PROPAGATION OF INTENSE CHARGED-PARTICLE BEAM IN COAXIAL MAGNETIC UNDULATOR

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The system of dynamics equations describing the propagation of high-current charged-particle beam in a coaxial drift-tube in combined longitudinal homogeneous (guide) and periodic magnetostatic undulator fields is formulated. The self-electric field and both longitudinal and azimuthal components of self-magnetic field of charged-particle beam averaged over the undulator period are obtained.

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INTRODUCTION

The estimates of limiting current of charged-particle beam propagating in the conducting grounded drift-tube in the longitudinal homogeneous (guide) strong (infinite) or finite magnetic fields are among the most studied in physical electronics (cf. [1 - 3]). In papers [4, 5] the authors discuss the limiting current of electron beam in the magnetostatic pump field of the hybrid coaxial free electron laser/maser (FEL/FEM). The authors of papers [3, 6, 7] estimated the limiting current of electron beam in the strong (infinite) longitudinal homogeneous magnetic field. The estimates made for the coaxial undulator [8] and values experimentally achieved in the Strathclyde hybrid coaxial FEL [9] exhibit propagating currents smaller by an order of magnitude than those in the longitudinal homogeneous magnetic field. The statement of problem is in Section 1. In Section 2 applying ideas of works [10 - 12] we obtain the self-electric and self-magnetic fields of charged-particle beam in a coaxial drift-tube including the longitudinal component of the self-magnetic field. In Section 3 we apply results of Section 2 to modelling electron beam propagation in hybrid coaxial FEL/FEM.

1. PROBLEM SETUP

The system of equations and initial conditions describing the dynamics of charged-particles beam under the assumption of laminar flow and circular cylindrical symmetry has the form:

$$\begin{aligned}
&\frac{dp_r}{dt}-\frac{p_\theta^2}{m_q r\gamma}=q\overline{E}_r^{self}+\frac{q}{m_q c\gamma}\left[p_\theta(\overline{B}_z^{self}+B_z^{ext}+B_{z0}^{ext})-p_z\overline{B}_\theta^{self}\right],\\
&\frac{dp_\theta}{dt}+\frac{p_r p_\theta}{m_q r\gamma}=-\frac{q}{m_q c\gamma}\left[p_r(\overline{B}_z^{self}+B_z^{ext}+B_{z0}^{ext})-p_z B_r^{ext}\right],\\
&\frac{dp_z}{dt}=\frac{q}{m_q c\gamma}\left[p_r\overline{B}_\theta^{self}-p_\theta B_r^{ext}\right],\qquad \frac{dr}{dt}=\frac{p_r}{\gamma},\qquad \frac{dz}{dt}=\frac{p_z}{\gamma},
\end{aligned}\tag{1}$$

$$\begin{aligned}
&p_r(0)=0,\qquad p_\theta(0)=0,\\
&p_z(0)=p_{z0},\quad r(0)=r_0,\qquad z(0)=z_0,
\end{aligned}\tag{2}$$

where $p_r(t)$, $p_\theta(t)$, $p_z(t)$ are r-, θ-, z- the components of relativistic momentum; $\gamma=\sqrt{1+p_r^2+p_\theta^2+p_z^2}$ is the relativistic factor; r and θ are radial and azimuthal coordinates; q and m_q are the charge and mass of particles (for electrons $q=-|e|$, e is the electron charge, $m_q=m_e$); c is the light velocity in vacuum. Here $\overline{E}_r^{self}(r)=-\partial\overline{\varphi}^{self}(r)/\partial r$ is the radial component of the self electric field ($\overline{\varphi}^{self}(r)$ is the scalar potential); $\overline{B}_\theta^{self}(r)$, $\overline{B}_z^{self}(r)$ are the θ-, z- components of the self-magnetic field; $\vec{B}_0^{ext}=(0,0,B_{z0}^{ext})$ is the external homogeneous static magnetic field produced by a solenoid. $\vec{B}_\perp^{ext}(r,z)=(B_r^{ext}(r,z),0,B_z^{ext}(r,z))$ is the undulator magnetostatic field produced by a system of permanent magnets [8, 13, 14]

