ACCIDENTAL RESONATORS: THEORY AND EXPERIMENT

Y.P. Bliokh
Department of Physics, Technion, Haifa 32000, Israel
E-mail: bliokh@physics.technion.ac.il:

Disorder-induced resonances in randomly-layered samples are studied theoretically and experimentally. An algorithm for the detection and characterization of the effective cavities that give rise to these resonances has been developed. This algorithm enables to find the eigenfrequencies and pinpoint the locations of the resonant cavities that appear in individual realizations of random samples. The association of any resonance with corresponding accidentally formed cavity allows determination of parameters of a given individual randomly-inhomogeneous sample by external measurements. Experimental results confirm the proposed theoretical model.

PACS: 42.55.Ah, 42.25.Fx, 71.55.Jv

1. THEORETICAL MODEL

Propagation of waves in a randomly-inhomogeneous media is also random in character. Reflection and transmission coefficients, which are only accessible for measurement under remote sensing, are also random. As a rule, statistically averaged values of these quantities are considered as the medium characteristics. Such averaged characteristics are not able to say something about individual realization of the medium. Moreover, sometimes we deal with a media which exists in a single exemplar only. It will be shown below that this problem – extraction information about individual exemplar of random medium from external measurements – is not so hopeless as it may seem at first glance.

This report is a short overview of results which have been obtained in a close collaboration with my colleagues and published in [2 - 8].

The most startling phenomenon related to waves propagation in a random media is Anderson localization (see [1] and references therein). This phenomenon manifests itself in reflection of an electromagnetic wave from a randomly layered dielectric medium. The wave amplitude decreases exponentially inside such disordered, locally-transparent medium, which results in high (exponentially close to unity) reflectivity.

When the thickness of disordered medium is large as compared with the localization length, the transmission coefficient is exponentially small, and the reflection coefficient is exponentially close to unity. This is correct for statistically-averaged values. The frequency-dependent transmission through any individual sample contains, among regions where transmission is exponentially small, a set of isolated frequencies, where the sample is almost transparent (Fig. 1).

Position and amplitudes of these resonances are individual fingerprints of the sample. Namely these resonances can be used for revealing the medium characteristics and spatial distribution of the incident wave inside the individual sample.

The basis assumption of the theory is the following: any resonant transmission line is associated with an accidentally transparent region inside the sample which is surrounded by semitransparent (due to Anderson localization) sections. This forms resonator whose cavity is associated with transparent region, and walls are formed by semitransparent sections of the sample. The procedure which is able to distinguish cavities and typical parts of the sample, whose structure (the layers thicknesses, reflection and transmission coefficients) is known, is presented below.

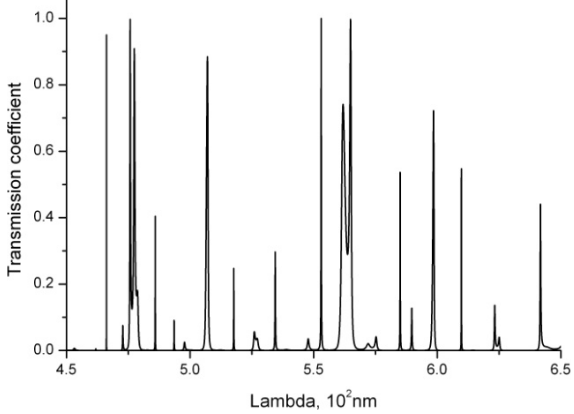

Fig. 1.

Let us consider the wave that propagates rightward from the point $z_j + 0$ (right side of the interface between the $(j-1)$-th and j-th layers) where its amplitude is $A_{j0}^{(+)}$ [thereafter superscripts $(\pm)$ denote amplitudes of the waves propagating to the right or to the left, accordingly]. This wave passes through the j-th layer, is partially reflected from the interface between the j-th and $(j+1)$-th layers, transmitted wave travels through $(j+1)$-th layer and partially reflected from the next interface, *etc*. Supposing, that reflection coefficients are small, $|r_j| \ll 1$, and, correspondingly, the absolute values of the transmission coefficients are close to unity, $|t_j| \approx 1$, the amplitude $\tilde{A}_j^{(-)}$ of the wave that was reflected from N_r layers and returned back in its starting point, is $\tilde{A}_j^{(-)} = r_j^{(+)}(k) A_{j0}^{(+)}$, where $r_j^{(+)}(k) = \sum_{m=1}^{N_r} r_{j+m} e^{2ik(z_{j+m}-z_j)}$ and k is the wave number. In the same fashion one can define the *left* reflection coefficient $r_j^{(-)}(k) = -\sum_{m=0}^{N_r} r_{j-m} e^{-2ik(z_{j-m}-z_j)}$. It is easy to see that $r_j^{(+)}(k)$ is -2k Fourier harmonics of the function $\sum_{m=1}^{N_r} r_{j+m}\delta[z-(z_{j+m}-z_j)]$, and $r_j^{(-)}(k)$ is +2k Fourier harmonics of the function

