distribution function. Let's consider the simple, square profile of a charge density of a bunch in the longitudinal and transverse directions. Then resultant expressions look like:

$$E_z^d = -2E_0\sum_s R_s(r_b)e_z^s(r)\Psi_\parallel^s(\tau), \tag{23}$$

$$E_r^d = 2E_0\sum_s R_s(r_b)e_r^s(r)\Psi_\perp^s(\tau), \tag{24}$$

$$H_\varphi^d = 2E_0\sum_s R_s(r_b)h_\varphi^s(r)\Psi_\perp^s(\tau), \tag{25}$$

$$E_z^C = E_0\sum_s \bar{R}_s(r_b)\bar{e}_z^s(r)\Psi_\parallel^C(\tau), \tag{26}$$

$$E_r^C = E_0\sum_s \bar{R}_s(r_b)\bar{e}_r^s(r)\Psi_\perp^C(\tau), \tag{27}$$

$$H_\varphi^C = E_0\sum_s \bar{R}_s(r_b)\bar{h}_\varphi^s(r)\Psi_\perp^C(\tau), \tag{28}$$

$$E_z^L = -\frac{4Q_0}{r_bL_b}\Psi_\parallel^p(\tau)\begin{cases}\dfrac{1}{k_pr_b}-\dfrac{I_0(k_pr)}{I_0(k_pa)}\Delta_1(k_pr_b,k_pa), & r<r_b\\ \dfrac{I_0(k_pr_b)}{I_0(k_pa)}\Delta_0(k_pa,k_pr), & r_b<r<a\end{cases} \tag{29}$$

$$E_r^L = -\frac{4Q_0}{r_bL_b}\Psi_\perp^p(\tau)\begin{cases}\dfrac{I_1(k_pr)}{I_0(k_p\mathrm{a})}\Delta_1(k_pr_b,k_pa), & r<r_b\\ \dfrac{I_1(k_pr_b)}{I_0(k_p\mathrm{a})}\Delta_1(k_p\mathrm{r},k_p\mathrm{a}), & r_b<r<a\end{cases} \tag{30}$$

In expressions (23)-(30) functions $\Psi_\parallel$ and $\Psi_\perp$ describe longitudinal profile of excited fields:

$$\Psi_\parallel^{p,s}(\tau) = \frac{v_0}{\omega_{p,s}L_b}\Big[\sin(\omega_{p,s}\tau)\Theta(\tau) - \sin\omega_{p,s}(\tau-\tau_b)\Theta(\tau-\tau_b)\Big], \tag{31}$$

$$\Psi_\parallel^C(\tau) = \frac{v_0}{\bar{\omega}_sL_b}\big[\exp(-\bar{\omega}_s|\tau|)-\exp(-\bar{\omega}_s|\tau-\tau_b|)\big], \tag{32}$$

$$\Psi_\perp^{p,s}(\tau) = \frac{v_0}{\omega_{p,s}L_b}\Big[(1-\cos\omega_{p,s}\tau)\Theta(\tau) -(1-\cos\omega_{p,s}(\tau-\tau_b))\Theta(\tau-\tau_b)\Big], \tag{33}$$

$$\Psi_\perp^C(\tau) = \frac{v_0}{\bar{\omega}_sL_b}\big[\mathrm{sgn}(\tau)(1-\exp(-\bar{\omega}_s|\tau|)) -\mathrm{sgn}(\tau-\tau_b)(1-\exp(-\bar{\omega}_s|\tau-\tau_b|)\big], \tag{34}$$

$\tau_b = L_b/v_0$, L_b is a bunch length, and functions R_s and $\bar{R}_s$ define a transverse form-factor of a solid bunch of radius r_b:

$$R_s(r_b) = \frac{2}{\kappa_p^sr_b}\left(\frac{a}{\omega_sD'(\omega_s)}\right)^{1/2}\frac{I_1(\kappa_p^sr_b)}{I_0(\kappa_p^sa)}, \tag{35}$$

$$\bar{R}_s(r_b) = \frac{2}{\bar{\kappa}_p^sr_b}\left(\frac{a}{\bar{\omega}_s\bar{D}'(\bar{\omega}_s)}\right)^{1/2}\frac{J_1(\bar{\kappa}_p^sr_b)}{J_0(\bar{\kappa}_p^sa)}. \tag{36}$$

Let's now proceed to numerical investigations of the obtained expressions (23) - (30). First of all the possible rate of acceleration (the value of longitudinal force behind a leading bunch) and possibility of focusing (value of transverse force) of accelerated bunch is in the interest.