$$\begin{aligned}
&B_r^{ext}(r,z)=-B_\perp^m\sum_{k=0}^{\infty}C_{2k+1}\times\\
&\times\sin((2k+1)(k_w z-\frac{\pi}{4}))F_{2k+1}^{(1)}((2k+1)k_w r),
\end{aligned}\tag{3}$$

$$\begin{aligned}
&B_z^{ext}(r,z)=-B_\perp^m\sum_{k=0}^{\infty}C_{2k+1}\times\\
&\times\cos((2k+1)(k_w z-\frac{\pi}{4}))F_{2k+1}^{(0)}((2k+1)k_w r),
\end{aligned}\tag{4}$$

where $B_\perp^m$ is the value of longitudinal component of the magnetic induction on cylindrical surfaces of the permanent magnets of the undulator (for simplicity we suppose that these surfaces are located at $r=r_1$ and $r=r_2$);

$$C_{2k+1}=\frac{4}{(2k+1)\pi}\sin\left[\frac{(2k+1)\pi}{4}\right];$$

$k_w=2\pi/l_w$ (l_w is the spatial period of undulator);

$$F^{(0)}_{2k+1}((2k+1)k_w r) = f_{2k+1} I_0((2k+1)k_w r) - g_{2k+1} K_0((2k+1)k_w r),$$

$$F^{(1)}_{2k+1}((2k+1)k_w r) = f_{2k+1} I_1((2k+1)k_w r) + g_{2k+1} K_1((2k+1)k_w r),$$

$$f_{2k+1} = \frac{K_0((2k+1)k_w r_1) + K_0((2k+1)k_w r_2)}{\Delta_{2k+1}}, \quad g_{2k+1} = \frac{I_0((2k+1)k_w r_1) + I_0((2k+1)k_w r_2)}{\Delta_{2k+1}},$$

$$\Delta_{2k+1} = I_0((2k+1)k_w r_1) K_0((2k+1)k_w r_2) - I_0((2k+1)k_w r_2) K_0((2k+1)k_w r_1),$$

$I_0(\cdot)$, $I_1(\cdot)$ are Bessel functions of 0- and 1-order; $K_0(\cdot)$, $K_1(\cdot)$ are modified Bessel functions of 0- and 1-order; z_0 is the longitudinal position of the injection plane; p_{z0} is the initial longitudinal momentum; r_0 is the initial radial position of a current beamlet (beam layer).

2. SELF-FIELDS OF CHARGED-PARTICLE BEAM

In the assumption of laminar flow and the average over the undulator period the components of the self-electric and self-magnetic fields of charged particle beam have form

$$\bar{e}_r^{self}(\rho) \equiv \frac{q r_2}{m_q c^2} \bar{E}_r^{self}(r) = -\frac{\partial \bar{f}^{self}(\rho)}{\partial \rho}, \tag{5}$$

$$\bar{b}_\theta^{self}(\rho) \equiv \frac{q r_2}{m_q c^2} \bar{B}_\theta^{self}(r) = -\frac{\partial \bar{f}^{self}(\rho)}{\partial \rho} \bar{\beta}_z(\rho) + \frac{1}{\rho} \int_{\rho_i}^{\rho} \rho' \frac{\partial \bar{f}^{self}(\rho')}{\partial \rho'} \frac{\partial \bar{\beta}_z(\rho')}{\partial \rho'} d\rho' +$$
$$+ \frac{1}{\rho \ln \rho_1} \int_{\rho_i}^{\rho_o} \bar{f}^{self}(\rho') \frac{\partial \bar{\beta}_z(\rho')}{\partial \rho'} d\rho' + \frac{1}{\rho \ln \rho_1} \int_{\rho_i}^{\rho_o} \frac{d\rho'}{\rho'} \int_{\rho_i}^{\rho'} \rho'' \frac{\partial \bar{f}^{self}(\rho'')}{\partial \rho''} \frac{\partial \bar{\beta}_z(\rho'')}{\partial \rho''} d\rho'' +$$
$$- \frac{\ln \rho_o}{\rho \ln \rho_1} \int_{\rho_i}^{\rho_o} \rho' \frac{\partial \bar{f}^{self}(\rho')}{\partial \rho'} \frac{\partial \bar{\beta}_z(\rho')}{\partial \rho'} d\rho', \tag{6}$$