$\sum_{m=0}^{N_r}(-r_{j-m})\delta[z-(z_{j-m}-z_j)]$. In other words, $r_j^{(\pm)}(k)$ are the Bragg reflection coefficients from N_r layers: $r_j^{(+)}(k)$ determines reflection of the wave propagating to the right from the *j*-th layer, whereas $r_j^{(-)}(k)$ determines reflection of the wave that propagates to the left of this layer.

The wave that made a closed path and returned back after consequent reflections from N_r layers located on the right, and N_r+1 layers located on the left from the chosen *j*-th layer, has the amplitude $A_{j1}^{(+)}$ which is note equal to the initial amplitude $A_{j0}^{(+)}$. The difference between these amplitudes is defined by the function $\Delta_j(k) = r_j^{(+)}(k) r_j^{(-)}(k)$:

$$A_{j0}^{(+)} - A_{j1}^{(+)} = A_{j0}^{(+)}[1-\Delta_j(k)].$$

It seems reasonable for our purpose to set the number of the layers N_r equal to number $N_{\rm loc}$ of layers on the localization length $\ell_{\rm loc}$.

Now we will show that properties of the function $\Delta_j(k)$ depend on whether the *j*-th layer is placed either in a cavity or not. To do this let us calculate this function for two extreme cases: for a conventional resonator, and for a Bragg grating.

The conventional resonator is formed by two semi-transparent walls spaced by a distance *d*. Selecting an arbitrary point *z* between the walls as a starting point, one can calculate the difference between the amplitudes of the waves: $A_{z0}^{(+)}\left[1-\Delta_z(k)\right]$, where corresponding reflection coefficients are $r^{(+)}(k) = r_r e^{2ik(z_r - z)}$ and $r^{(-)}(k) = r_l e^{2ik(z-z_l)}$, and z_r and z_l are positions of the right and left resonator walls. For the resonator eigenmode the difference between the amplitudes is minimal, i.e., $\mathrm{Im}\Delta_z(k)=0$ and $\mathrm{Re}\Delta_z(k)>0$. Both these conditions can be written as $\arg\Delta_z(k) \equiv \arg r_r + \arg r_l + 2kd = 2\pi n$, that is a conventional definition of the resonator eigenfrequency. It is easy to see, that for eigenfrequency the resonator *Q*-factor is defined as follows: $1/Q = \left[1-\mathrm{Re}\Delta_z(k)\right]$.

Now let us calculate the function $\Delta_z(k)$ for a regular periodic sample (Bragg grating) assuming that all reflection coefficients are equal and the layer thicknesses *d* are equal too. When the Bragg reflection condition $kd=\pi$ is satisfied, then $\mathrm{Im}\Delta_z(k)=0$ and $\mathrm{Re}\Delta_z(k)<0$.

Thus, functions $\Delta(k)$ calculated for both conventional resonator and Bragg grating differ by the signs of their real parts: $\mathrm{Re}\Delta(k)>0$ for resonator, and $\mathrm{Re}\Delta(k)<0$ for Bragg grating. It seems reasonable to suppose that namely the sign of $\mathrm{Re}\Delta_j(k)$ distinguishes cavities from typical parts of the random layered medium. The wave number k_{res} of the cavity eigenmode can be defined as a wave number is defined as a root of equation $\mathrm{Im}\Delta_j(k)=0$. These statements are confirmed by numerical simulations in the following way.