The numerical calculations in the gigahertz frequency range of excited wakefield waves have shown that it is rather easy to place an accelerated bunch in a focusing phase. In this case a focusing is provided by a field of the plasma wave (30) in which there is no component of a magnetic field, and acceleration is provided by a field of eigenmodes of a dielectric waveguide (23). One of such variants is given in the paper [11]: $a = 1.1\,\mathrm{cm}$, $b = 4.3$ cm, $\varepsilon_d = 2{,}1$, $n_p = 10^{10}\,\mathrm{cm}^{-3}$, energy of bunch electrons is 5 MeV, $Q_0 = 0.32\,\mathrm{nC}$, $L_b = 1.7\,\mathrm{cm}$, bunch cross-section area is $0.91\,\mathrm{cm}^{-2}$. The vacuum wavelength resonant with a bunch of the fundamental mode of dielectric structure is equal to ~ 11cm, and plasma wavelength is ~ 33 cm. Just essential difference in lengths of two types of waves provides possibility of focusing of an accelerated bunch at its corresponding placing behind a drive bunch.

The case investigated in paper [11] corresponds to an approximation of the linear plasma, as $n_b/n_p = 0.04$. As the field frequency increases, it becomes more and more difficult to fulfill the strong inequality $n_b/n_p \ll 1$. The reason consists in the unequal growth rate of density of electrons in a bunch n_b and plasma density n_p. Really, the frequency of dielectric mode grows in inverse proportion to thickness of a dielectric tube

$$\omega_s \sim 1/(b-a). \tag{37}$$

The electron bunch sizes must be less than a half of wavelength therefore the required bunch density at decreasing of the sizes of structure grows by the cubic law

$$n_b \sim 1/(b-a)^3. \tag{38}$$

At the same time the density of electrons of plasma changes in inverse proportion to a square of the size of the accelerating channel

$$n_p \sim 1/a^2. \tag{39}$$

The condition (39) follows from necessity to work near a maximum of amplitude of a transverse electric field of a plasma wave that is carried out under condition $\omega_pa \sim 1$ [8, 13].

If to refuse the requirement of fulfillment the strong inequality $n_b/n_p \ll 1$, it is possible to provide an focusing of an accelerated bunch also in more high-frequency range, than it is investigated in paper [12]. Longitudinal and transverse profiles of forces for one of such possible cases are shown in Fig. 1 and Fig. 2.

For the results of calculations presented in these figures the quartz tube $(\varepsilon_d = 3.75)$ with an outer radius b =0.6 mm and inner radius b =0.5 mm was used. The energy of bunch electrons was 5 GeV, the bunch charge was 3 nC, its length $L_b = 0.2$ mm, the bunch radius $r_b = 0.45$ mm. The sizes of a bunch and its charge give density of electrons in it $n_b = 1.47 \cdot 10^{14}$ cm^{-3}. Plasma density used in calculations $n_p = 4.41 \cdot 10^{14}$ cm^{-3}. In that way, $n_b / n_p = 1/3$. In the following section we will show that this dimensions is sufficient for validity of the results of the linear calculations given in Figs. 1-2. For the presented calculations the wavelength of the fundamental dielectric mode is equal to ~1 mm, and a plasma wavelength is 1.6 mm.

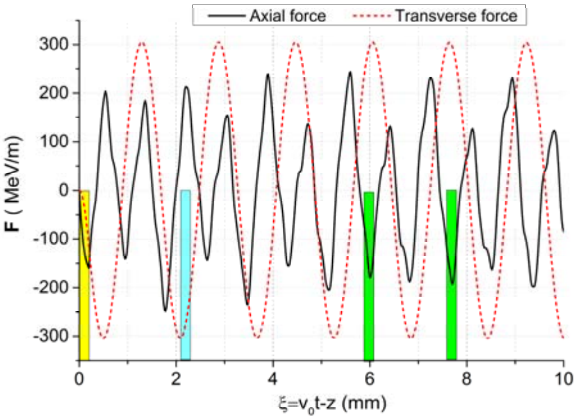

Fig. 1. Axial profile of the axial force (black line) and axial profile of transverse force (red line) at the distance r=0.45 mm from waveguide axis. Drive bunch (yellow rectangle) moves from right to left. Cyan rectangle shows possible location of electron witness bunch and green rectangles show possible location of positron witness bunch. Plasma density $n_p = 3n_b$