$$\bar{b}_z^{self}(\rho) \equiv \frac{q r_2}{m_q c^2} \bar{B}_z^{self}(r) = \frac{1-\rho_o^2}{1-\rho_1^2} \frac{\partial \bar{f}^{self}(\rho_o)}{\partial \rho} \bar{\beta}_\theta(\rho_o) + \frac{\rho_i^2 - \rho_1^2}{1-\rho_1^2} \frac{\partial \bar{f}^{self}(\rho_i)}{\partial \rho} \bar{\beta}_\theta(\rho_i) - \frac{\partial \bar{f}^{self}(\rho)}{\partial \rho} \bar{\beta}_\theta(\rho) +$$
$$- \int_{\rho}^{\rho_o} \rho' \frac{\partial \bar{f}^{self}(\rho')}{\partial \rho'} \frac{\partial}{\partial \rho'} \left(\frac{\bar{\beta}_\theta(\rho')}{\rho'} \right) d\rho' + \frac{\rho_i^2 - \rho_1^2}{1-\rho_1^2} \int_{\rho_i}^{\rho_o} \rho' \frac{\partial \bar{f}^{self}(\rho')}{\partial \rho'} \frac{\partial}{\partial \rho'} \left(\frac{\bar{\beta}_\theta(\rho')}{\rho'} \right) d\rho' + \tag{7}$$
$$+ \frac{2}{1-\rho_1^2} \int_{\rho_i}^{\rho_o} \rho' \frac{\partial \bar{f}^{self}(\rho')}{\partial \rho'} \bar{\beta}_\theta(\rho') d\rho' + \frac{2}{1-\rho_1^2} \int_{\rho_i}^{\rho_o} d\rho' \rho' \int_{\rho_i}^{\rho'} \rho'' \frac{\partial \bar{f}^{self}(\rho'')}{\partial \rho''} \frac{\partial}{\partial \rho''} \left(\frac{\bar{\beta}_\theta(\rho'')}{\rho''} \right) d\rho'',$$

where $\bar{e}_r^{self}$, $\bar{b}_\theta^{self}$, $\bar{b}_z^{self}$ are the radial component of the dimensionless self-electric and the θ- and z-components of the dimensionless self-magnetic fields of beam, respectively; $\bar{f}^{self}(\rho) = q\bar{\varphi}^{self}(r)/(m_q c^2)$ is the dimensionless scalar potential; $\rho = r/r_2$ is the dimensionless radial coordinate, $\rho_1 = r_1/r_2$ is the dimensionless inner radius of the drift-tube, $\rho_i = r_i/r_2$, $\rho_o = r_o/r_2$ are the dimensionless inner and outer radii of the beam boundaries; $\bar{\beta}_\theta(\rho) = \bar{\pi}_\theta(\rho)/\bar{\gamma}(\rho)$ $\bar{\beta}_z(\rho) = \bar{\pi}_z(\rho)/\bar{\gamma}(\rho)$ are the dimensionless θ-, z-components of velocity ($\bar{\pi}_\theta(\rho) = \bar{p}_\theta(\rho)/(m_q c)$, $\bar{\pi}_z(\rho) = \bar{p}_z(\rho)/(m_q c)$ are the dimensionless θ-, z-components of momentum). For thin beams the assumptions of independence of beam density on radial coordinate and of a "rigid rotator" ($\omega_b \equiv \bar{\upsilon}_\theta(r)/r = const$, ω_b are the angular velocity) are natural. In this case all integration in expressions (5) - (7) disappear and the system of equations becomes system of just differential equations so it is not difficult to find its numerical solutions (compare with papers [8, 13]). Further we shall consider this important case.