If the peaks in the transmission spectrum are associated with the accidently formed cavity, the wave intensity maximum should be placed in regions inside the sample, where $\mathrm{Re}\Delta_j(k)>0$, and cannot be found in regions, where $\mathrm{Re}\Delta_j(k)<0$. Fig. 2 shows result of calculation of the function $\mathrm{Re}\Delta_j(k)$ along the sample (horizontal axis) for different wave numbers (vertical axis). Regions where $\mathrm{Re}\Delta_j(k)>0$ are marked by black, and regions where $\mathrm{Re}\Delta_j(k)<0$ are marked by gray. The wave intensity (if it is more than half of its maximum) is marked by light gray; regions, where $\mathrm{Im}\Delta_j(k)=0$, are shown by light-light gray lines. This picture confirms that the field intensity is concentrated in cavities, and the resonant frequency is determined by the condition $\mathrm{Im}\Delta_j(k_{\rm res})=0$.

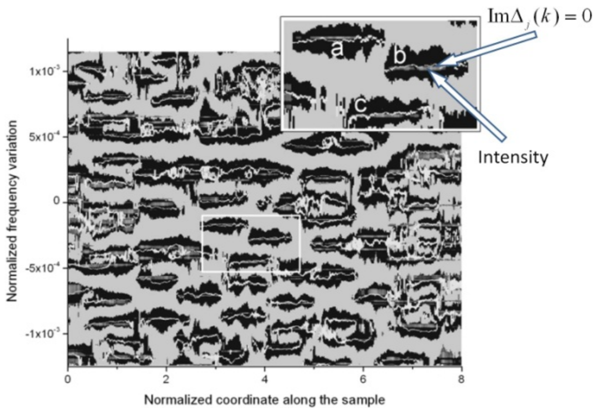

Fig. 2.

There is also a strong correlation (shown in Fig. 3.) between spatial distributions of $\mathrm{Re}\Delta_j(k_{\rm res})$ (light line) and the wave intensity $I_j(k_{res})$ (dark line).

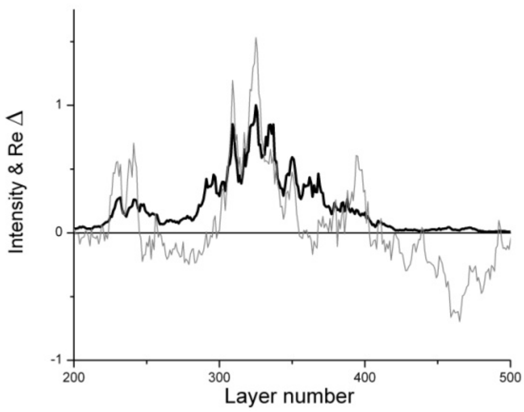

Fig. 3.

Thus, any resonant line in the transmission spectrum (resonant hole in the reflection spectrum) is associated with resonator. The resonator walls are formed by "typical" parts where the wave amplitude decreases exponentially due to Anderson localization. Therefore, the walls transmission coefficients depend on the position x_c of the cavity in the interior of the sample of the total length L:

$$T_1 \sim e^{-2x_c/l_{\text{loc}}},\ T_2 \sim e^{-2(L-x_c)/l_{\text{loc}}}. \quad (1)$$

The "typical" transmission and the reflection coefficients are

$$T_{\text{typ}} \sim e^{-2L/l_{\text{loc}}} \quad (2)$$

$$R_{\text{typ}} = 1 - T_{\text{typ}} - \Gamma \ell_{\text{loc}} \quad (3)$$

The resonant transmission and reflection coefficients, and the resonant line width, can be easily calculated using the analogy with a resonator:

$$T_{\text{res}} = \frac{4T_1T_2}{\left(\Gamma\ell_{\text{res}} + T_1 + T_2\right)^2}, \quad (4)$$

$$R_{\text{res}} = 1 - T_{\text{res}} - \Gamma\ell_{\text{res}}T_{\text{res}} / T_2, \quad (5)$$

$$\delta\omega_{\text{res}} = \frac{c}{4\pi\ell_{\text{res}}}\left(\Gamma\ell_{\text{res}} + T_1 + T_2\right), \quad (6)$$

where $\ell_{\text{res}} \sim 2l_{\text{loc}}$ is the cavity length. The parameter Γ characterizes the dissipation in the medium. The normalized intensity of the resonant wave inside the cavity is

$$I_{\text{res}} = T_{\text{res}} / T_2. \quad (7)$$

In principle, the values in the left-hand sides of Eqs. (2) - (6) can be found experimentally using external measurements only. These values allow determination of the cavity position, the localization length, and the wave field enhancement in the cavity. In practice, some measurements are difficult if not possible to realize. For example, sometimes only reflected wave can be registered. In this case certain characteristic of the medium and the wave intensity distribution can be revealed, and others remain unknown.