Fig. 1 shows axial dependence of the longitudinal force, acting on the test particle. It follows from the dependence, given in Fig. 1, that we can ensure acceleration of charged particles with their simultaneous radial focusing by placing the testing bunch at some distance from the drive bunch head. As it can be seen in the Figure, the radial force almost harmoniously depends on the axial coordinate with the period of approximately 0.16 cm, i.e. the Langmuir wave makes the greatest contribution into the radial force. At the same time, its contribution into the axial force, accelerating test particles, is predominantly small. The axial force is predominantly determined by the eigen modes of the dielectric waveguide; its complex behavior from the axial coordinate is caused by excitation several radial modes of the dielectric waveguide.

Fig. 2 shows the radial dependence of axial and transverse forces, acting on the test particle, placed in the first of the maximums of the accelerating field, at the distance of 2.3 mm behind the drive bunch head. The axial force changes insufficiently in the transport channel cross section, while the radial force remains focusing along all the channel section.

For the used in analytical calculations parameters of the dielectric waveguide, bunch and plasma, the focusing force amplitude is approx. 300 MeV/m, which equals the focusing magnetic field induction ~1T. The acceleration gradient, G > 200 MeV/m, is attractive, and also is the prospect of possibly stabilizing the motion of both drive and witness bunches, hitherto a difficulty for the single-channel DWA [14].

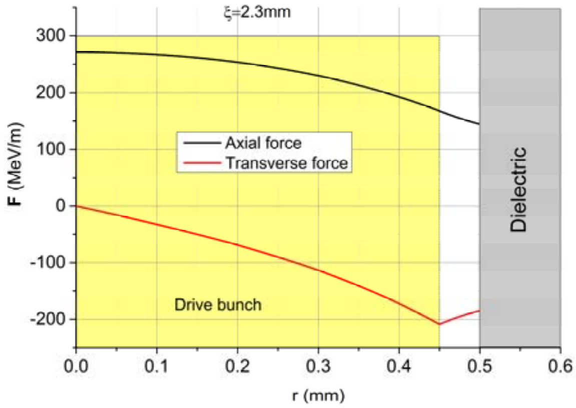

Fig.2. Transverse profile of the longitudinal force (black line) and transverse one (red line), acting on a witness electron, located at a distance of 2.3 mm ($\xi \equiv v_0\tau = v_0 t - z$) *from the head of the drive bunch*

It should be noted the undoubted advantage of the considered plasma-dielectric wakefield accelerator: it can be used both for acceleration of electron bunches, and for acceleration of positron bunches. From Fig. 1 it is seen that for this purpose accelerated electron and positron bunches must have different delay relative to drive bunch. Focusing will be provided for both bunches. This property of charge symmetry is topical one for perspective electron-positron colliders.

3. NUMERICAL SIMULATIONS

The analytical expressions and the calculations which have been carried out with using them and have been presented in previous section are true for the approximation of linear plasma $n_b \ll n_p$ (overdense regime). As pointed above, this strong inequality difficult to carry out in high frequency range of the excited wake fields. Therefore the terahertz example given of previous section which used for the calculation the simple inequality $n_b / n_p = 1/3 < 1$ requires further verification of the validity of the numerical results. With this object in view we have made full particle-in-cell (PIC) numeric simulation of wakefields excitation in the plasma dielectric structure under investigation. For calculations we used both our own PIC code and XOOPIC code realized for Linux [15]. The results of modeling, made with both codes, coincide well.

In Fig. 3 the longitudinal distributions of transverse and longitudinal forces acting on a test electron, calculated by XOOPIC code are presented. For numerical simulation the same parameters of structure and a bunch, as in the previous section were used. The input and output ends of the waveguide of length $l = 8$ mm was short-circuited by the conductive planes, i.e. the boundary condition $E_r |_{z=0,l} = 0$ was used. Electrons and the ions (hydrogen) of plasma striking the dielectric surface were moved away from calculation domain.

Comparison of the curves in Fig. 3 with corresponding curves in Fig. 1 confirms acceptable coincidence of the results of PIC modeling at self-consistent account of the plasma dynamics and analytical results.