3. ENVELOPE APPROXIMATION

Following [8], we suppose that the averaged over the period of undulator focusing force compensates the defocusing by the self-electric and self-magnetic fields of charged-particle beam and take into account only the first (leading) harmonic of the undulator magnetostatic field (see expressions (3), (4)). Assuming that the above conditions are realized for the envelopes of the beam boundary (see [15, p. 197]), we require that the relations following from (1) and (5) - (7) hold:

$$\frac{(b_\perp^m)^2 C_1^2 F_1^{(1)}(\kappa_w\rho_{i,eq})}{2\kappa_w}\left\{\frac{F_1^{(1)}(\kappa_w\rho_{i,eq})}{\kappa_w\rho_{i,eq}}\left[1+\frac{i_0(2\rho_{i,eq}^2-\rho_1^2-1)}{(1-\rho_1^2)\pi_{z0}}\right]-F_1^{(0)}(\kappa_w\rho_{i,eq})\right\}+$$
$$+b_{z0}^{ext}\frac{b_\perp^m C_1 F_1^{(1)}(\kappa_w\rho_{i,eq})}{\kappa_w\sqrt{2}}=-\frac{i_0 G}{\pi_{z0}\rho_{i,eq}\ln\rho_1},$$
$$\frac{(b_\perp^m)^2 C_1^2 F_1^{(1)}(\kappa_w\rho_{o,eq})}{2\kappa_w}\left\{\frac{F_1^{(1)}(\kappa_w\rho_{o,eq})}{\kappa_w\rho_{o,eq}}\left[1+\frac{i_0(2\rho_{o,eq}^2-\rho_1^2-1)}{(1-\rho_1^2)\pi_{z0}}\right]-F_1^{(0)}(\kappa_w\rho_{o,eq})\right\}+$$
$$+b_{z0}^{ext}\frac{b_\perp^m C_1 F_1^{(1)}(\kappa_w\rho_{o,eq})}{\kappa_w\sqrt{2}}=-\frac{i_0[G+2\ln\rho_1]}{\pi_{z0}\rho_{o,eq}\ln\rho_1}, \quad (8)$$

where $i_0 = I_0/I_A$ is the dimensionless beam current ($I_A = -17.05$ kA for electrons); $I_0 = qn_0\upsilon_{z0}\pi(r_o^2 - r_i^2)$ is the beam current; n_0 is the beam density in assumption that it does not depend on the radial coordinate; $\rho_{i,eq}$, $\rho_{o,eq}$ are the dimensionless equilibrium inner and outer beam radii; $\kappa_w = k_w r_2$; $\pi_{z0} = p_{z0}/(m_q c)$ is the dimensionless initial longitudinal momentum; $b_{z0}^{ext} = qr_2B_{z0}^{ext}/(m_qc^2)$, $b_\perp^m = qr_2B_\perp^m/(m_qc^2)$ are the dimensionless induction of longitudinal homogenous magnetic field and the longitudinal component of magnetic induction on cylindrical surface of permanent magnets, correspondently. In the derivation of the equilibrium equations (8) we use the following estimate for the average of the equilibrium θ-component of momentum over the undulator period

$$\overline{\pi}_\theta \approx \sqrt{\overline{\pi_\theta^2}} \quad (9)$$

in such a way accounting for the influence of longitudinal homogenous magnetic field. Alternatively, we could have used another estimate for the average of the equilibrium θ-component of momentum over the undulator period

$$\overline{\pi}_\theta = 0. \quad (10)$$

In this case, in the equilibrium system (8) the last summands on the left hand sides would vanish, so the solutions to the system would not depend on the dimensionless magnetic induction of the longitudinal homogenous magnetic field (see Table).