2. EXPERIMENTS

The model described above was confirmed experimentally. The experiments were carried out using the following setups:

1. single-mode waveguide randomly filled with dielectric plates (14…16 GHz);
2. the random stack of dielectric plates (75…110 GHz);
3. single-mode optical fiber with randomly-located Bragg gratings (1540 nm).

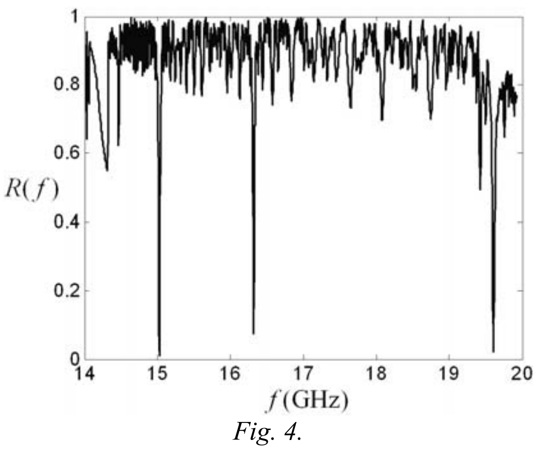

Fig. 4.

In the first set of experiments only reflected signal can be measured. Example of the reflected signal amplitude is shown in Fig. 4. The transmitted signal intensity was below the experimental noise.

The narrow slot along the waveguide allows the measurement of the intensity distribution along the sample (see Fig. 5.).

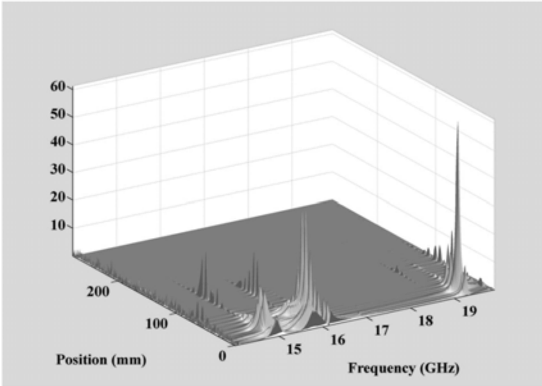

Fig. 5.

We calculated position of the intensity maxima inside the waveguide and the intensity enhancement factor, and compared these values with the experimentally measured ones. The results are presented in Tabl. 1.

Table 1

	x_{res}	$I_{\text{res}} / I_{\text{in}}$
Theory	4.0, 4.8, 3.6	190, 150, 123
Experiment	4.2, 5.5, 3.6	86, 201, 96

In the second experiment both reflected and transmitted signals were registered and analyzed. Because the position of the intensity maxima and the enhancement coefficient could not be measured, the calculated and actual loss factors of the medium were compared. Results are presented in Tabl. 2. The value of the tangent of the loss angle, averaged over 5 analyzed resonances, is $8.35 \cdot 10^{-4}$, whereas the actual value is $5.2 \cdot 10^{-4}$. It is important to note that for non-resonant frequencies the difference $1-(R+T)$ is too small to be measured.

Table 2

Frequency (GHz)	1-R	T	δf	$\tan\alpha \cdot 10^{-4}$
f_1=83.5	0.978	0.75	0.40	4.77
f_2=92.0	0.998	0.33	0.39	13.45
f_3=105.7	0.993	0.31	0.34	10.14
f_4=101.8	0.87	0.18	0.25	6.22
f_5=99.8	0.77	0.30	0.45	7.16

The aim of the third experiment was determination of the number of resonances in a given frequency interval as a function of number of scatterers (number of Bragg gratings). The analytical dependence is shown in Fig. 6 by dashed line, black points mark experimental results.

When the sample length is large enough, there is the chance that eigenfrequencies of two or more resonant cavities are close. These cavities interact with each over by their evanescent fields. Such chain of coupled cavities (resonators) forms so-called necklace state.