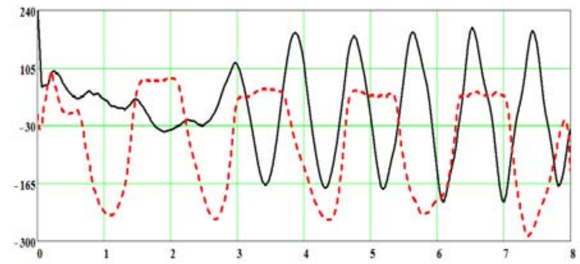

Fig. 3. Axial profile of the axial force (black line) and axial profile of transverse force (red line) at the distance r=0.45 mm from waveguide axis at time t=26.688 psec. Drive bunch moves from left to right, its head is located at z=8 mm. Distance (x-axis) is measured in mm, forces (y-axis) are in MeV/m

The accelerating and focusing fields coincide quite well. The greatest difference is observed in defocusing areas of the wake field. This difference can be explained for by the pushing out of plasma electrons to the dielectric surface. The remaining plasma ions reduce the defocusing field. This is confirmed by configuration space of plasma electrons (see Fig. 4) and configuration space of ions. Outer boundary of plasma electrons is deeply modulated with space period equal to plasma wavelength. At that time outer boundary of ions remains smooth. Outer boundary of electrons is deeply modulated with space period equal to plasma wavelength. At the same time outer boundary of ions remains smooth. Thus near the dielectric surface the periodic cavities, having total positive charge, arise. These positive charged cavities decrease defocusing force acting on test electrons. It should be noted that in this simulation the loss of plasma electrons was not very high, about ~8%.

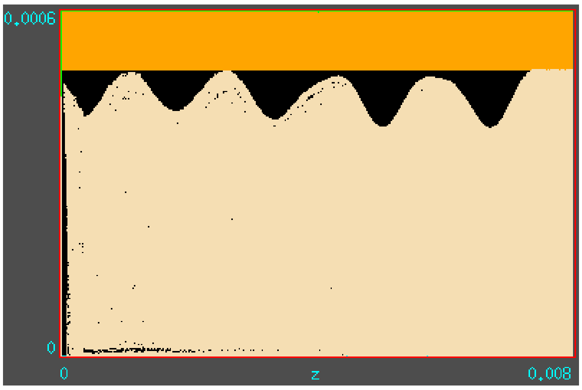

Fig. 4. Configuration space (z,r), of plasma electrons (yellow dots) at time t=26.688 psec. Dielectric tube is shown by orange color

Fig. 5 shows axial profiles of the accelerating and focusing forces created with the drive bunch of 1nC charge (ratio $n_b/n_p = 1/9 \ll 1$). As it followed from Fig. 4 we observe a more exact coincidence with the analytical calculations (taking into account the normalization for the bunch charge), given in Fig. 1.

Another, in comparison with Figs. 3, 4, extreme case $n_b \gg n_p$ (underdense or blowout regime [9, 10]) with a focusing provided by plasma ions, remaining in the transport channel after plasma electrons have been pushed out of it by the intense drive bunch.

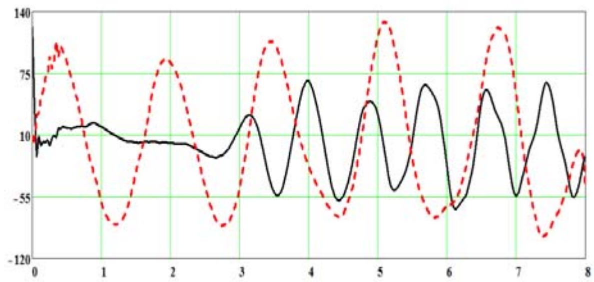

Fig. 5. The same as in Fig. 3 only for the drive bunch charge equal to 1nC

In Fig. 6 longitudinal profiles of longitudinal and transverse forces with a case $n_b/n_p = 30$ are shown. For these calculations the quartz tube with an outer radius $b = 0.5$ mm and inner radius $a = 0.2$ mm was used. Radius of an electron bunch was equal to 0.2 mm, rest parameters correspond to Figs. 1 - 4. Density of plasma used in calculations, given in Fig. 6, $n_p = 10^{14}$ cm^{-3}.