I_0, kA	$\rho_{i,eq}$ Strath.	$\rho_{o,eq}$ Strath.	$\rho_{i,eq}$ papers [13, 14]	$\rho_{o,eq}$ papers [13, 14]
0.5	0.890	0.905		
1.0			0.752	0.767
3.0	0.864	0.951	0.740	0.783
5.0	0.848	0.981	0.728	0.800
10.0			0.700	0.831

Dependence of the equilibrium inner and outer radii of the beam on the beam current under the assumption (10)

In Fig. 1 the dependence of the relativistic electron beam equilibrium radii on the value of longitudinal homogeneous magnetic field is shown. We see that for all values of the longitudinal homogeneous magnetic field with the increase of the beam current the inner equilibrium radius decreases and the outer equilibrium radius increases. Also, one can see that with the increase of the longitudinal homogeneous magnetic field the beam thickness decreases if the beam current is fixed. It is obvious that for the coaxial geometry and tube parameters of the Strathclyde FEL/FEM the zone of admissible values of guide magnetic field is much narrower (see Fig. 1,a).

Fig. 2,a,b shows the dependence of the inner and outer radii of the beam on the longitudinal coordinate obtained by numerical solution of the system of equations (1) with the initial conditions (2). The initial inner and outer radii of the electron beam are found from the equilibrium equations (8); such choice of initial conditions allows us to ensure minimal possible oscillations of outer and inner boundaries of the beam (see [13]).

In Fig. 3,a,b for different injection points along the radial coordinate we present the dependence of the radii of the respective beamlets (beam layers) of relativistic electron beam on the longitudinal coordinate in the absence of guide magnetic field obtained by numerical solutions of system (1) with initial conditions (2). One can see quite rapid destruction of the relativistic electron beam laminar flow (compare with [16, p. 205]).

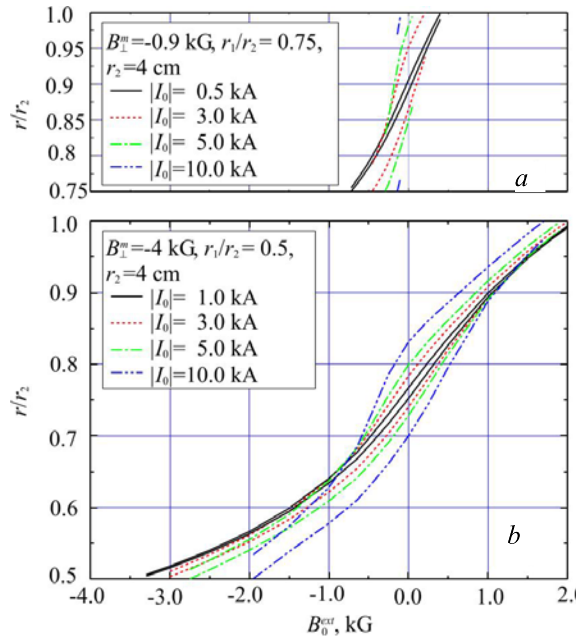

Fig. 1. Dependence of the equilibrium radii on the induction of longitudinal homogenous magnetic field for different values of beam injection current: a) Strathclyde FEL/FEM [4, 9] and b) ubitron considered in [13, 14]

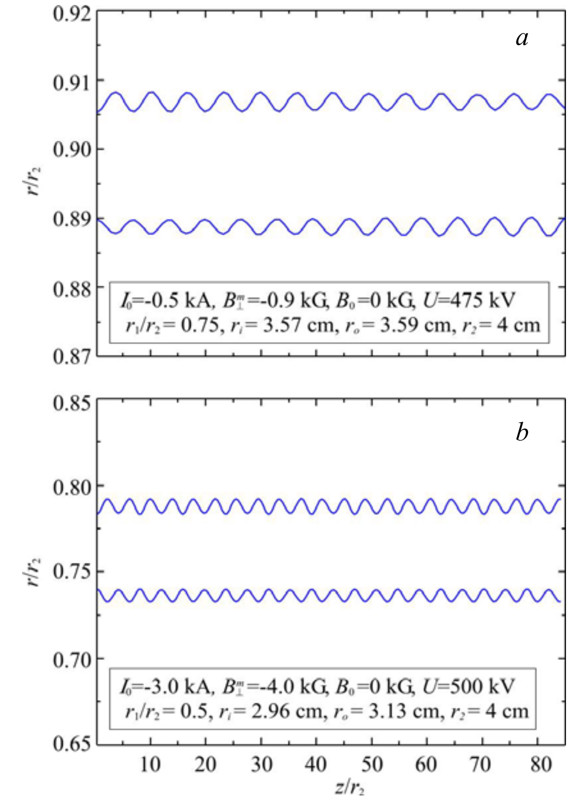

Fig. 2. Dependence of the inner and outer radii of the beam on the longitudinal coordinate in the absence of guide magnetic field: a) Strathclyde FEL/FEM [4, 9] and b) ubitron considered in [13, 14]