ISSN 1562-6016. ВАНТ. 2013. №4(86)

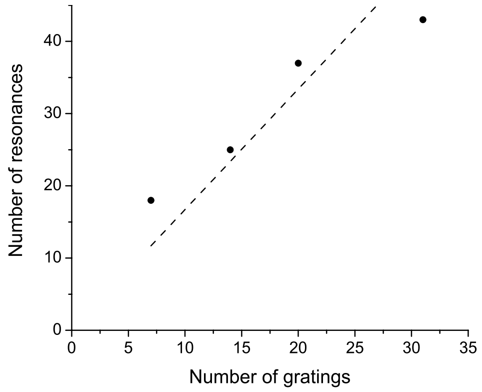

Fig. 6.

The experimental setup 1 allows investigation of necklace state formation. First of all, positions of resonances in the disordered stack of dielectric plates were found. Second, the air gap with tunable dimension was inserted into the stack. The position at which the air gap is introduced was chosen to correspond to the peak of a single Anderson localized mode of unperturbed random sample. This allowed us to tune the eigenfrequency of the selected mode. The spectral positions of the localized states as functions of the air gap thickness are plotted in Fig. 7.

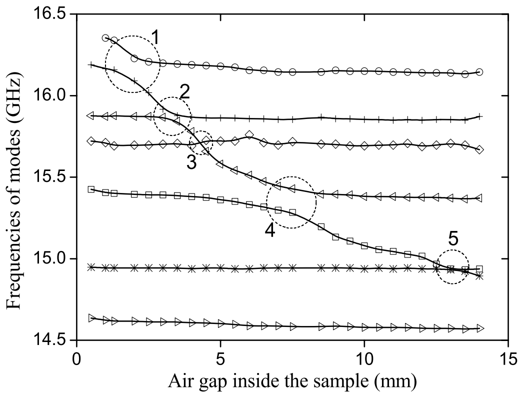

Fig. 7.

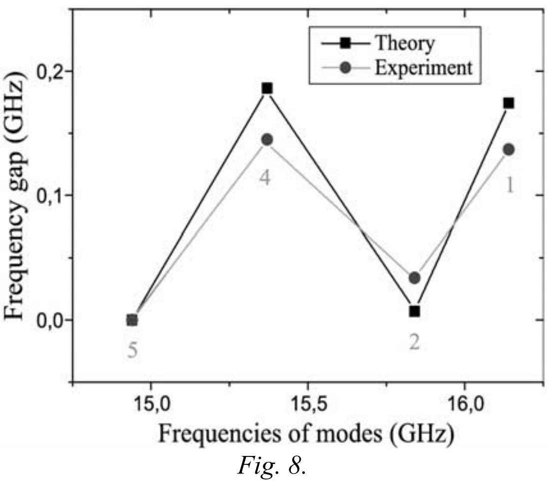

Fig. 8.

The frequencies of Anderson localized modes may cross or exhibit anticrossing (level repulsion). The resonator model allows prediction what will happened: either the level repulsion or crossing. The resonator parameters and the coupling coefficients were calculated using experimental data. Next, using these characteristics, we calculated the minimal frequency differences for interacting pairs of resonances, and compared them with experimentally measured ones. Result of this comparison is shown in Fig. 8.

Among experimental investigations, the numerical simulations were done in order to expand the model to nonlinear random media. The theory showed that measurements of the transmission spectra for only two different intensities of the incident wave is required for quantitative description of such properties of nonlinear disordered sample as bistability, hysteresis, and nonreciprocal transmission.

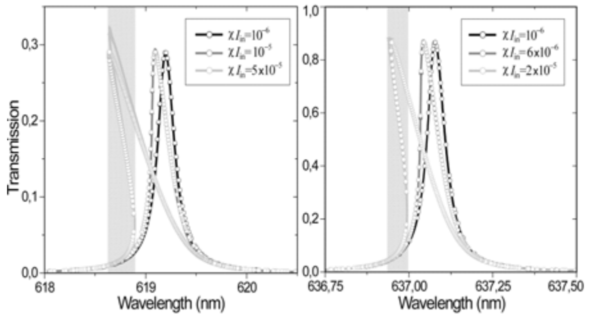

Fig. 9.

Fig. 9. demonstrates comparisons between results of numerical experiments (circles) and spectral characteristics which were calculated using two different intensities of the incident wave (solid lines). Left and right panels correspond to two resonances.