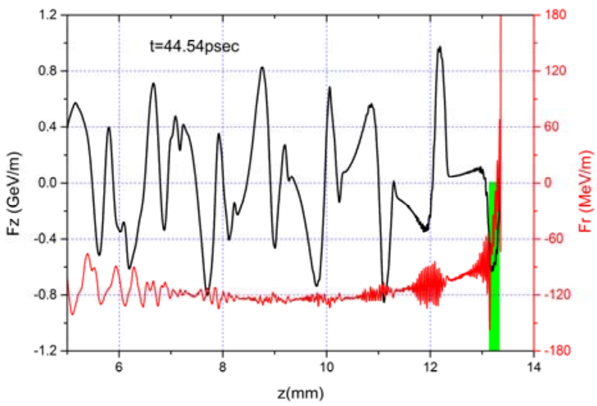

Fig. 6. Axial profile of the axial force (black line) and axial profile of transverse force (red line) in the case of blowout excitation regime of plasma-dielectric waveguide: n_b/n_p =30. Forces are recorded at the distance r=0.1 mm from waveguide axis at time t=44.54 psec. Drive bunch moves from left to right, its location is shown green rectangle

As follows from Fig. 6 the focusing force is almost axially homogeneous behind the drive bunch and is equal ~ 120 MeV/m. Longitudinal force weakly changed in comparison with a vacuum case, i.e. it, as expected, is formed by eigen waves of a dielectric tube. If to increase plasma density to the value corresponding to Figs. 1 - 4, it is possible to expect amplitude of the focusing force ~ 530 MeV/m, that corresponds to an induction of a focusing magnetic field ~1.8 T.

At last, we will consider transverse motion of the test electrons accelerated by a wakefield wave. In Fig. 7 are shown the positions of trailing electrons during moving under action of longitudinal and transverse forces presented in Figs. 1, 2. Initial longitudinal positions of test electrons are within the electron bunch shown in Fig. 1 $(z = 2.2 \pm 0.1\,\text{mm})$. In a transverse direction the test electrons are uniformly distributed from 0 to r_b.

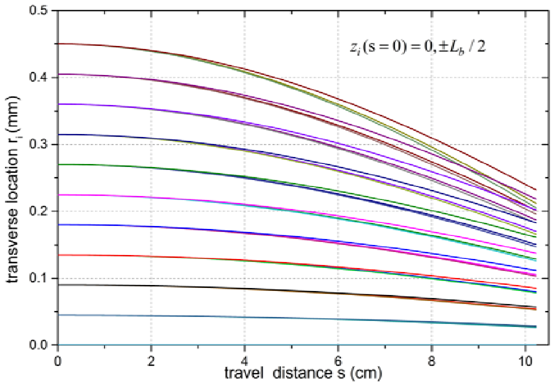

Fig. 7. Radii of witness bunch electrons during the acceleration by wakefield of drive bunch. Initially (at s=0) center of witness bunch is located at the distance of 2.2 cm behind of drive bunch head

From Fig. 7 follows that for the distance ~10 cm the accelerated electron bunch is focused almost twice. At that a spread of the transverse coordinates of particles in the head and in the tail of bunch is less 12%.

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ФОКУСИРОВКА ЭЛЕКТРОННЫХ И ПОЗИТРОННЫХ СГУСТКОВ В ПЛАЗМЕННО-ДИЭЛЕКТРИЧЕСКОМ КИЛЬВАТЕРНОМ УСКОРИТЕЛЕ

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Представлены результаты аналитических и численных исследований возбуждения кильватерных полей в плазменно-диэлектрической структуре. В линейном приближении (сверхплотная плазма) показано, что при определенной плотности плазмы суперпозиция плазменной и диэлектрической волн позволяет ускорять тестовый сгусток с его одновременной фокусировкой. Также мы выполнили моделирование методом "частица в ячейке" возбуждения кильватерных полей для случаев разреженной и сверхплотной плазмы.

ФОКУСУВАННЯ ЕЛЕКТРОННИХ І ПОЗИТРОННИХ ЗГУСТКІВ В ПЛАЗМОВО-ДІЕЛЕКТРИЧНОМУ КІЛЬВАТЕРНОМУ ПРИСКОРЮВАЧІ

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Представлені результати аналітичних досліджень і чисельного моделювання збудження кільватерних полів у плазмово-діелектричної структурі. У лінійному наближенні (надщільна плазма) показано, що при певній щільності плазми суперпозиція плазмової і діелектричної хвиль дозволяє прискорювати тестовий згусток з його одночасної фокусуванням. Також ми виконали моделювання методом "частинка в комірці" збудження кільватерних полів для випадків розрідженій і надщільної плазми.