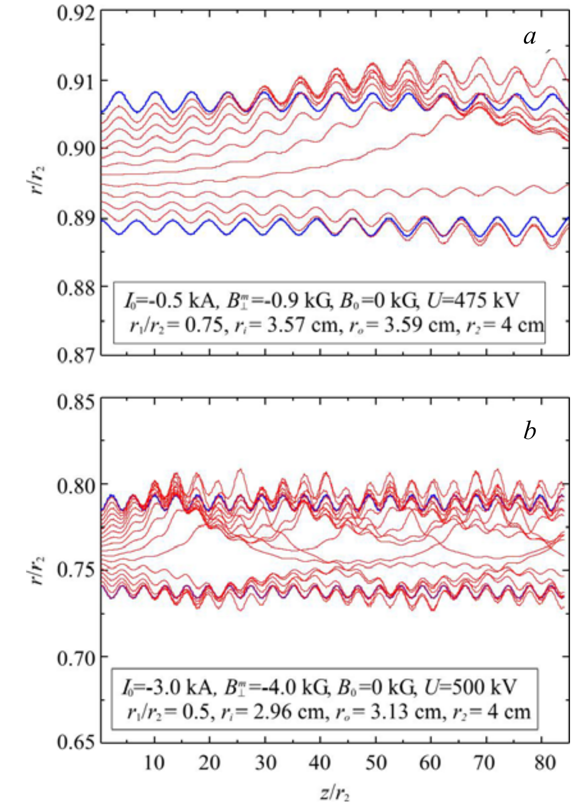

Fig. 3. Dependence of the radii of various beamlets (beam layers) on the longitudinal coordinate in the absence of guide magnetic field. Initially (in the injection plane) inner and outer beamlets are shown in blue; initially internal beamlets are shown in red

CONCLUSIONS

The steady propagation of high-current electron beam in a coaxial drift tube is studied in the approximation of continuous medium in the combined longitudinal homogeneous (guide) magnetic and periodic magnetostatic undulator fields. The closed system of dynamical equations, which describes propagation of charged-particle beam in coaxial drift-tube without an assumption of constant beam density and/or "rigid rotor" type of rotation, is derived. Expressions for the averaged over the period of the undulator self-electric and self-magnetic fields of the charged-particle beam in a coaxial drift-tube including the longitudinal component of the self-magnetic field are obtained.

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ПРОХОЖДЕНИЕ ИНТЕНСИВНОГО ПУЧКА ЗАРЯЖЕННЫХ ЧАСТИЦ В КОАКСИАЛЬНОМ МАГНИТНОМ ОНДУЛЯТОРЕ

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Сформулирована замкнутая система динамических уравнений, описывающая транспортировку пучка заряженных частиц в коаксиальной камере дрейфа в комбинированных конечном продольном однородном (ведущем) и периодическом ондуляторном магнитостатических полях. Получены выражения для усредненных по периоду ондулятора собственных электрических и магнитных полей пучка заряженных частиц.

ПРОХОДЖЕННЯ ІНТЕНСИВНОГО ПУЧКА ЗАРЯДЖЕНИХ ЧАСТИНОК У КОАКСІАЛЬНОМУ МАГНІТНОМУ ОНДУЛЯТОРІ

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Сформульовано замкнену систему динамічних рівнянь, що описує транспортування пучка заряджених частинок у коаксіальній камері дрейфу в комбінованих скінченому поздовжньому однорідному (ведучому) та періодичному ондуляторному магнітостатичних полях. Отримано вирази для усереднених за періодом ондулятора власних електричних і магнітних полів пучка заряджених частинок у коаксіальній камері дрейфу.