CONCLUSIONS

The non-typical, resonant peaks in the transmission spectrum, or deeps in the reflection spectrum from disordered one-dimensional layered structure, can be identified with resonator-like structures into the medium. This mapping allows adequate interpretation of external measurements and determination of the individual sample parameters. The comparisons between the calculated and experimentally measured characteristics of the samples and the wave intensity spatial distributions show that they are in a rather good agreement. One might think that the accuracy of determining the characteristics of media is not high enough. It is worth to remind here that we deal with media containing dozens of randomly arranged layers, and there are no any real resonators in these media. The developed model allows mapping the system which contains several dozen or even hundreds parameters onto the simple resonator. I want to emphasize that, in some cases, the definition of the medium parameters is possible only in the presence of both disorder and dissipation.

REFERENCES

1. *50 Years of Anderson Localization*, Ed E. Abrahams (World Scientific, Singapore 2010).
2. K.Y. Bliokh, Y.P. Bliokh, and V.D. Freilikher. Resonances in 1D disordered systems: localization of energy and resonant transmission // *J. Opt. Soc. of America B.* 2004, v. 21, p. 113-120.
3. K.Y. Bliokh, Y.P. Bliokh, V. Freilikher, A.Z. Genack, B. Hu, and P. Sebbah. Localized modes in open one-dimensional dissipative random systems // *Phys. Rev. Lett.* 2006, v. 97, p. 243904-(1-4).
4. J.A. Scales, L.D. Carr, D.B. McIntosh, V. Freilikher, and Y.P. Bliokh. Millimeter waves localization:

slow light and enhanced absorption // *Phys. Rev. B*. 2007, v. 76, p. 085118-(1-7).
5. K.Y. Bliokh, Y.P. Bliokh, V. Freilikher, A.Z. Genack, and P. Sebbah. Coupling and Level Repulsion in the Localized Regime: from Isolated to Quasi-Extended Modes // *Phys. Rev. Lett.* 2008, v. 101, p. 133901-(1-4).
6. K.Y. Bliokh, Y.P. Bliokh, V. Freilikher, S. Savel'ev, and F. Nori. Unusual Resonators: Plasmonics, Metamaterials, and Random Media // *Rev. Mod. Phys.* 2008, v. 80, p. 1201-1213.
7. I.V. Shadrivov, K.Y. Bliokh, Y.S. Kivshar, Y.P. Bliokh, and V. Freilikher. Bistability of Anderson localized states in nonlinear random media // *Phys. Rev. Lett.* 2010, v. 104*,* p. 123902-(1-4).
8. Y. Bliokh, E.I. Chaikina, N. Lizárraga, E.R. Méndez, V. Freilikher, and F. Nori. Disorder-induced cavities, resonances, and lasing in randomly-layered media // *Phys. Rev. B.* 2012, v. 86*,* p. 054204-(1-10).

Article received 17.05.2013.

СЛУЧАЙНЫЕ РЕЗОНАТОРЫ: ТЕОРИЯ И ЭКСПЕРИМЕНТ

Ю.П. Блиох

Резонансы слоистой случайной среды исследуются теоретически и экспериментально. Разработан алгоритм, позволяющий выявить области, образующие резонатор, ответственный за резонансное прохождение волны через случайно-неоднородный образец. Этот алгоритм позволяет определить резонансную частоту и характеристики случайно сформированного резонатора. Ассоциация каждого резонанса с соответствующим случайно сформированным резонатором позволяет определить параметры отдельного случайно-неоднородного образца с помощью внешних измерений. Предложенная теоретическая модель подтверждена экспериментально.

ВИПАДКОВІ РЕЗОНАТОРИ: ТЕОРІЯ І ЕКСПЕРИМЕНТ

Ю.П. Бліох

Резонанси шаруватого випадкового середовища досліджуються теоретично і експериментально. Розроблено алгоритм, що дозволяє виявити області, що утворюють резонатор, відповідальний за резонансне проходження хвилі через випадково-неоднорідний зразок. Цей алгоритм дозволяє визначити резонансну частоту і характеристики випадково сформованого резонатора. Асоціація кожного резонансу з відповідним випадково сформованим резонатором дозволяє визначити параметри окремого випадково-неоднорідного зразка за допомогою зовнішніх вимірювань. Запропонована теоретична модель підтверджена експериментально.